

Projective Geometry: Homogeneous Points

Recall: "math hack"

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \begin{bmatrix} X \\ Y \\ W \end{bmatrix}$$

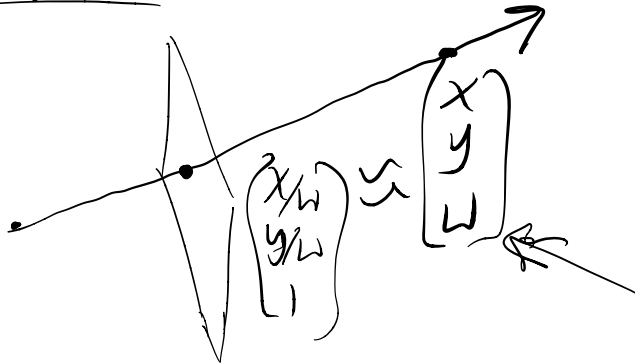
Enables linear-plus-translation

$\begin{bmatrix} X \\ Y \\ W \end{bmatrix}$ 2D homogeneous representing 2D pt:

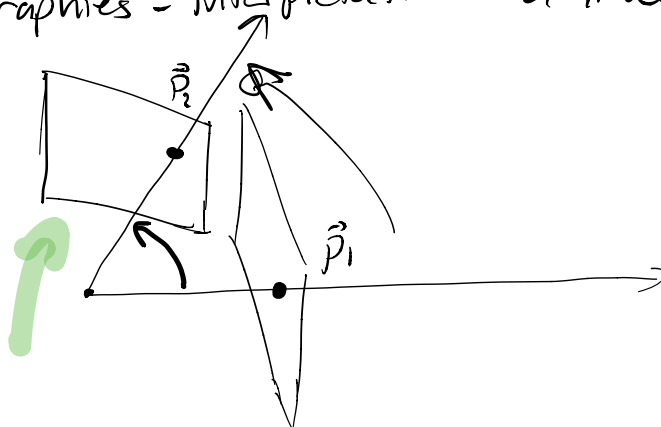
Formally, a 2D homogeneous pt is an object in \mathbb{P}^2 (projective space)
 \uparrow
 2D

$$\begin{bmatrix} x/w \\ y/w \end{bmatrix}$$

A geometric interpretation: objects in \mathbb{P}^2 are the projection onto a plane of objects in \mathbb{R}^3 .



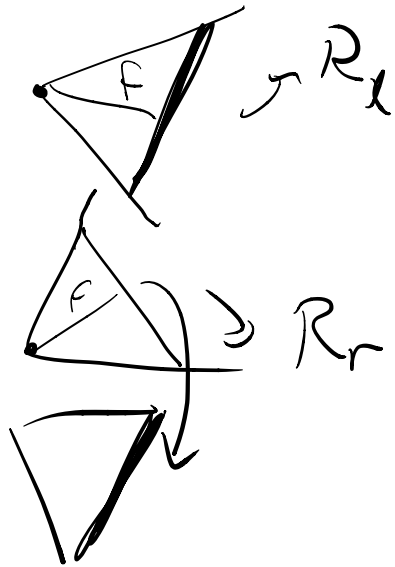
Homographies - interpretation: a linear transform on the \mathbb{R}^3 objects
 e.g. image plane!



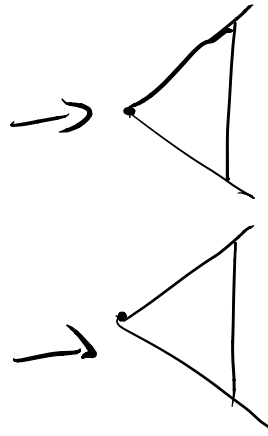
To project into a different (rotated) plane, apply a (3D) rotation.

Stereo Rectification

Have:



Want:



$$x_{\text{pixel}} = K x_{\text{cam}}$$

$$\vec{p}_2 = R \vec{p}_1$$

↑

$$q_{\text{pix}} = K_l R R_r^{-1} K_r^{-1} q_{\text{pix}}$$

↑ ↑ ↑

$$\left[K_l R R_r^{-1} K_r^{-1} \right]_{3 \times 3} \text{ aka } H$$