CSCI 497P/597P: Computer Vision

Lecture 16
Fitting Transformations with Outliers: **RAN**dom **SA**mple **C**onsensus (RANSAC)
Announcements

• P2 is out

• Do you want the option to work in pairs?
Goals

• Understand the Random Sample Consensus (RANSAC) algorithm.

• Be prepared to implement RANSAC to fitting image coordinate transforms using matches that may contain outliers.
Warping

We've found correspondence. How do we fit a transformation to a given set of matches?
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How do we fit a transformation to a given set of matches?
Analogy: fit a line to a given set of points?
We've found correspondence.
How do we fit a transformation to a given set of matches?
Analogy: fit a line to a given set of points?

This is a model-fitting problem.
Problem Statement: Last time

( Imperfect )

Given a set of feature matches, how do I find the transformation that relates the two images? translation? affine? homography?

* all points may be off by a little
Fitting a Homography: TL;DM

\[
\begin{bmatrix}
  x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\
  0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\
  \vdots & & & & & & & & & \\
  x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\
  0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \\
\end{bmatrix}
\begin{bmatrix}
  h_{00} \\
  h_{01} \\
  h_{02} \\
  h_{10} \\
  h_{11} \\
  h_{12} \\
  h_{20} \\
  h_{21} \\
  h_{22}
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0 \\
  \vdots \\
  0 \\
\end{bmatrix}
\]

\[
A_{2n \times 9} \quad h_{9 \times 2n} = 0
\]
Fitting a Homography: TL;DM

For each feature match \((x_i, y_i) \rightarrow (x'_i, y'_i)\), fill in 2 rows of \(A\) as in:

\[
\begin{bmatrix}
    x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\
    0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\
    0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n
\end{bmatrix}
\]

Solve the homogeneous least squares problem:

\[
\min_h \|Ah\|^2
\]

Take the SVD of \(A\) to get \(U, S, \text{and} V\)

- Let \(h\) — the right singular vector of \(A\) whose singular value is smallest.
- Let \(h\) = the column of \(V\) (row of \(V^T\)) whose column (row) index is the same as that of the smallest diagonal entry of \(S\).

Reshape \(h\) into \(H_{3 \times 3}\) and divide by the bottom-right entry.
Problem Statement: Last time

(imperfect*)

Given a set of feature matches, how do I find the transformation that relates the two images?

translation?
affine?
homography?

* all points off by a bit
When does least squares work well?
Problem Statement: Today

(imperfect*)

Given a set of feature matches, how do I find the transformation that relates the two images?

* a few points are outliers
How could we fit a line to this data?
An idea

- If I have a hypothesis, I can tell how "good" it is:
  - Count the number of points that are close to the line (inliers)
An idea

• If I have a hypothesis, I can tell how "good" it is:
  • Count the number of points that are close to the line (inliers)

• Algorithm: generate all possible lines and pick the one with the most inliers
  • Runtime: $O(\infty)$
Another idea

• If I have a hypothesis, I can tell how "good" it is:
  • Count the number of points that are close to the line (inliers)
  • Algorithm: generate many random lines and pick the one with the most inliers.

• Questions:
  • How many lines? Which ones? How do I measure "inlierness"?
\[ k = 5 \]
The key

• Points that fit the model will agree.

• Points that don't fit the model will all be wrong in their own unique ways, and there won't be a very large set of them that agree with each other.

“All good matches are alike; every bad match is bad in its own way.”
-Tolstoy, as misquoted by Alyosha Efros
The Algorithm

\[
\text{for } i = 0 \ldots K:
\]
\[
d_i \leftarrow \subseteq \text{ random data points}
\]
\[
M_i \leftarrow \text{fit-model}(d_i)
\]
\[
\text{inlier-count} \leftarrow \sum \left( \mathbf{1}(|M(x_i) - y_i| < \delta) \right)
\]
\[
\text{if} \quad \text{inlier-count} > \text{best-count},
\]
\[
\text{best-count} \leftarrow \text{inlier-count}
\]
\[
\text{best-M} \leftarrow M_i
\]
\[
\text{best-data} \leftarrow \{ x_i, y_i : |M(x_i) - y_i| < \delta \}
\]
\[
M_{\text{final}} \leftarrow \text{fit-model}(\text{best-data})
\]
Choose Parameters:

\[ \delta \] - inlier threshold

assume Gaussian noise \( \delta \geq 5, 25 \)

\[ k \] : iterations - guess, or assume fraction of inliers and acceptable probability of picking a set of all inliers.
Questions Remain

• How do we generate a hypothesis for transformations?
Questions Remain

• How do we generate a hypothesis for transformations?

\[ S : \text{ smallest number of points that can fully determine your model. Or: } \# \text{ of degrees of freedom.} \]

**Linear Regression?** 2 points fit a line.

\[ (x_1, y_1) \quad (x_2, y_2) \]
Translation? I match determines \((t_x, t_y)\)

\[(x_i, y_i) \rightarrow (x'_i, y'_i)\]

Affine?

\[A \cdot t = b\]

smallest \(n\) without being underdetermined: 3
Homography? Awkward: $A$ is $2n \times 9$ but has only $8$ DOF.

Turns out: SVD works on $8 \times 9 A$:  

$$A \in \mathbb{R}^{a \times 9} = U \Sigma V^T$$

Same argument applies, so

$A_{\text{88}}$ uses 4 matches, 2 residuals per match.
\[ s = \lceil \text{DOF} / 2 \rceil \]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Name} & \text{Matrix} & \# \text{D.O.F.} & \text{Preserves:} & \text{Icon} \\
\hline
\text{translation} & \begin{bmatrix} I & t \end{bmatrix}_{2 \times 3} & 2 & \text{orientation} + \cdots & \square \\
\hline
\text{rigid (Euclidean)} & \begin{bmatrix} R & t \end{bmatrix}_{2 \times 3} & 3 & \text{lengths} + \cdots & \diamond \\
\hline
\text{similarity} & \begin{bmatrix} sR & t \end{bmatrix}_{2 \times 3} & 4 & \text{angles} + \cdots & \diamond \\
\hline
\text{affine} & \begin{bmatrix} A \end{bmatrix}_{2 \times 3} & 6 & \text{parallelism} + \cdots & \square \\
\hline
\text{projective} & \begin{bmatrix} \tilde{H} \end{bmatrix}_{3 \times 3} & 8 & \text{straight lines} & \square \\
\hline
\end{array}
\]
Questions Remain

- How do we choose the parameters?
  - $k$ (# iterations)
  - $s$ (# data points needed to fit a model)
  - $\delta$ (inlier threshold)

(See above - page 22 for $d$, $k$
25 - 28 for $s$)