Conditional Probability and Independence

Roll 2 dice, and let X = the # on the first die
Y = the # on the second die

The joint probability distribution $P(X,Y)$ gives the prob. of each combination of values of X and Y.

and

In the case of two dice, $P(X)$ doesn't depend on $P(Y)$, so we have independence, which implies $\boxed{P(X,Y) = P(X)P(Y)}$

(!)

Let's roll two 3-sided dice to see why:

X	Y
1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$P(X=1) = \frac{1}{3}$

"and"

$P(X=1, Y=3) = \frac{1}{9}$

$P(Y=2 \mid X=3) = \frac{1}{3}$

given that (conditioned on)

This is a conditional probability.

Equivalent def. of independence: $P(Y=1 \mid X=1) = P(Y=1)$

↗ $P(Y)$ doesn't depend on X

Ex. 3-6

$\to H: \frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

$\to T: 0.6, 0.2, 0.2$

C	D	P(C, D)
H	1	1/6
H	2	1/6
H	3	1/6
T	1	3/10
T	2	1/10
T	3	1/10

given

4. $P(D=1 \mid C=T)$

$\frac{3}{16}$ $\frac{3}{5}$

$P(D=1, C=T)$

$P(D=1) = \frac{1}{6} + \frac{3}{10}$