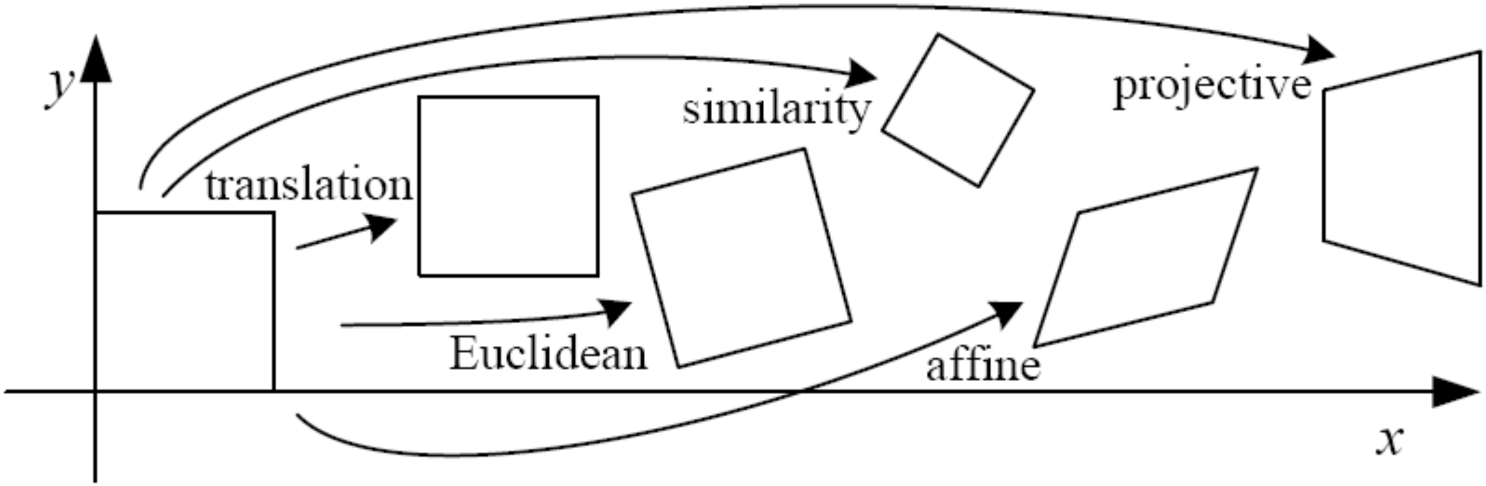
CSCI 497P/597P: Computer Vision



Lecture 12: Transformations

2D Linear and affine Transformations

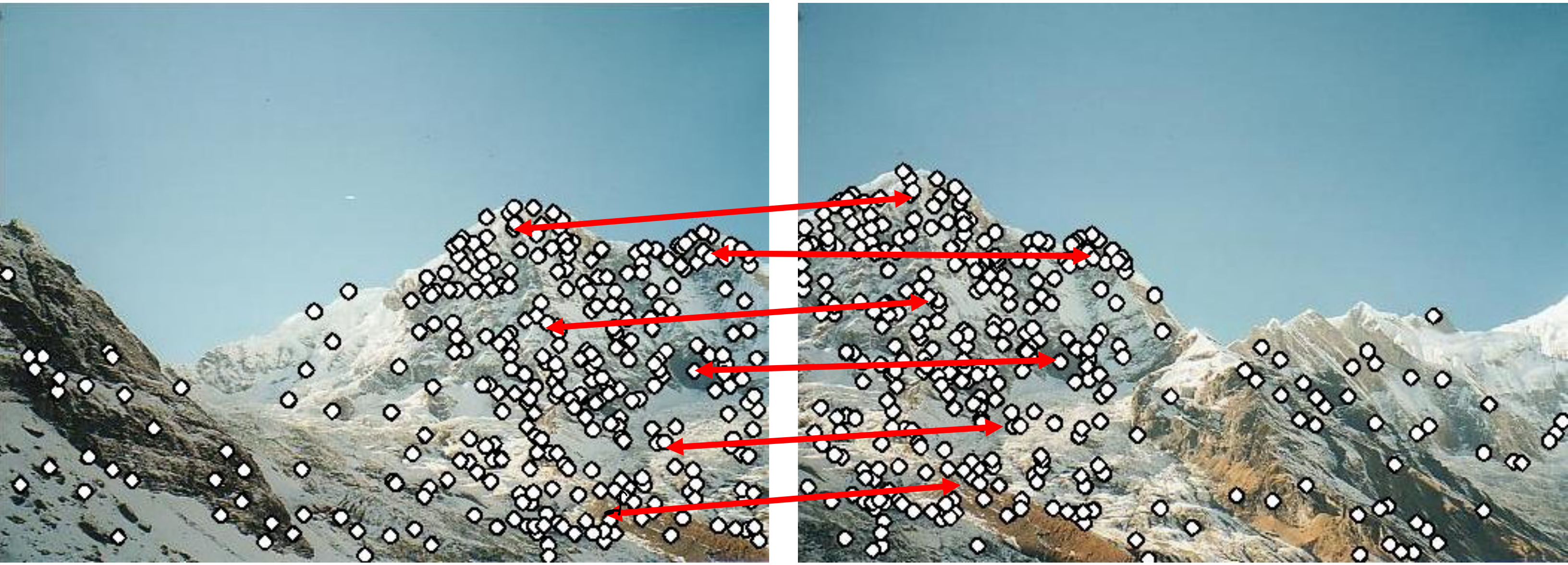
Announcements

- Reminder: fill out feedback survey by tonight.
- HW1 due Wednesday, P1 due Friday

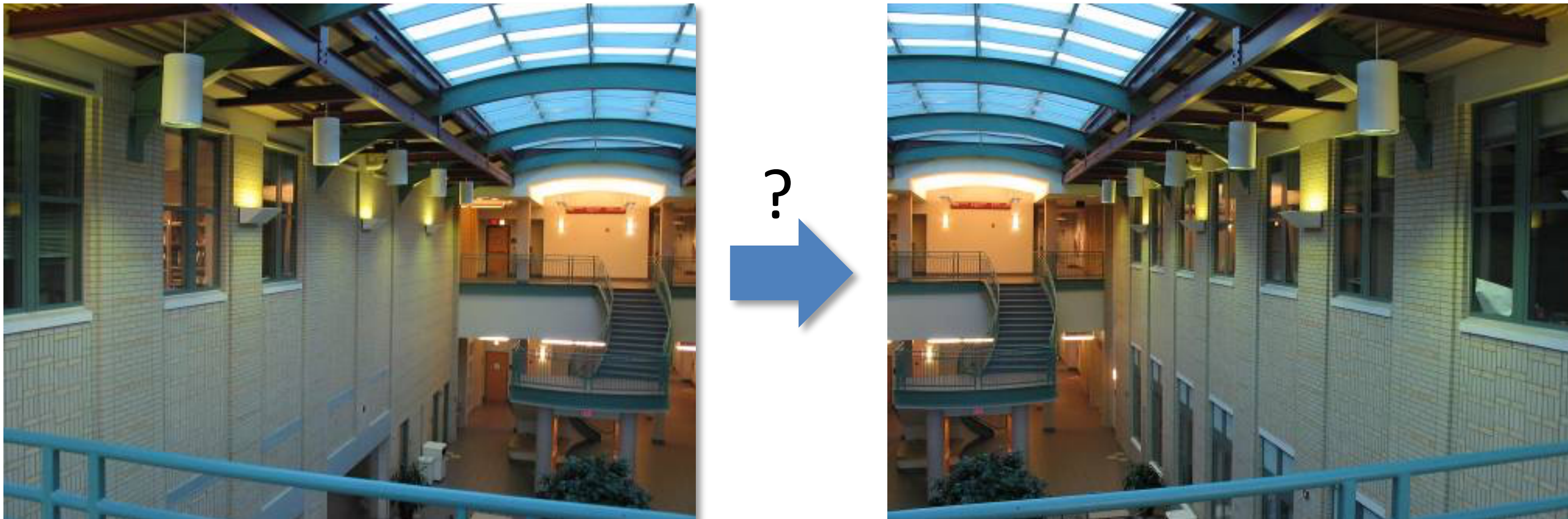
Goals

- Understand the mathematical framework for geometric transformations on images (image warping).
- Know what is possible with 2D linear transformations: (scale, shear, rotation)
- Understand the motivation and math behind homogeneous coordinates.
- Know what is possible with 2D affine transformations: (all of the above, plus translation)

Running motivational example: Panorama Stitching



How are these images related?



Given a set of feature matches, how can we warp one image to fit the other?

Where are we?

1. How do we describe the transformation?

   affine, projective transformations

2. How do we find an accurate transformation?

   least squares
   RANSAC

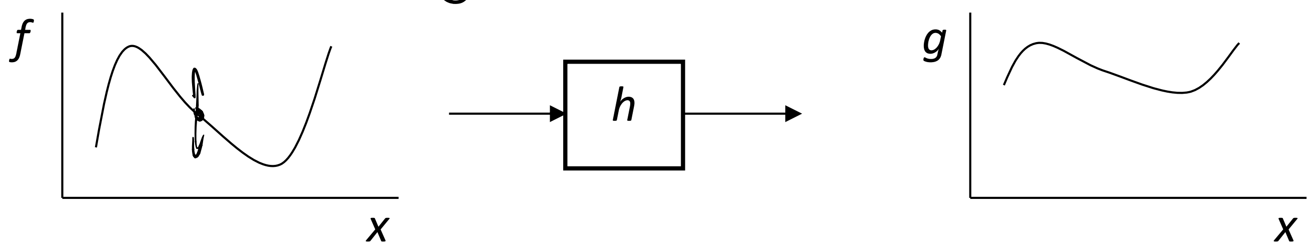
3. How do we actually warp the image?

   inverse warping

This is a geometric transformation.

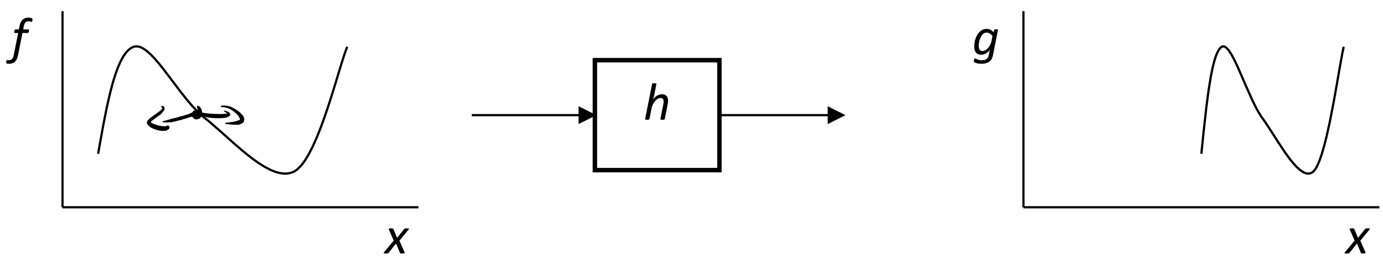
- **Filtering** was a transformation on the *range:*

$$g(x) = h(f(x))$$

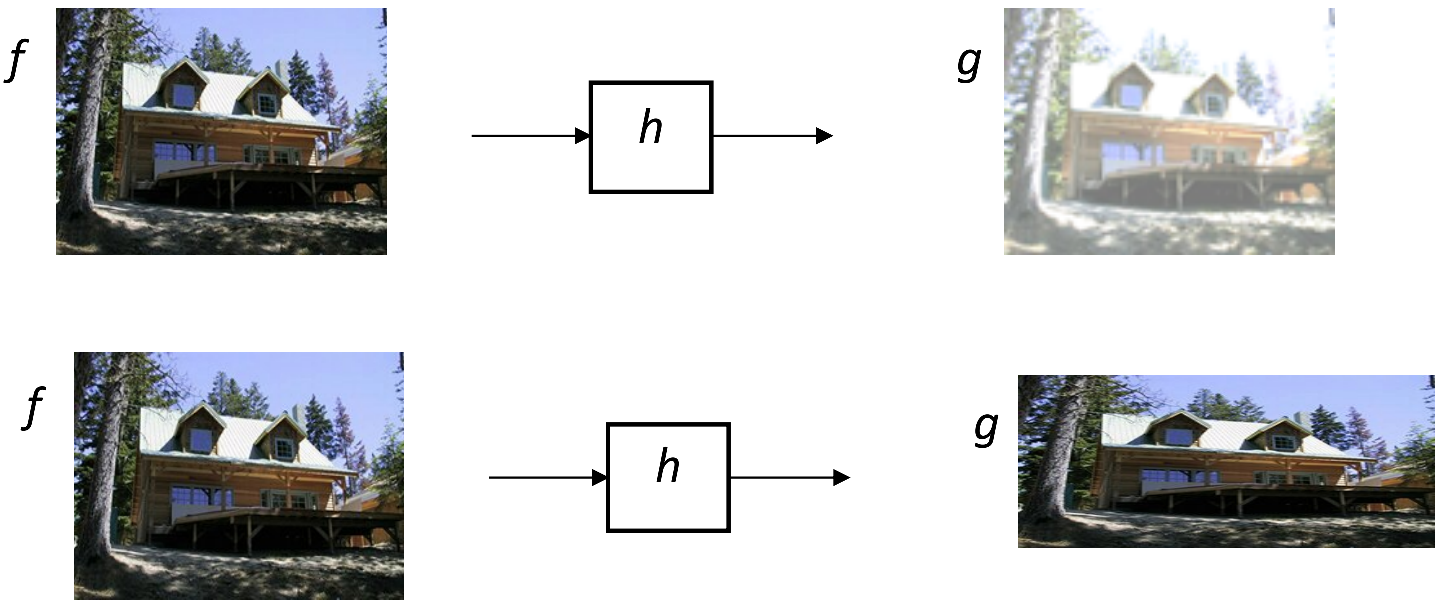


- **Warping** is a transformation on the domain:

$$g(x) = f(h(x))$$

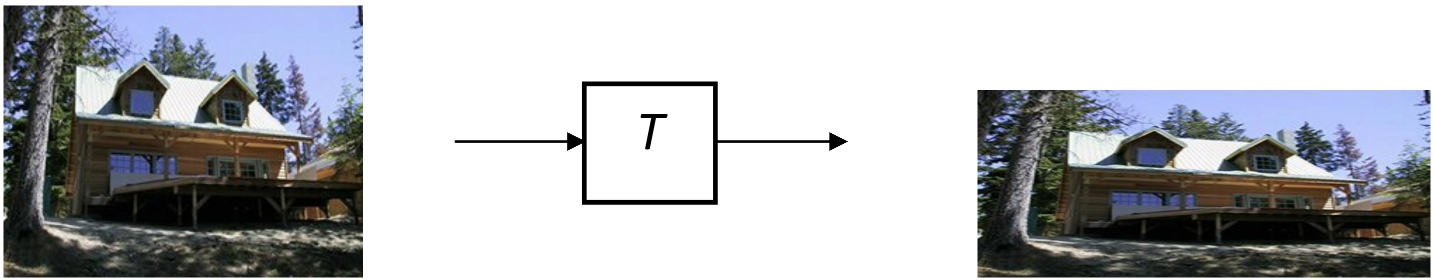


Filtering vs Warping



Parametric (global) Warping

- Apply the same function to all coordinates.



p = (x,y) T transforms *image coordinates* **p'** = (x',y')

$$x', y' = T(x, y)$$

Self-imposed restriction: T is a matrix.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \overbrace{\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}}^{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

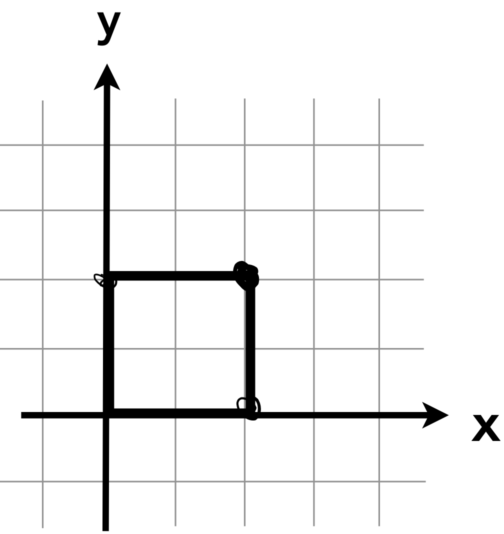
What can we do with this?

2x2 Matrices

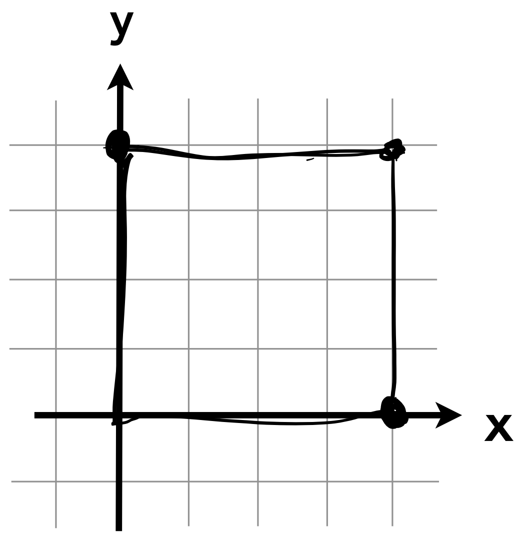
T

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t_{11}x + t_{12}y \\ t_{21}x + t_{22}y \end{bmatrix}$$

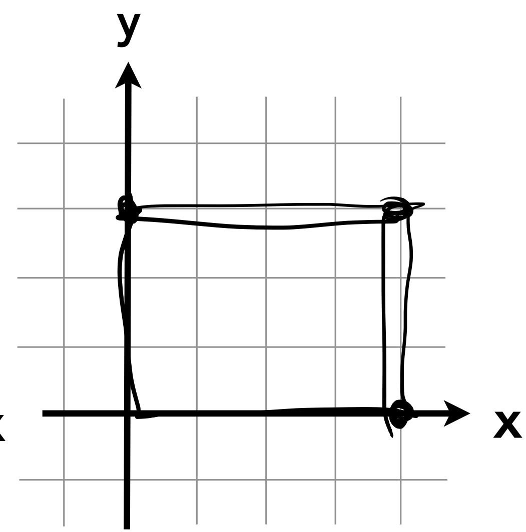
Scale



Input



Uniform Scale

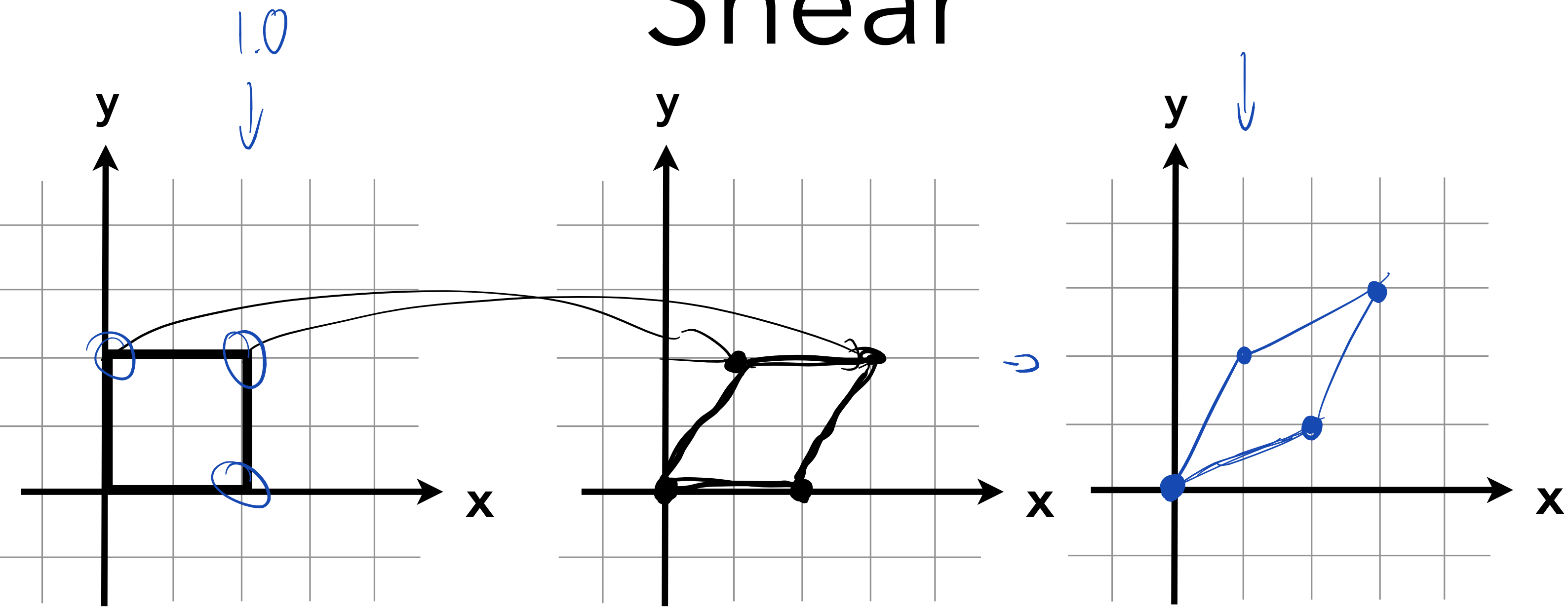


Nonuniform Scale

$$\begin{pmatrix}2x\\2y\end{pmatrix} = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{bmatrix}2 & 0\\0 & 1.5\end{bmatrix}$$

Shear

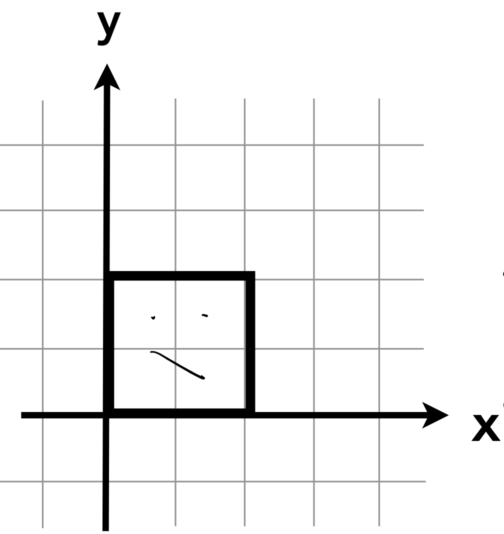


Input | Shear (x) | Shear (both)

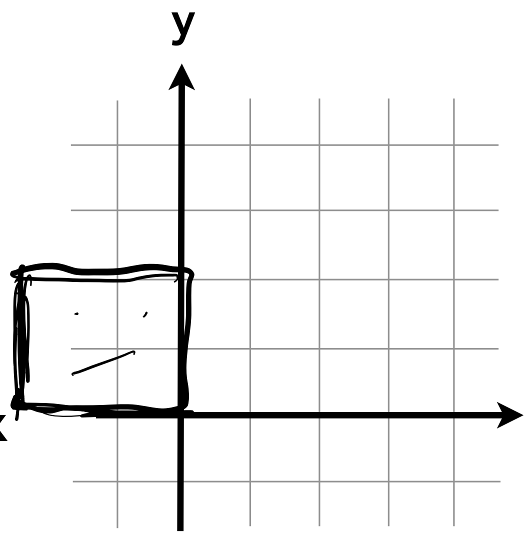
$$\begin{bmatrix} x+0.5y \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x+0.5y \\ 0.5x+y \end{bmatrix} \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Reflection

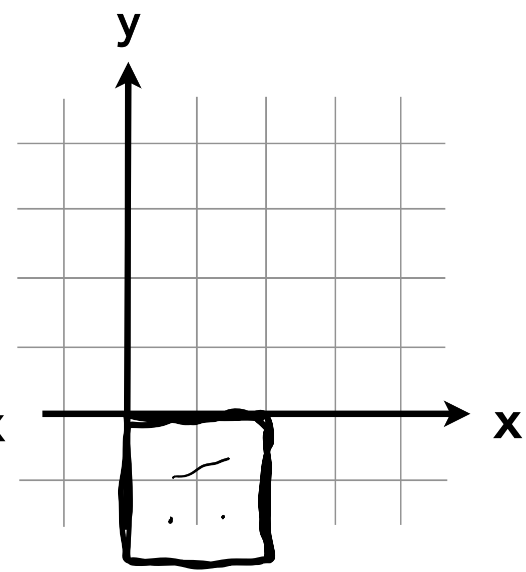


Input



Reflection (x)

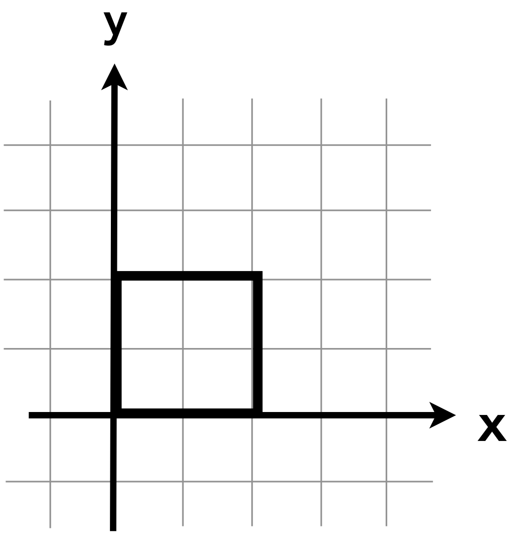
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



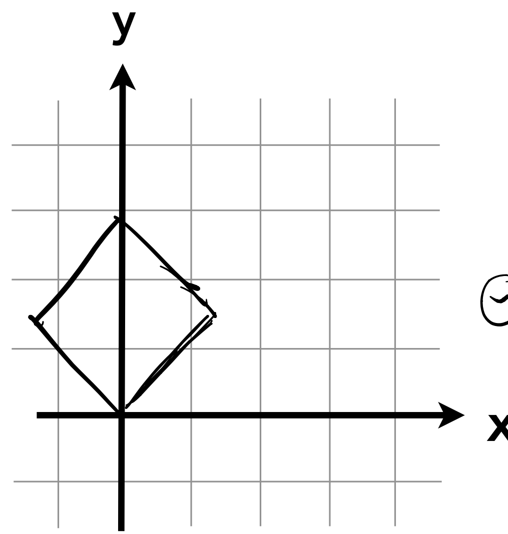
Reflection (y)

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Rotation



Input

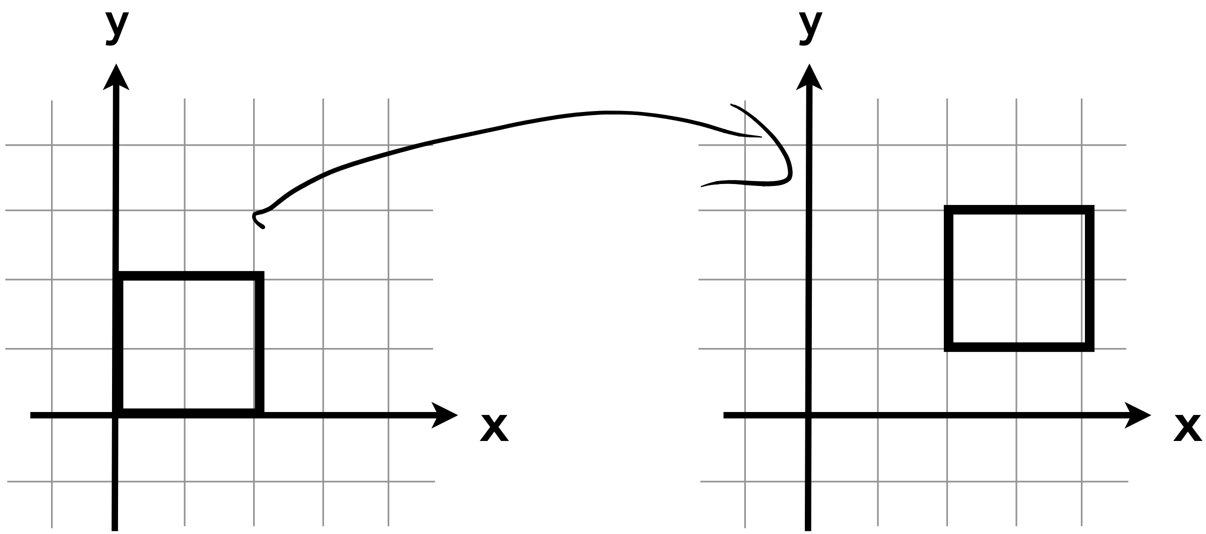


$\theta = 45^\circ$

Rotation about the origin

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Translation



Input

Translation by (u, v)

$$\begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \leftarrow \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Matrices can't translate.

We'll use a clever math hack to make them do it anyway:

Homogeneous Coordinates

Use a **3D** vector to represent a **2D** point.
Always put a **1** in the third dimension.

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Homogeneous Coordinates

Use a **3D** vector to represent a **2D** point.
Always put a **1** in the third dimension.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Matrices can't translate.

We'll use a clever math hack to make them do it anyway:

Homogeneous Coordinates

Use a **3D** vector to represent a **2D** point.
Always put a **1** in the third dimension.
How do we transform these?

1. 2.

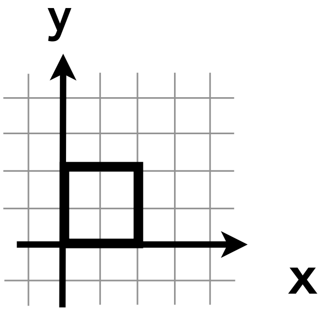
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_x \\ t_{21} & t_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} t_{11}x + t_{12}y + t_x \\ t_{21}x + t_{22}y + t_y \\ 1 \end{bmatrix}$$

Interactive Demo

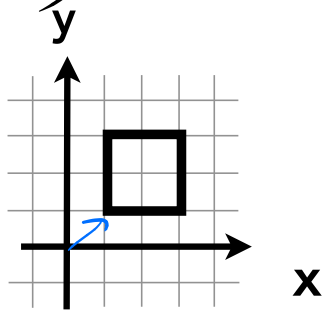
- https://iis.uibk.ac.at/public/piater/courses/demos/homography/homography.xhtml

Which of these **can** be done by a **2D linear** transformation?

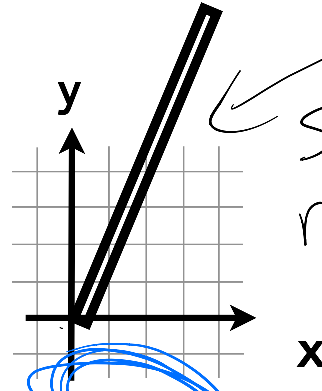
(2x2 matrix)



Input



A



B

Scale
rotate

socrative.com

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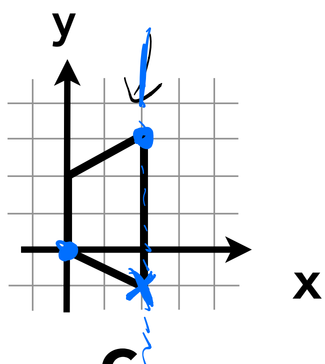
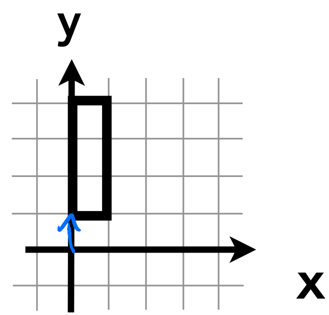


C



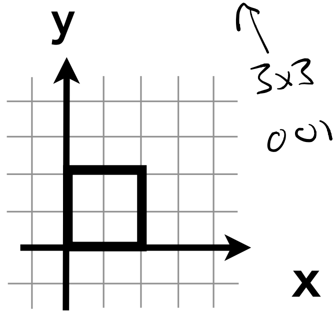
D

Which of these **can't** be done by a **2D affine** transformation?

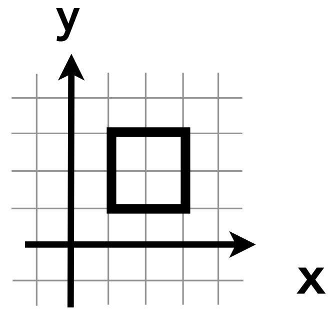
(2D homogeneous)

3x3

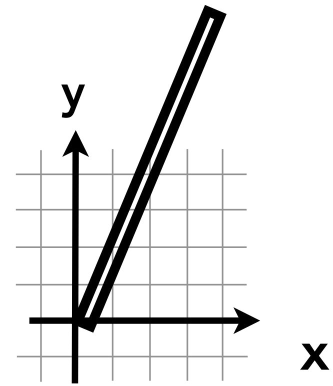
0 0 1



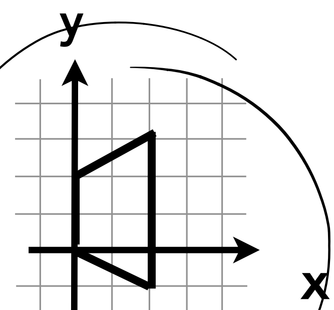
Input



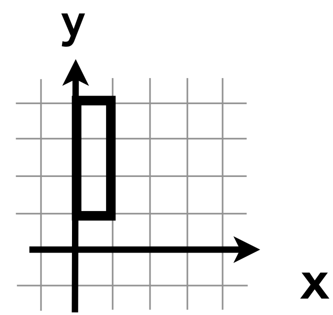
A



B



C



D

Transformations: Properties

- Same as matrix multiplication!
  - Associative $A(Bx) = (AB)x$
  - **Not** commutative $ABx \neq BAx$

$T_1 \to$ scale

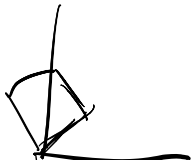
$T_2 \to$ rotation

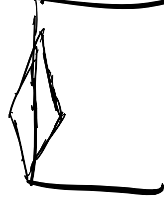
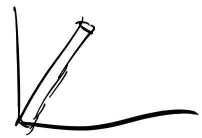
$$\underbrace{(T_2(T_1 \vec{x}))}_{} = (\overbrace{T_2 T_1}^{\text{scale and rotate}})\vec{x}$$

Rotation scale

$\downarrow \quad \downarrow$

$$T_1 T_2 \vec{x} \neq T_2 T_1 \vec{x}$$

$T_2 \vec{x}$  $T_1 x$ 

$T_1 T_2 \vec{x}$  $\neq$ $T_2 T_1 x$ 

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} & & \\ & T & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{x'}{w} \\ \frac{y'}{w} \\ 1 \end{bmatrix}$$