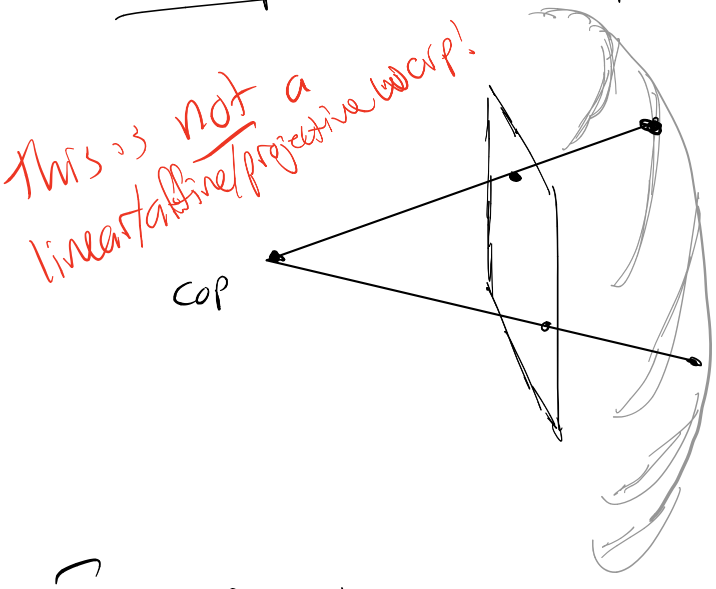
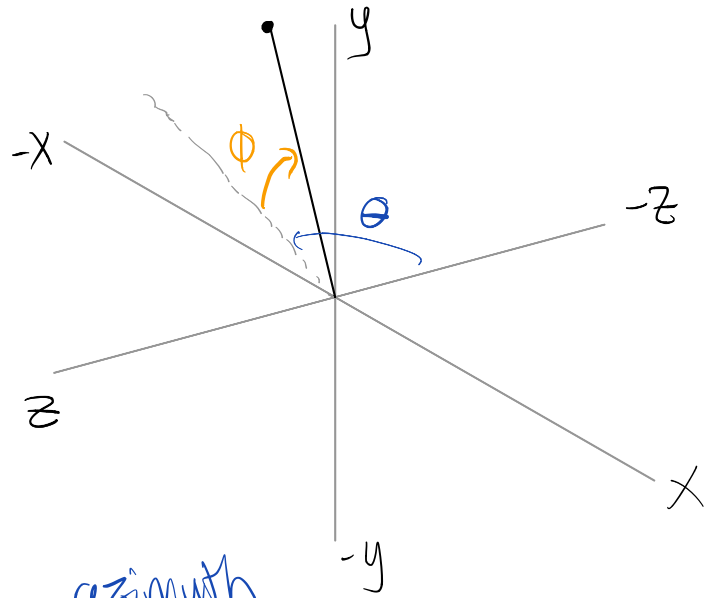
Spherical Warp: project image plane onto a sphere.





azimuth
θ : ccw from -z in xz plane

elevation
ϕ : up towards +y from xz plane

In principle:

each point $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ in img coords

and map to angular coords (θ, ϕ).

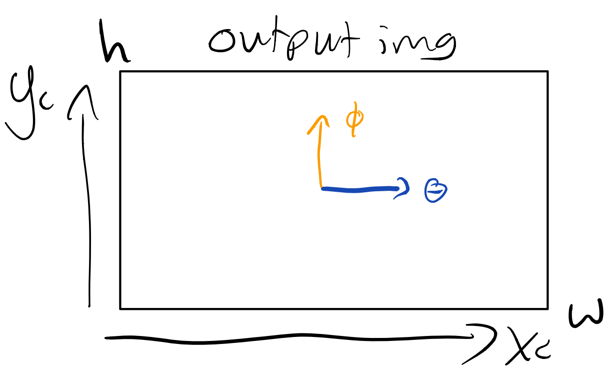
In practice:

inverse warping - fill in spherical coord output image with interpolated values from planar image.

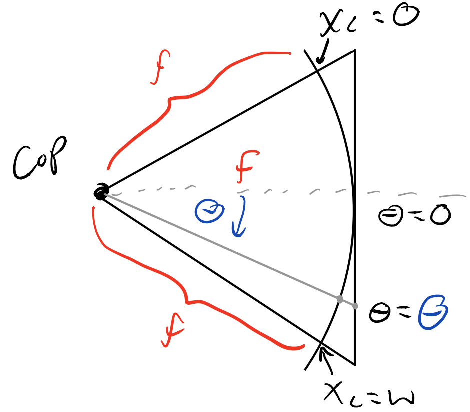
Given spherical pixel coords (x_c, y_c):
find image coords (x, y):

1. Move $\theta = 0$, $\phi = 0$ to origin:

$$\left(x_c - \frac{w}{2} \, , \, y_c - \frac{h}{2} \right)$$



2. Convert to angles



$$\text{Arc length} = \frac{\theta}{2\pi} \cdot \text{circumference}$$

$$= \frac{\theta}{\cancel{2\pi}} \cdot \cancel{2\pi} \underbrace{r}_{=f}$$

$$\text{Arc length} = \theta r \qquad \theta = \frac{\text{arc length}}{r}$$

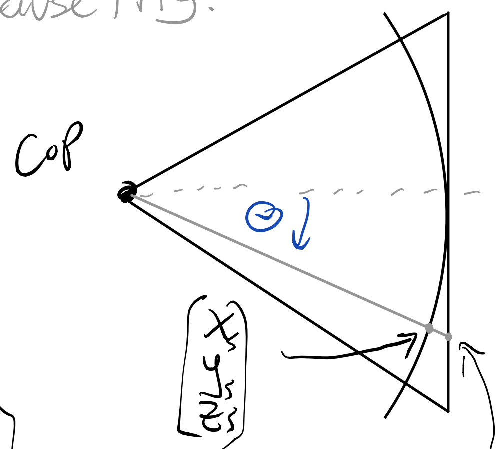
$$\boxed{\begin{aligned} \theta &= \frac{1}{f}\left(x_c - \frac{w}{2}\right) \\ \phi &= \frac{1}{f}\left(y_c - \frac{h}{2}\right) \end{aligned}}$$

(cartesian)

3. Convert (θ, ϕ) to a 3D point on unit sphere:

$$x_s = \sin\theta \cos\phi$$
$$y_s = \sin\phi$$
$$z_s = \cos\theta \cos\phi$$

because trig.



$$\begin{bmatrix} \frac{x_s}{z_s} \\ \frac{y_s}{z_s} \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

must be 1 - normalize

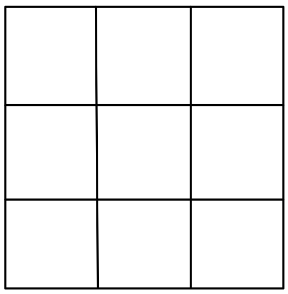
One more detail...

Radial Distortion

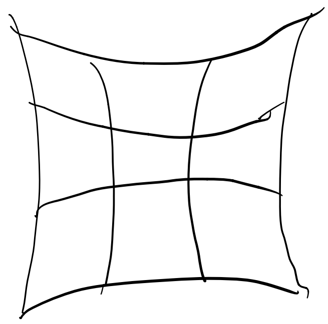
The pinhole model is great, but in reality...
we have lenses.

The thin lens model is great, but in reality...
497Cam, MkIII

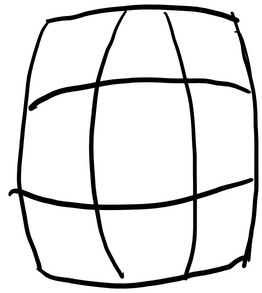
Lenses introduce distortion.



No distortion



Pincushion distortion

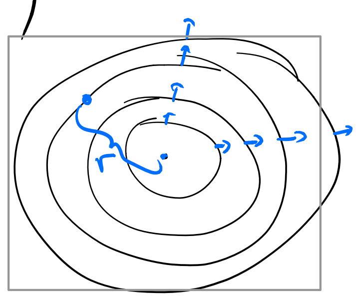


Barrel distortion

These are both kinds of Radial distortion:
Function of distance from the center of the lens (img)

To model this:
assume pixels are "nudged" radially in position by a predictable amount, $f(r^2)$

Assume low-order polynomial for f:



Suppose "Ideal" (undistorted) coordinates (x_i, y_i)

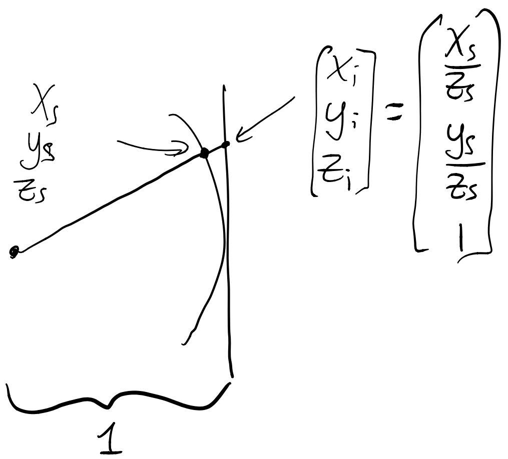
Then distorted position is modeled as:

Let $r^2 = x_i^2 + y_i^2$ (with (0,0) at img center)

distorted coords
$$\begin{cases} x_d = x_i(1 + k_1 r^2 + k_2 r^4) \\ y_d \;\; y_i(1 + k_1 r^2 + k_2 r^4) \end{cases}$$

parameters - can be fit by taking a photo of a checkerboard/grid

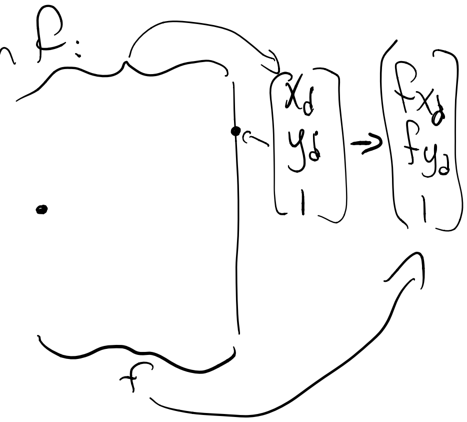
Back to our spherical warp...



4. Apply radial distortion:

$$r^2 = x_i^2 + y_i^2$$
$$x_d = x_i\left(1 + k_1 r^2 + k_2 r^4\right)$$
$$y_d = y_i\left(1 + k_1 r^2 + k_2 r^4\right)$$

5. Camera actually has focal length f:



$$x_p = f x_d + \frac{w_0}{2}$$
$$y_p = f y_d + \frac{h_0}{2}$$

6. Pixel origin is in the corner