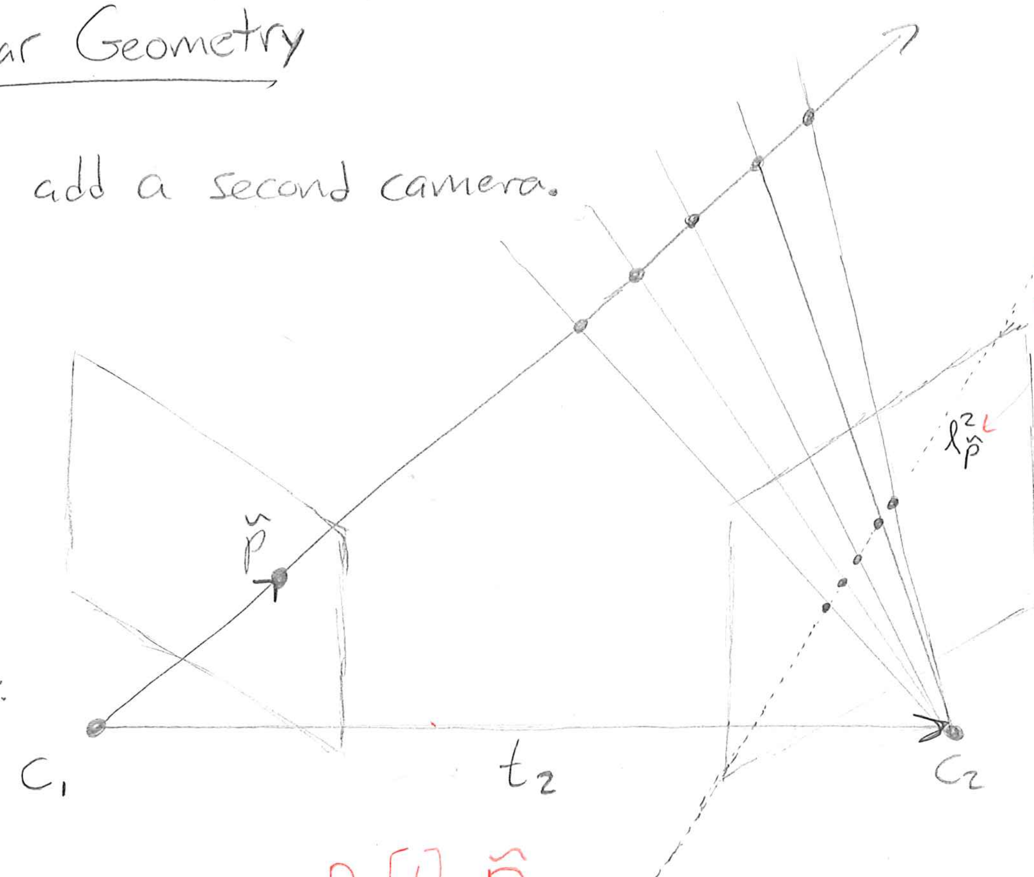
Epipolar Geometry

Let's add a second camera.



For now, assume:

$K_1 = I_{3\times3}$

$R_1 = I_{3\times3}$

$t_1 = \vec{0}$

For now, assume:

$K_2 = I_{3\times3}$

R_2, t_2 known

$R_2 [t]_\times \tilde{p}$

For a given (homogeneous) point $\tilde{p}$ in image 1, where might it appear in image 2?

- The 3D point that projected to $\tilde{p}$ could be anywhere along the ray from C_1 in the direction of $\tilde{p}$.
- Those points all project into image 2 along a line! What's the equation (homog. coordinates) of that line?

The 3D plane projecting to that line is spanned by $\tilde{p}$ and t_2, so its coordinates are $t_2 \times p$.

However, those coordinates are wrt image 1! What does camera 2 see?

Observation: the plane (line) $t_2 \times \tilde{p}$ goes through C_2's CoP, so all we need to see the line from C_2's view is to rotate it (or equivalently, the camera): $\ell^2_{\tilde{p}} = R_2(t_2 \times \tilde{p}) = R_2 [t]_\times \tilde{p}$

We have $\ell^2_{\hat{p}} = R_2 [t]_\times \hat{p}$. (7)

The line in image 2 where $\hat{p}$'s 3D point must project onto.

Rotation of C_2

Cross-product matrix for translation of camera 2 WRT C_1

A point in image 1

Any point $\hat{q}$ in image 2 that lies on this line satisfies:

$$\hat{q}^T \ell^2_{\hat{p}} = 0$$

or: $$\underbrace{\hat{q}^T R_2 [t]_\times}_{} \hat{p} = 0$$

E = the essential matrix

If $K_1 = K_2 = I_{3\times3}$, then for any pixel $\hat{p}$ in image 1, the corresponding point $\hat{q}$ in image 2 satisfies

$$\hat{q}^T E \hat{p} = 0$$

What if K_1 and K_2 are not $I_{3\times3}$, but are known?

Pixel p in image 1: $\hat{p} = K_1^{-1} p$

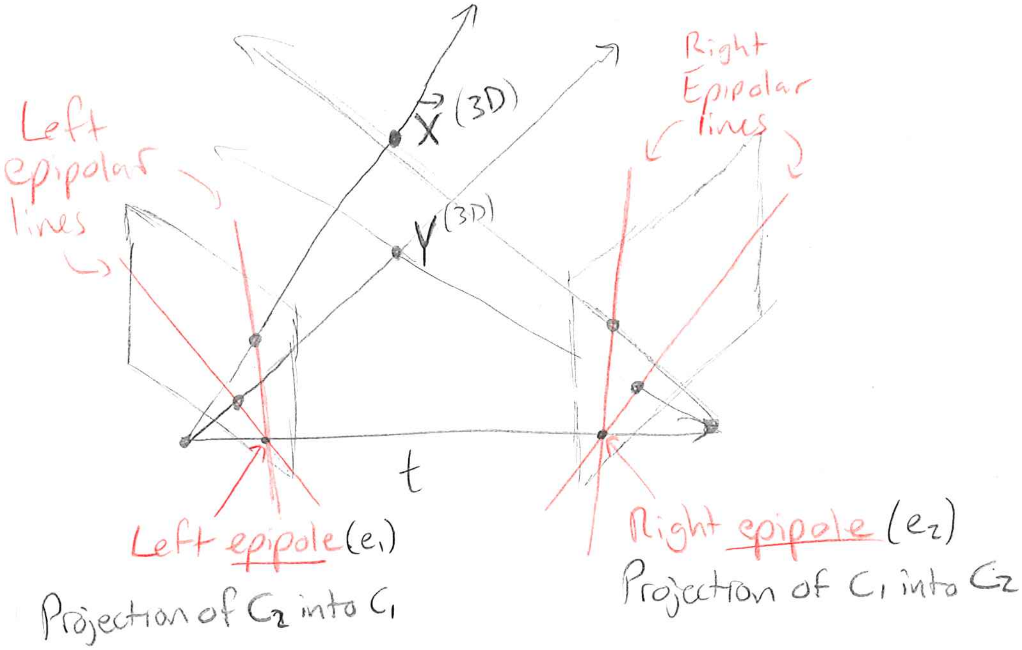
q in image 2: $\hat{q} = K_2^{-1} q$

Plug into the above:

$$\hat{q}^T R_2 [t]_\times \hat{p} = 0$$

$$q^T \underbrace{K^{-T} R_2 [t]_\times K^{-1}}_{F \text{ = the fundamental matrix}} p = 0$$

For pixel coordinates p, q, if they correspond to the same 3D point, then $\boxed{q^T F p = 0}$



Properties of F and epipolar geometry:

- F has rank 2: $F\vec{p}$ maps to a 1D solution space.
- All epipolar lines go through the epipole; the baseline vector t spans all epipolar planes and passes through both epipoles.
- e_1 spans the null space of F: $Fe_1 = 0$
- e_2 spans the null space of F^T: $F^T e_2 = 0$

We derived all this assuming $K_1, K_2, R_2\ t_2$ are known.
Can we find F, then use it to determine camera parameters?
→ Sort of. (i.e. yes, but it's hard.)

Finding F: Like finding H, we can set up a homogeneous leas squares system using equations from $q^T F p = 0$ to solve for the entries of F. } The 8-point algorithm

K, R, t from F: this is harder, but there are strategies.

In general, reconstructing camera pose and 3D geometry is done jointly (chicken-and-egg); this is called "Structure from motion"
(3D geom.) ⟷ (camera pose)