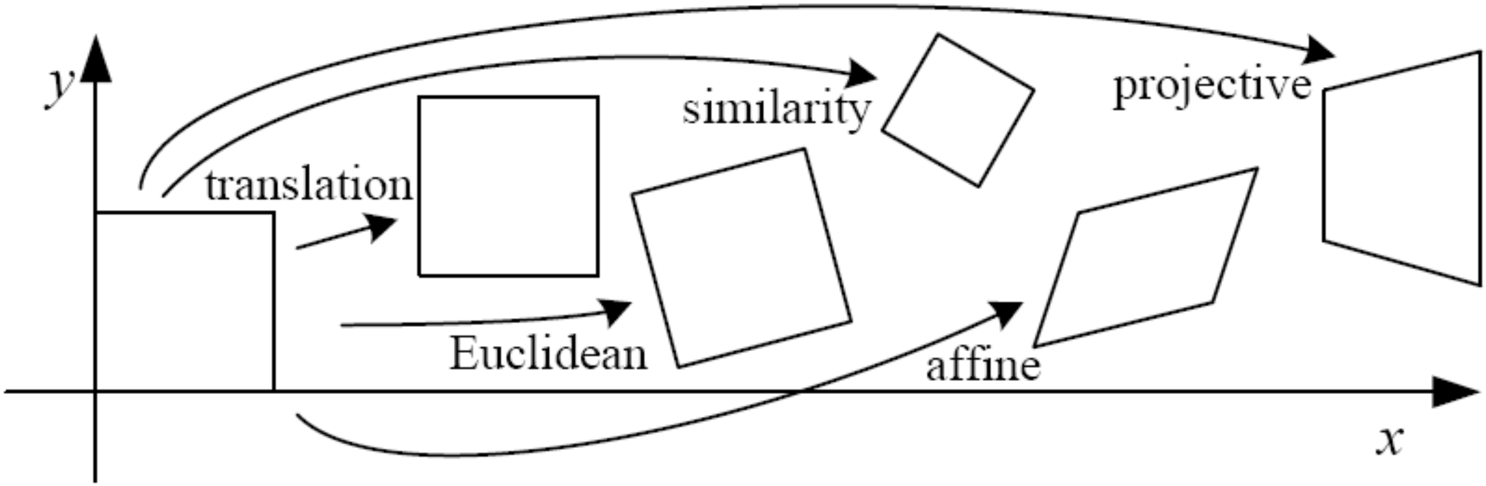
CSCI 497P/597P: Computer Vision



Lecture 13

Forward and Inverse Warping

Projective Transformations

Announcements

- Please add a **`username.txt`** to the base directory of your P1 repository, containing (only) your WWU username.
- Reminder: HW1 due Wednesday; P1 due Friday

Goals

- Know how to resample images using forward and inverse warping
- Know the definition of a projective (homography) transformation, and gain some geometric intuition for what it represents in 2D.
  - Understand the intuition behind a "point at infinity"
  - Know why a homography has 8 degrees of freedom, not 9

Last time: Affine Transformations

- Affine transformations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} a & b \\ d & e \end{matrix}} & \begin{matrix} c \\ f \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

translation

1

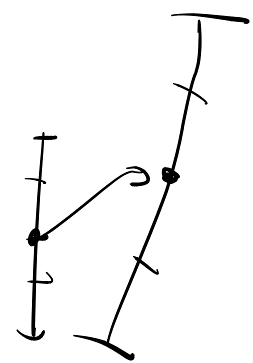
Last time: Affine Transformations

- Affine transformations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties:
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios of lengths along lines are preserved
  - Closed under composition
  - Origin maps to origin

linear



Last time: Affine Transformations

- Affine transformations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties:
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios of lengths along lines are preserved
  - Closed under composition

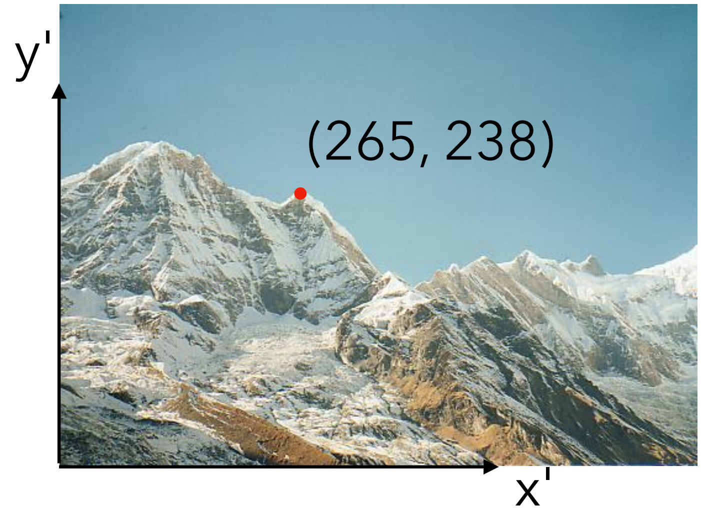
  linear

  - Origin **does not** necessarily map to origin

  affine

Warping

translation



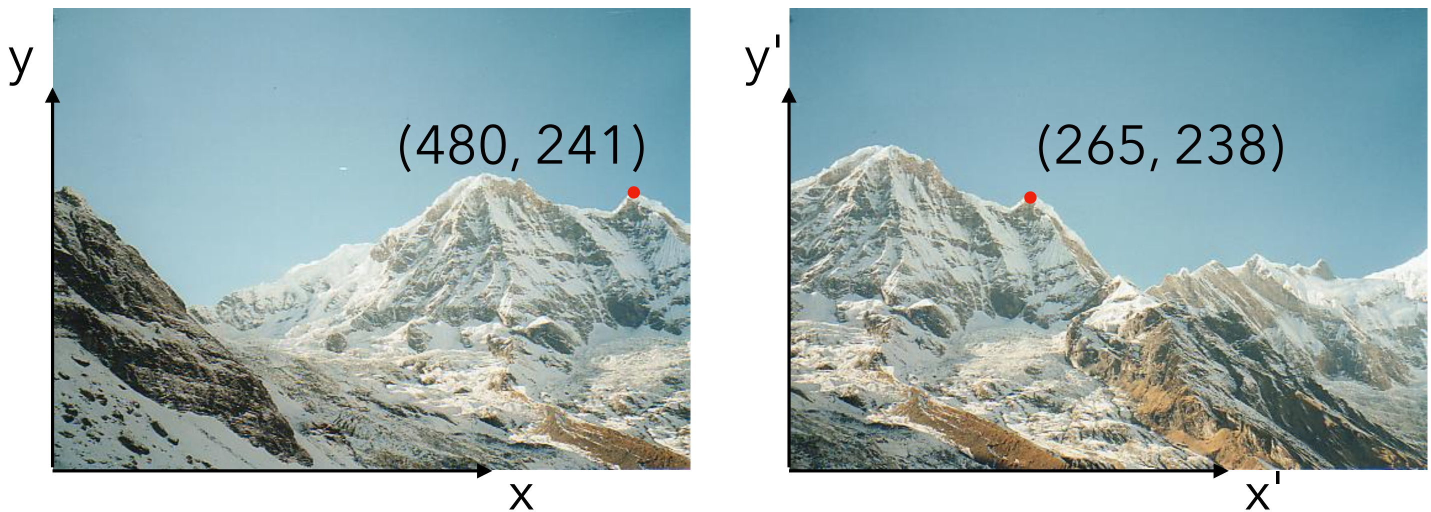
We've found correspondence. What's the transformation?

$x' = x + t_x$

$y' = y + t_y$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -215 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

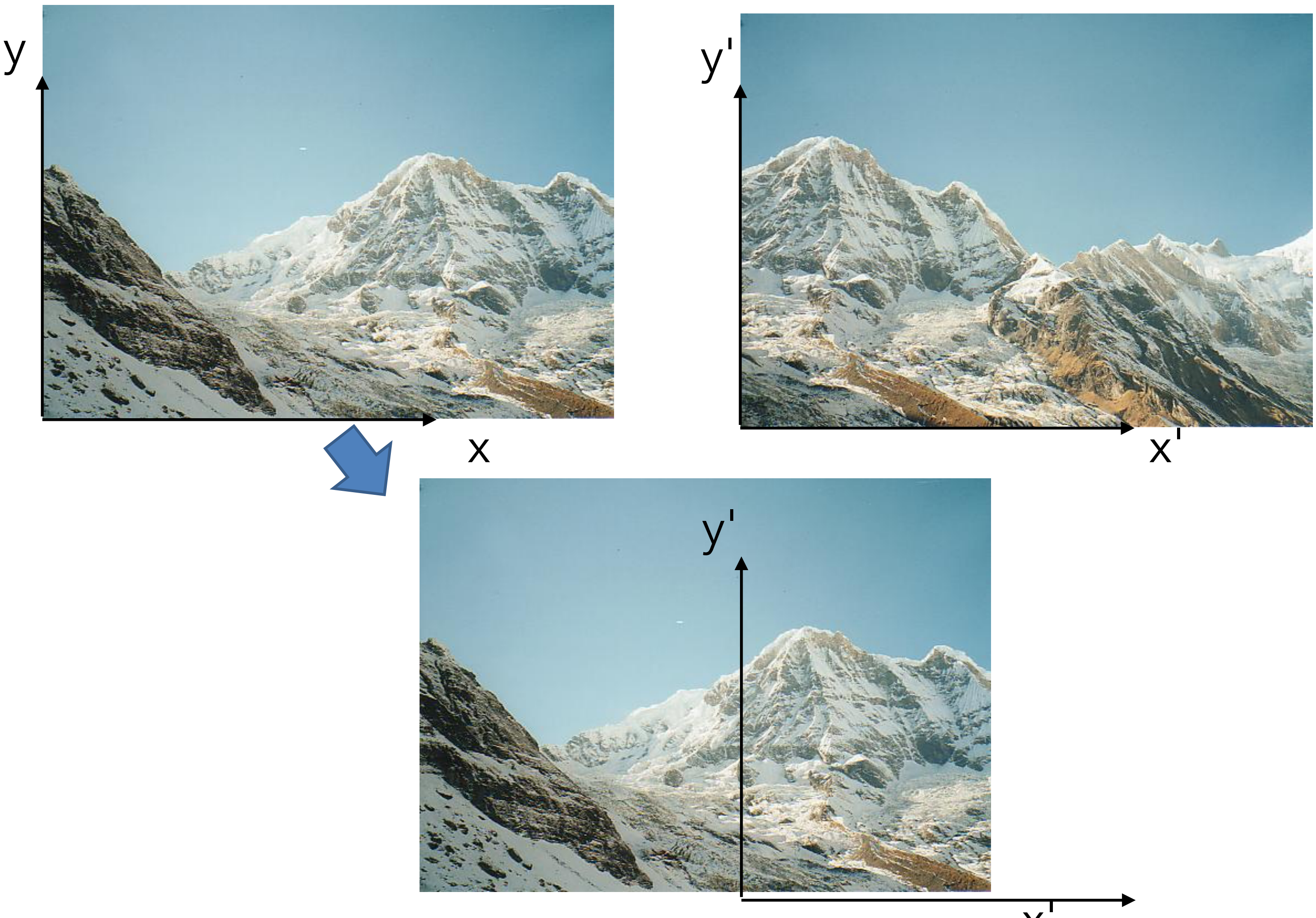
Warping



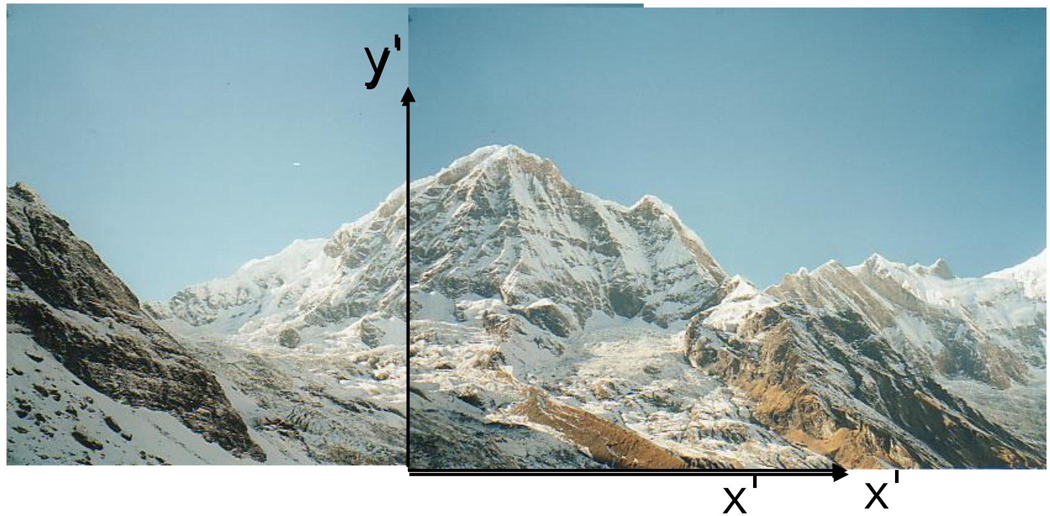
We've found the transformation. How do we warp the image?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -215 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Warping



Warping

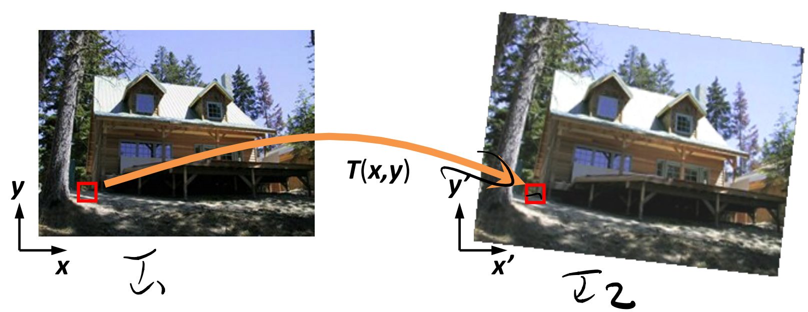


Forward Warping

for (x, y) in I_1

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = T \begin{bmatrix} x \\ y \end{bmatrix}$$

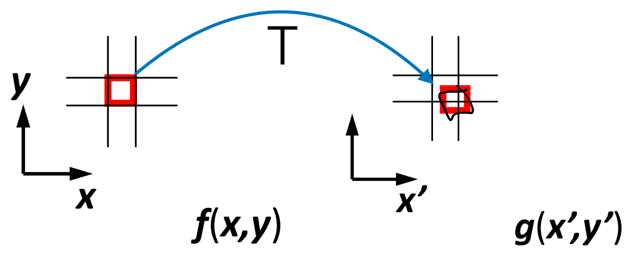
$$I_1[x', y'] = I[x, y]$$

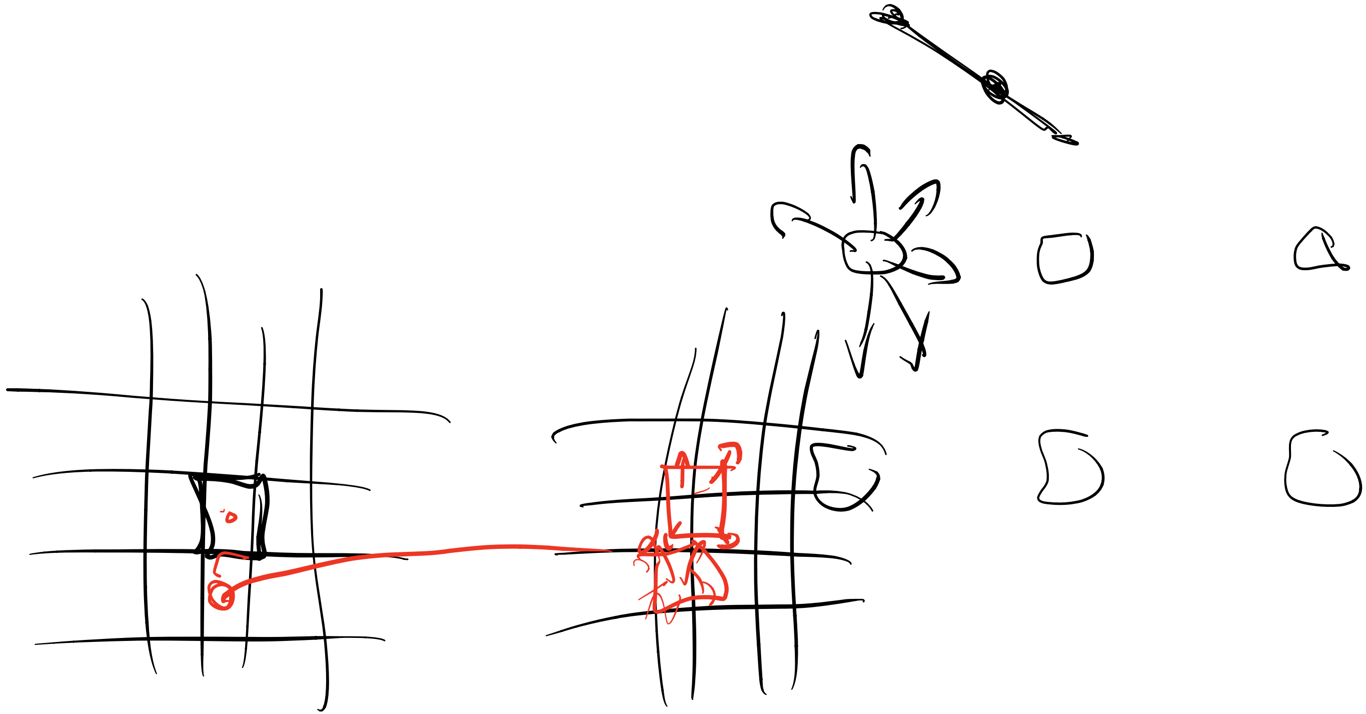


Warping

We've found the transformation. How do we warp the image?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -214.8 \\ 0 & 1 & -2.9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

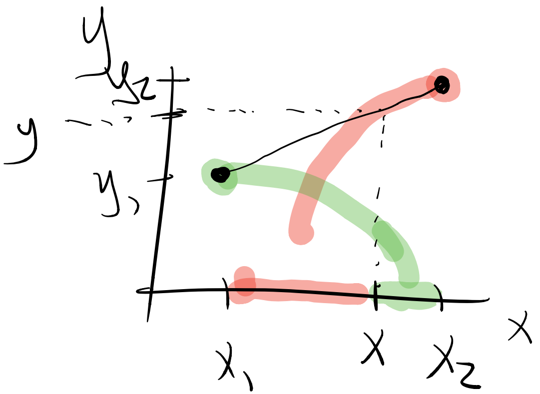




$$T = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$$

Inverse Warping

<u>Linear interpolation (1D)</u>



$x_1 = 0$
$x_2 = 1$

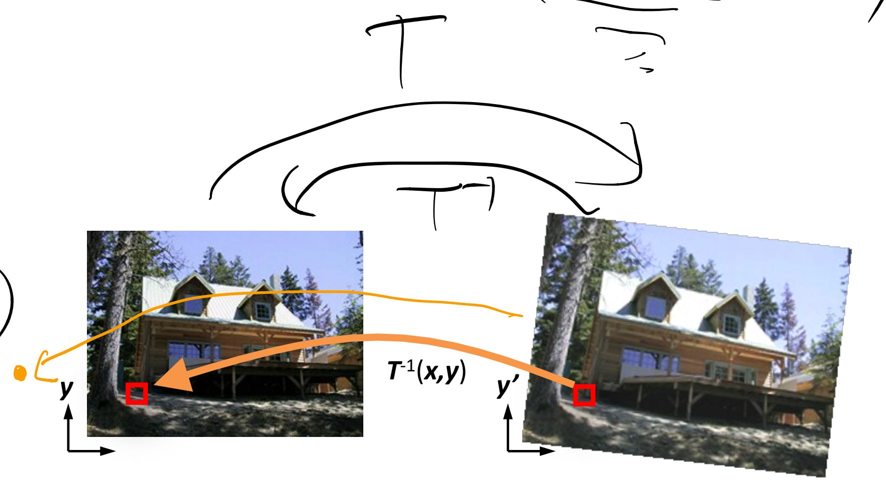
$$y = y_1(x_2 - x) + y_2(x - x_1)$$

$$y = y_1(1 - x) + y_2(x)$$

for x', y' in I_1':

$$x, y = T^{-1}\begin{bmatrix} x' \\ y' \end{bmatrix}$$

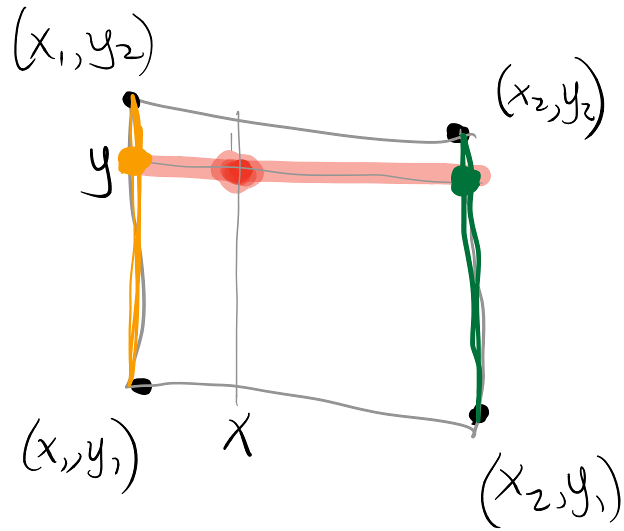
$$I_1'[x', y'] = \underline{\text{interpolate}}\left(I_1, \begin{bmatrix} x \\ y \end{bmatrix}\right)$$

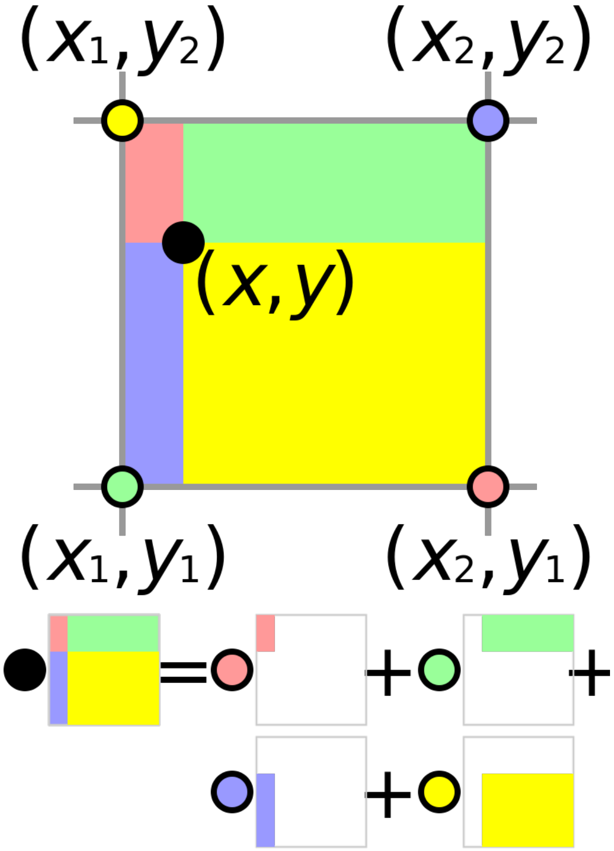


Bilinear Interpolation

Interpretations:

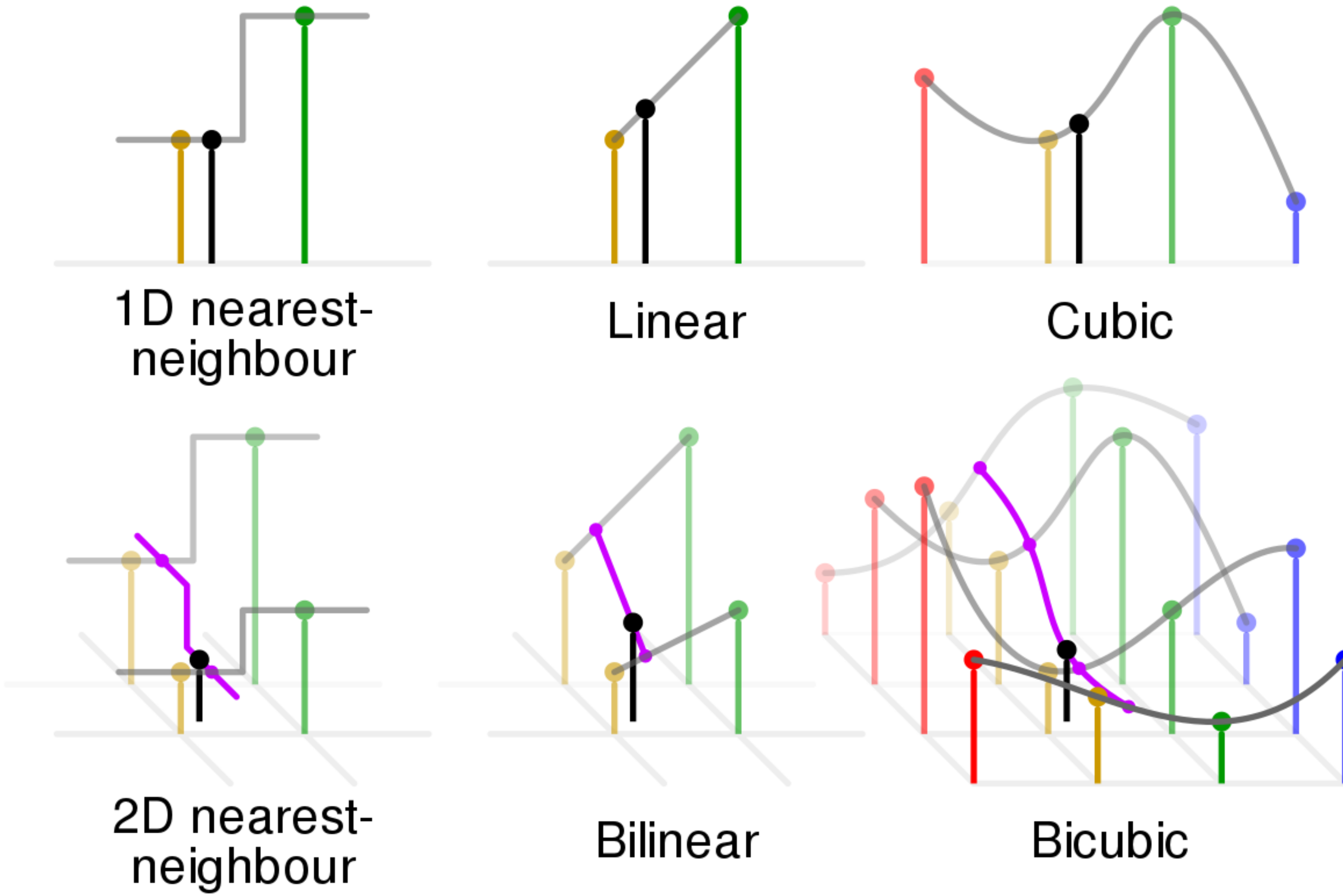
- Tent filter centered at (x,y)
- weights are areas of opposite corner rectangles
- linear interp on 2 sides then interp those





(x1, y2)
(x2, y2)
(x, y)
(x1, y1)
(x2, y1)

Other interpolation filters exist



Are these related by an affine transformation?

Affine Transformations

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

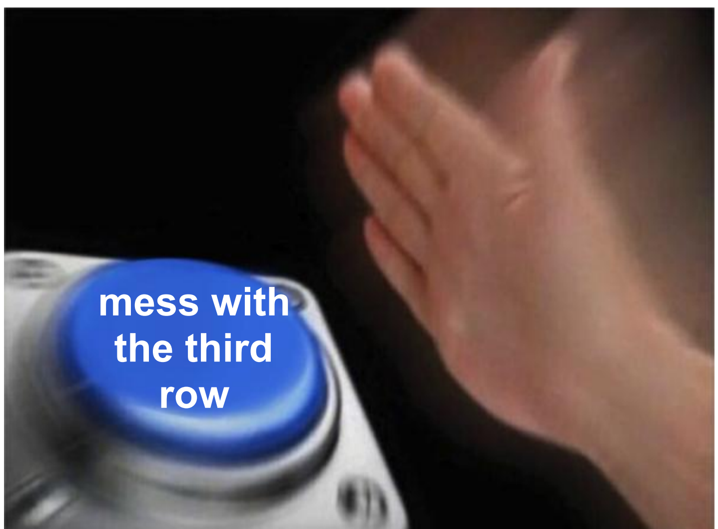
Scott: No messing with the third row!

Affine Transformations

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

Scott: No messing with the third row!

Students:



Well... you asked.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} ax + by + c \\ dx + ey + f \\ gx + hy + 1 \end{bmatrix}$$

Well… you asked.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Hang on a minute: what does it even mean if w != 1?

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} \approx \begin{bmatrix} x'/w \\ y'/w \\ 1 \end{bmatrix} \rightarrow 2D \begin{bmatrix} x'/w \\ y'/w \end{bmatrix}$$

To get the 2D coordinates, we need to **normalize**.

Projective Transformation (also known as Homograpy)

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

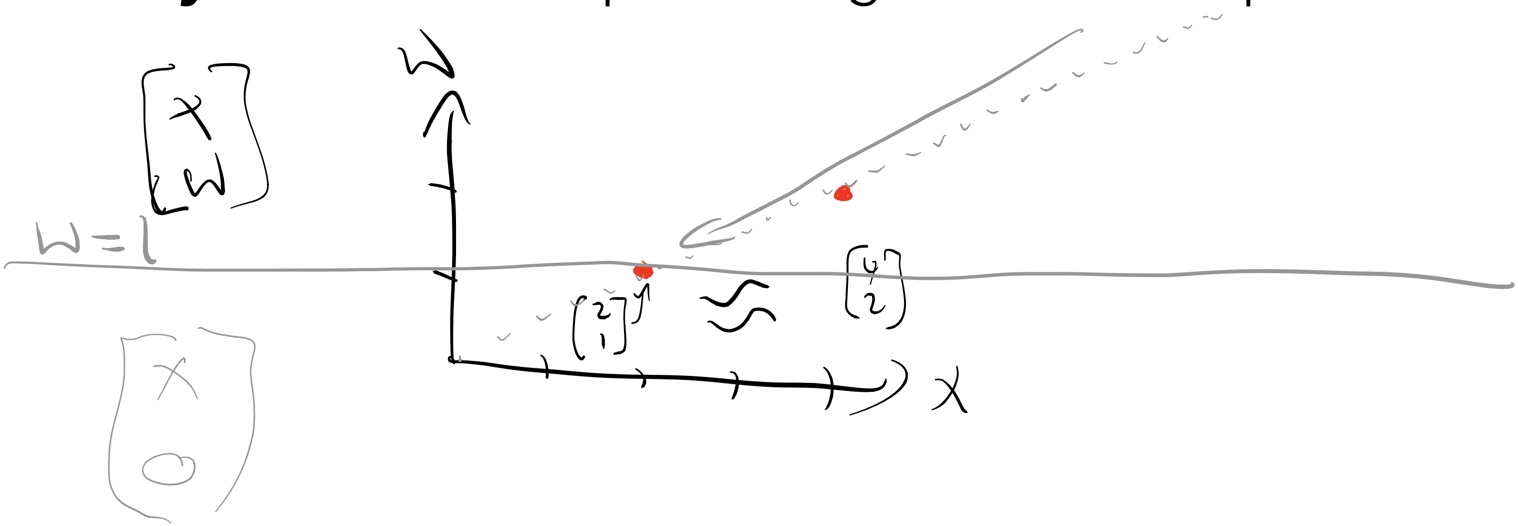
What does this do?

https://iis.uibk.ac.at/public/piater/courses/demos/homography/homography.xhtml

Homogeneous Coordinates: Intuition for our math hack

A 3-vector belongs to a **family** of 3-vectors representing the same 2D point.

1D example, for intuition: A 2-vector belongs to a **family** of 2-vectors representing the same 1D point.



Homographies for image alignment