Spherical Warp:

In principle:
- Each point \([X \ Y]\) in img cords
- and map to angular coords \((\Theta, \Phi)\).

In practice: inverse warping - fill in spherical coord
- output image with interpolated values from
- planar image.

Given spherical pixel coords \((X_c, Y_c)\):
- find image cords \((X, Y)\):
  1. Move \(\Theta=0, \Phi=0\) to origin:
     \[ \left( X_c - \frac{W}{2}, \ Y_c - \frac{h}{2} \right) \]
2. Convert to angles

Arc length = \( \frac{\theta}{2\pi} \cdot \text{circumference} \)
= \( \frac{\theta}{2\pi} \cdot 2\pi r \)

Arc length = \( \theta r \)
\( \theta = \frac{\text{arc length}}{r} \)

\[
\Theta = \frac{1}{r} (x_c - \frac{w}{2})
\]

\[
\phi = \frac{1}{r} (y_c - \frac{h}{2})
\]

(cartesian)

3. Convert \((\Theta, \phi)\) to a 3D point on unit sphere:

\[
x_s = \sin \Theta \cos \phi
\]

\[
y_s = \sin \Theta
\]

\[
z_s = \cos \Theta \cos \phi
\]

because tris.

\[
\begin{bmatrix}
x_s \\
y_s \\
z_s
\end{bmatrix} = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

must be 1 - normalize
One more detail...

**Radial Distortion**

The pinhole model is great, but in reality...

We have lenses.

The thin lens model is great, but in reality...

Lenses introduce distortion.

| No distortion | Pincushion distortion | Barrel distortion |

These are both kinds of **radial distortion**: function of distance from the center of the lens (ing)

To model this:

- Assume pixels are "nudged" radially in position by a predictable amount, $F(r^2)$

Assume low-order polynomial for $F$:
Suppose "Ideal" (undistorted) coordinates \((x_i, y_i)\)
Then distorted position is modeled as:

\[
\begin{aligned}
X_d &= x_i(1 + k_1 r^2 + k_2 r^4)
\quad (\text{with (0,0) at img center}) \\
y_d &= y_i(1 + k_1 r^2 + k_2 r^4)
\end{aligned}
\]

Parameters can be fit by taking a photo of a checkerboard/grid.

Back to our spherical warp...

4. Apply radial distortion:
\[
r^2 = x_i^2 + y_i^2 \\
X_d = x_i(1 + k_1 r^2 + k_2 r^4) \\
y_d = y_i(1 + k_1 r^2 + k_2 r^4)
\]

5. Camera actually has focal length \(f\):
\[
\begin{aligned}
x_p &= f x_d + \frac{w_0}{2} \\
y_p &= f y_d + \frac{h_0}{2}
\end{aligned}
\]

6. Pixel origin is in the corner

\[
(x_0, y_0) \quad \text{at} \quad (0, 0)
\]