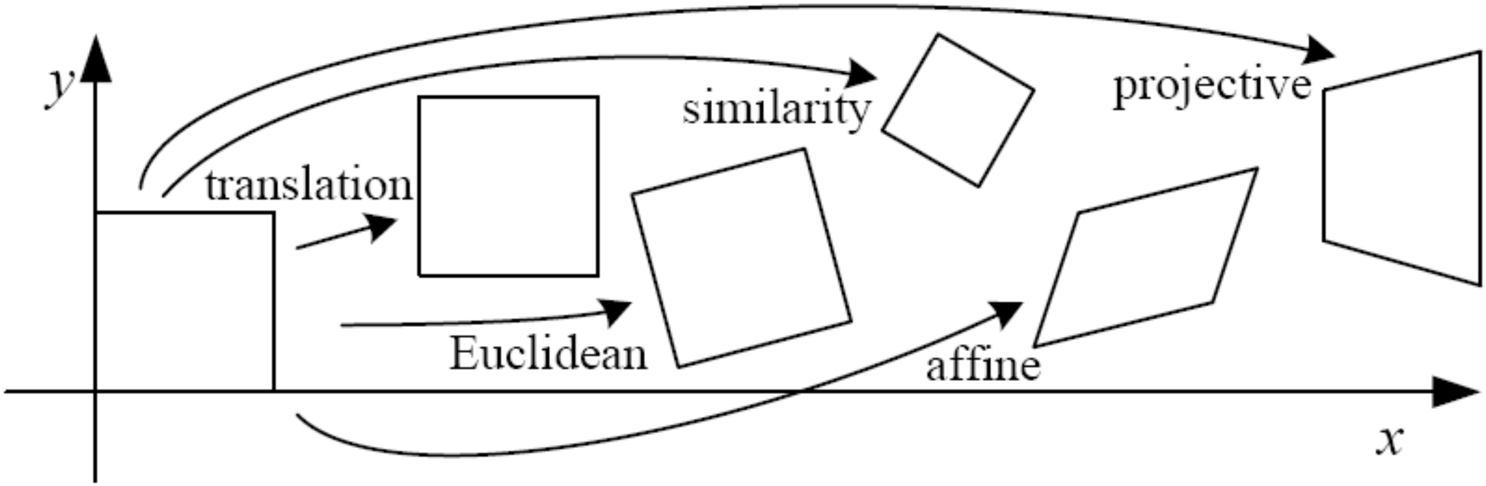
CSCI 497P/597P: Computer Vision



Lecture 12: Transformations

2D Linear and Affine Transformations

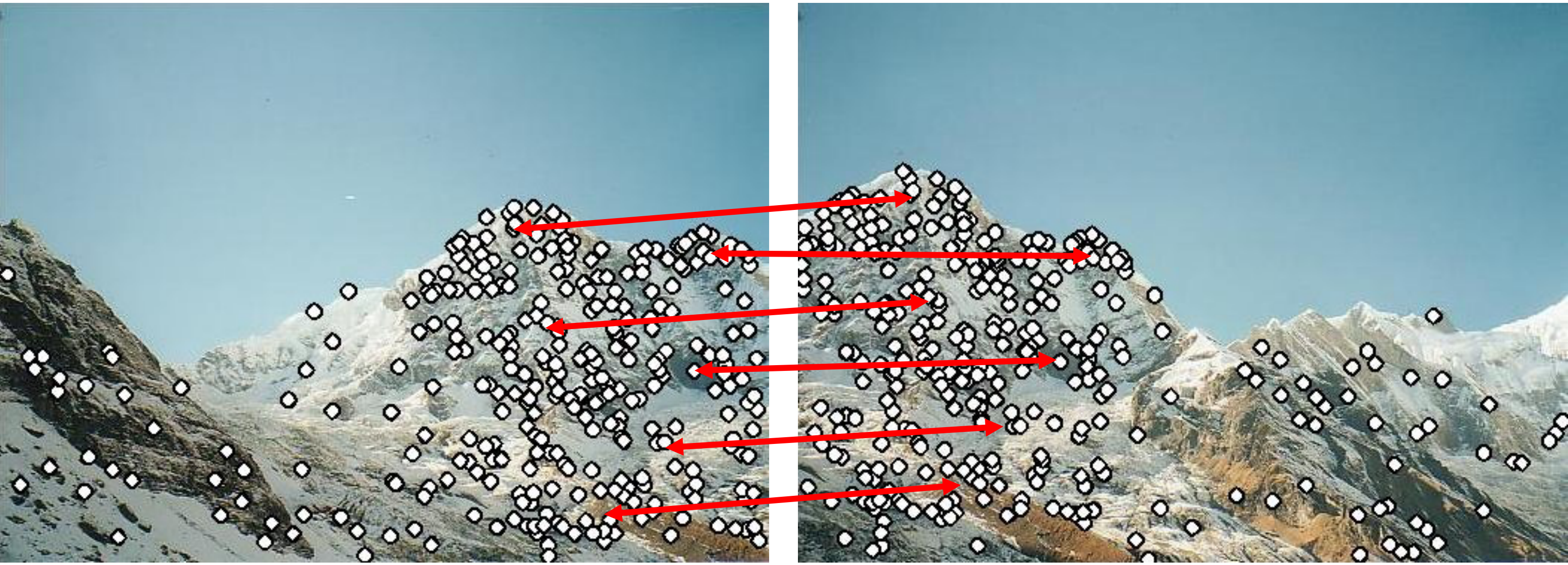
Announcements

- To use slip days, send me email **after** you've submitted late.

Goals

- Know what is possible with 2D linear transformations: (scale, shear, rotation)
- Understand the motivation and math behind homogeneous coordinates.
- Know what is possible with 2D affine transformations: (all of the above, plus translation)

Running motivational example: Panorama Stitching



Where are we?

linear, affine, or projective transformations

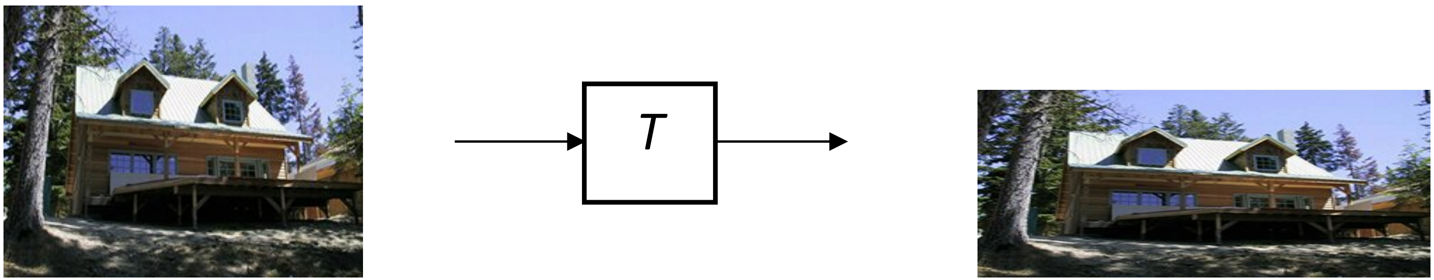
1. How do we describe the transformation?
2. How do we find an accurate transformation?
3. How do we actually warp the image?

- least squares
- RANSAC

inverse warping

Parametric (global) Warping

- Apply the same function to all coordinates.



p = (x,y) T transforms *image coordinates* **p'** = (x',y')

$$x', y' = T(x, y)$$

Self-imposed restriction: T is a matrix.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What can we do with this?

2x2 Matrices

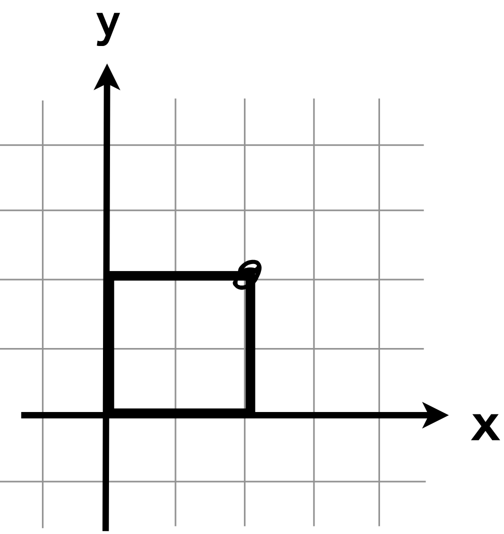
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} t_{11} & t_12 \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

A 2x2 matrix can represent all possible linear transformations on input coordinates.

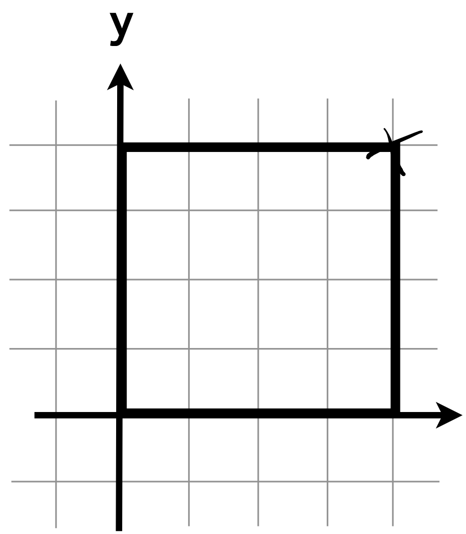
(each output coordinate = linear function of input coordinates)

$f(x, y) = ax + by$ ← linear fn

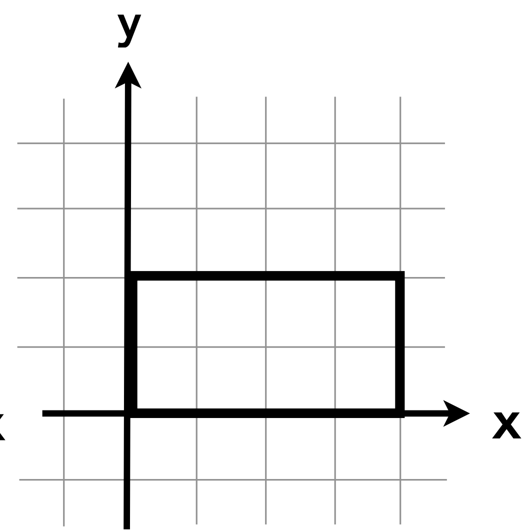
Scale



Input



Uniform Scale



Nonuniform Scale

$x' = 2x$

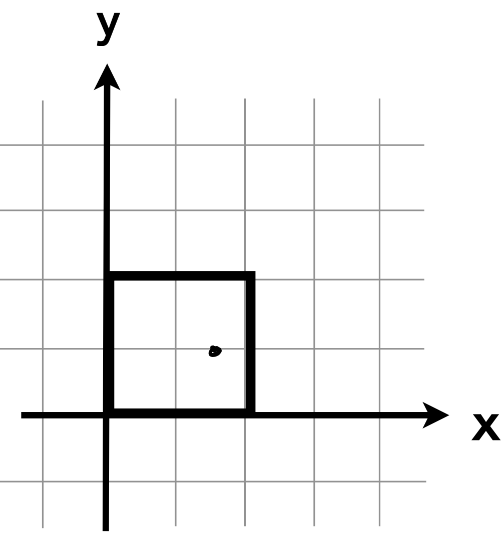
$y' = 2y$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

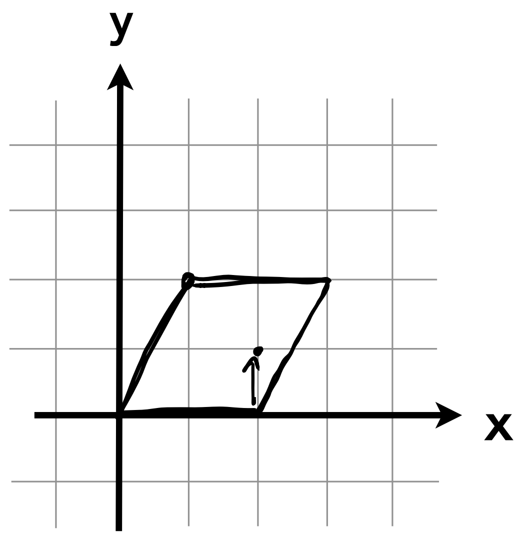
$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

↑ leave y unchanged

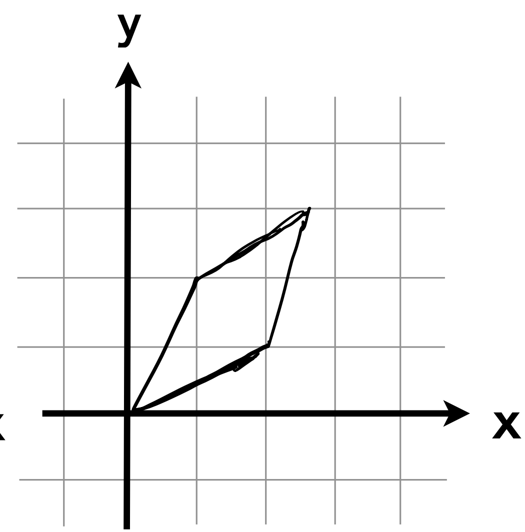
Shear



Input



Shear (x)



Shear (both)

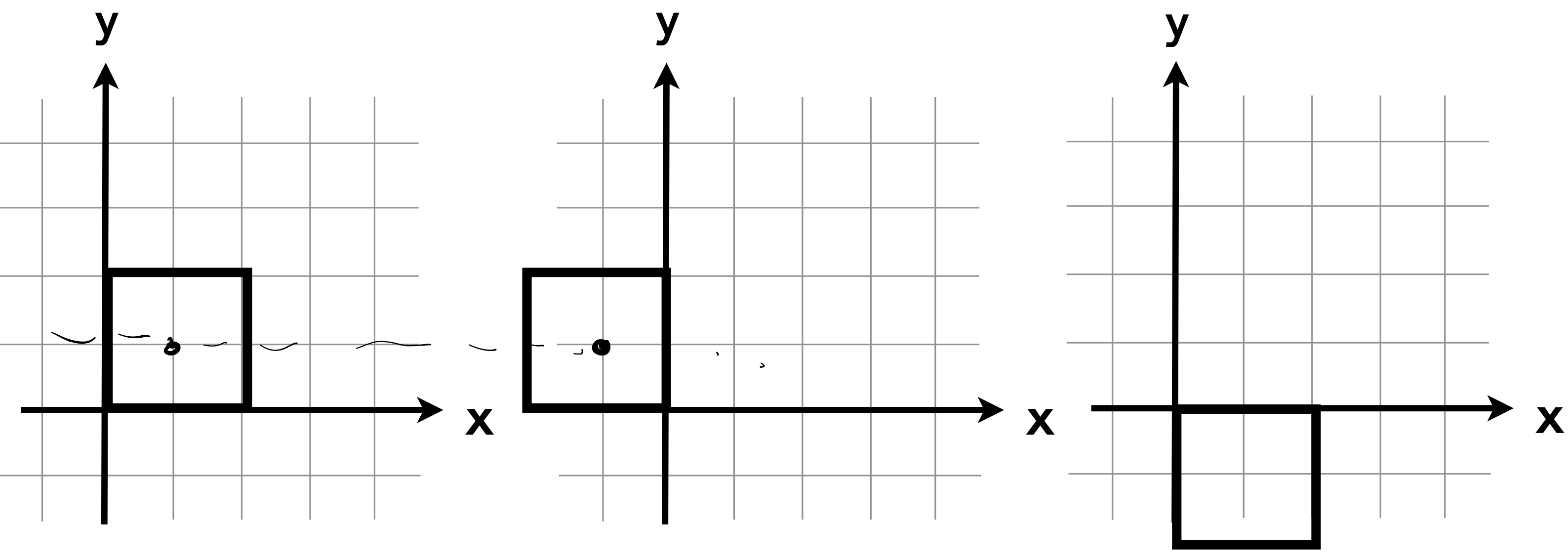
$$x' = x + 0.5y$$

$$y' = y$$

$$\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 1 & s_x \\ s_y & 1 \end{bmatrix}$$

Reflection



Input

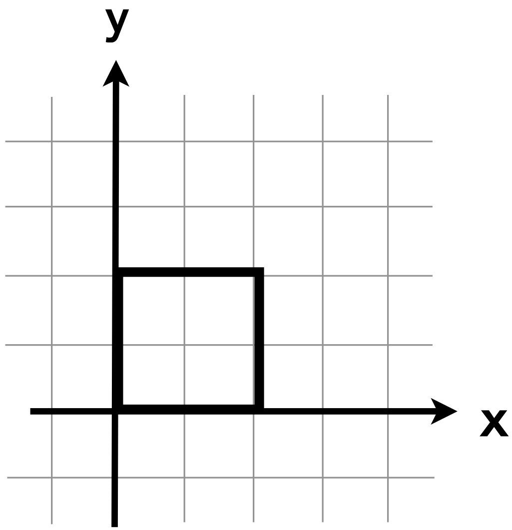
Reflection
(across y)

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

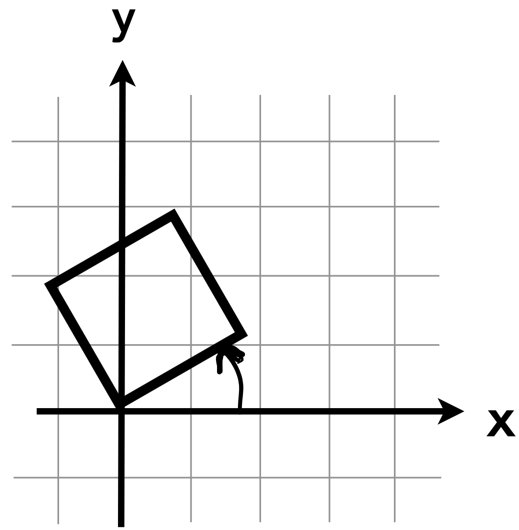
Reflection
(across x)

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Rotation



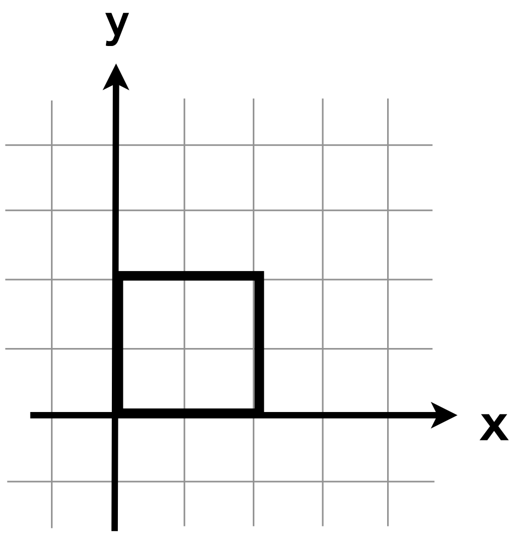
Input



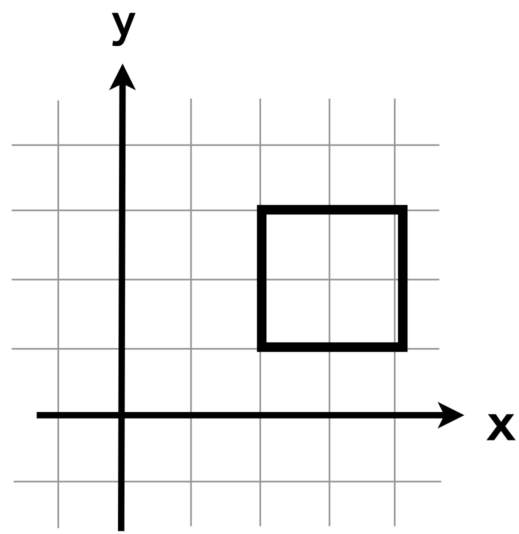
Rotation about the origin

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Translation



Input

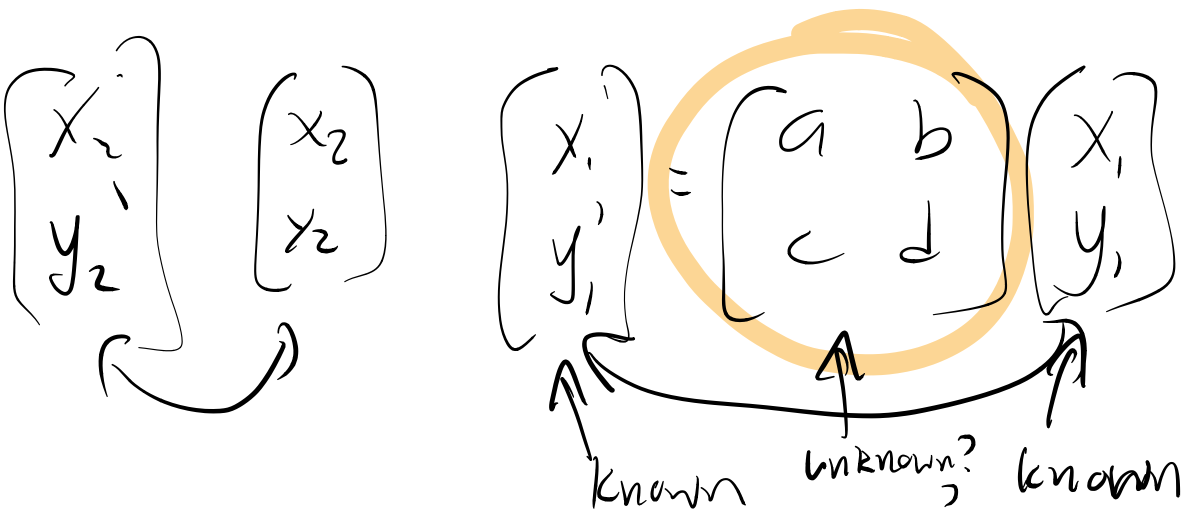


Translation by (u, v)

Today's Problems

1. What's the fewest correspondences you could use to (unambiguously) find a 2x2 linear transformation T?

2. Come up with a matrix that represents a *translation*.



Matrices can't translate.

We'll use a clever math hack to make them do it anyway:

Homogeneous Coordinates

Use a **3D** vector to represent a **2D** point.
Always put a **1** in the third dimension.

Matrices can't translate.

We'll use a clever math hack to make them do it anyway:

Homogeneous Coordinates

Use a **3D** vector to represent a **2D** point.

Always put a **1** in the third dimension.

3x3 ??

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} & & \\ & T & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Matrices can't translate.

We'll use a clever math hack to make them do it anyway:

Homogeneous Coordinates

Use a **3D** vector to represent a **2D** point.
Always put a **1** in the third dimension.
How do we transform these?

linear translation

$$\begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{22} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} t_{11}x + t_{12}y + t_{13} \\ t_{21}x + t_{22}y + t_{23} \\ 1 \end{bmatrix}$$

Interactive Demo

- https://iis.uibk.ac.at/public/piater/courses/demos/homography/homography.xhtml

Affine Transformations

The transformations possible with a 3x3 matrix like this

$$\begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

are called affine transformations.

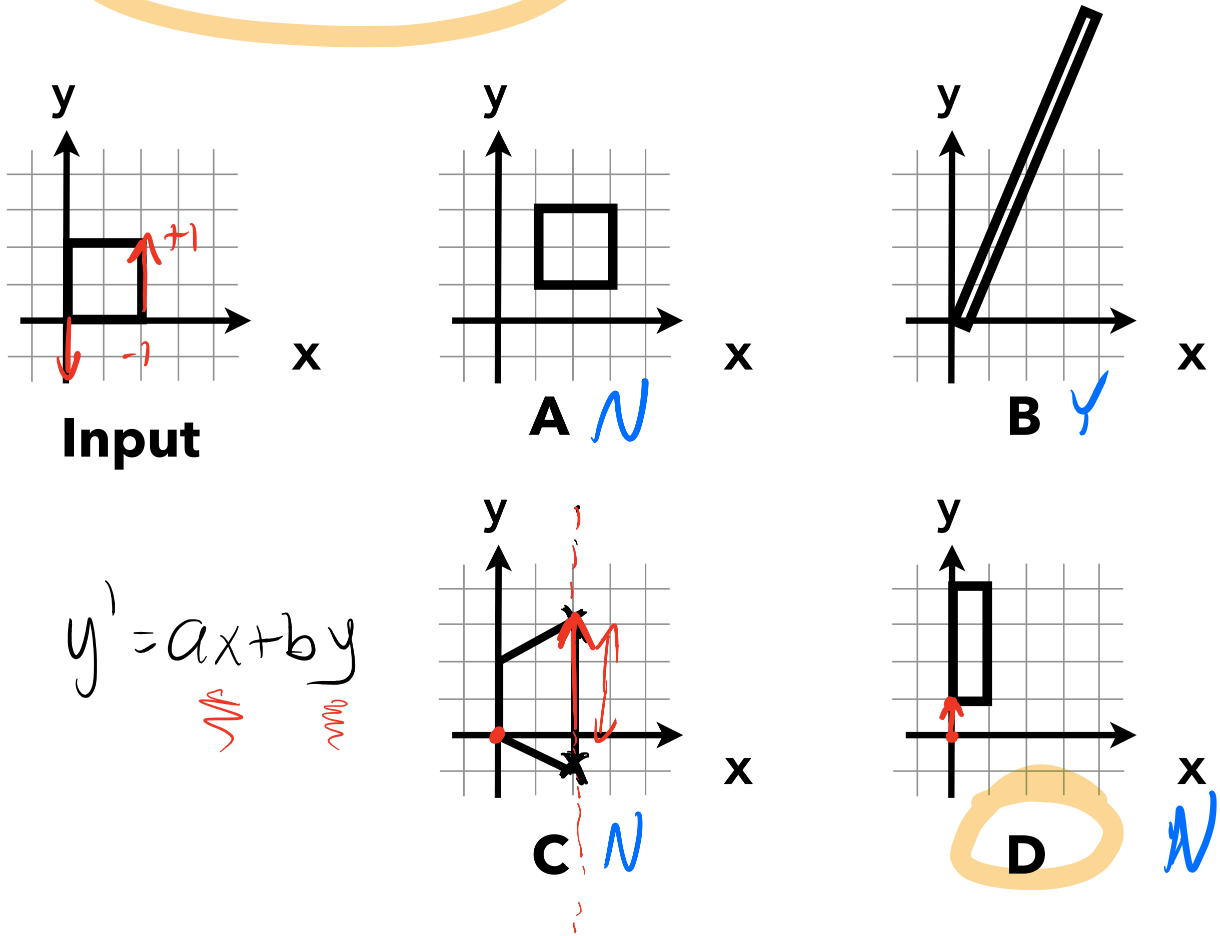
each input coordinate is an affine function of the input coordinates

Affine basically means linear plus shift:

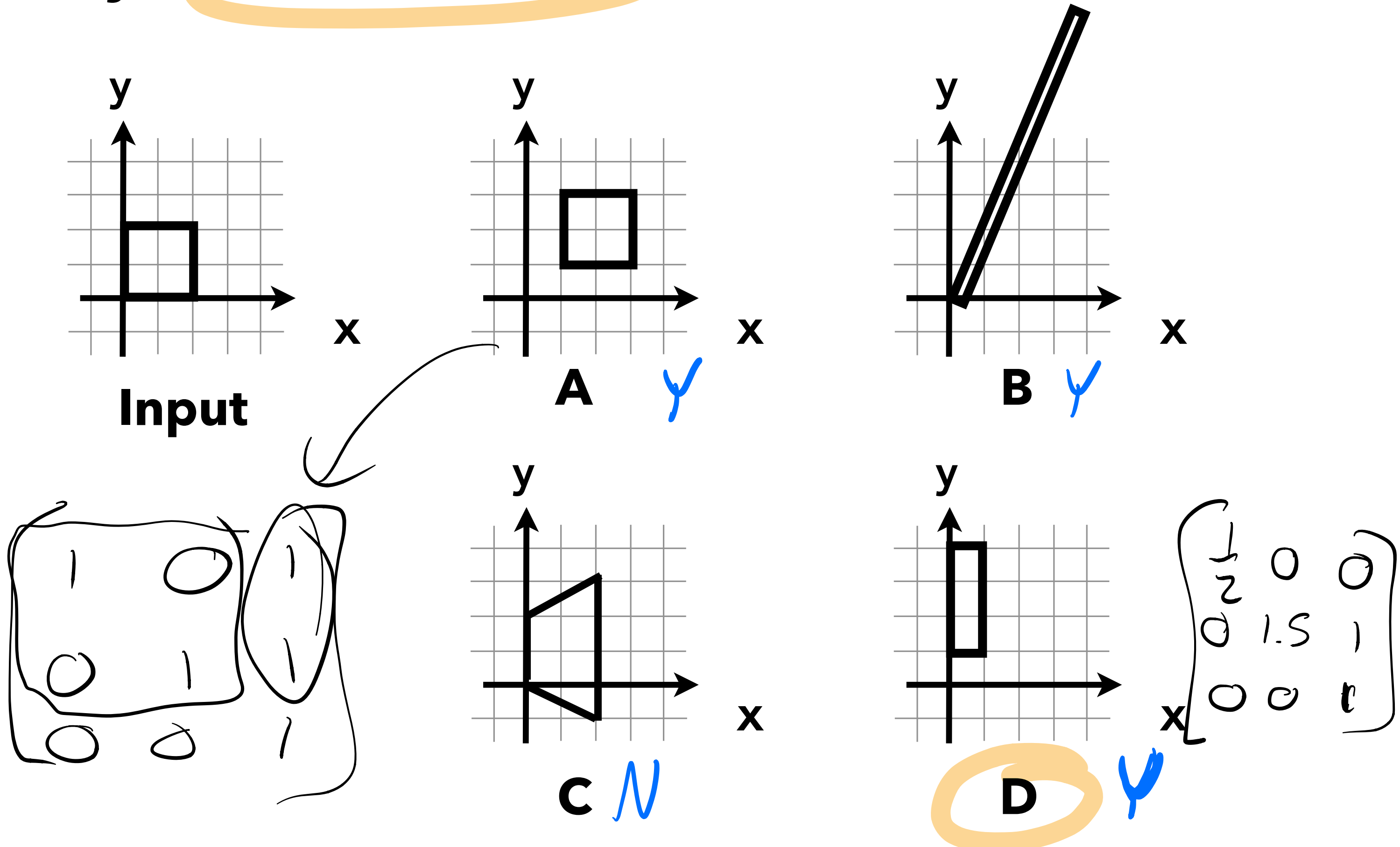
$f(x, y) = ax + by$ is linear

$f(x, y) = ax + by + c$ is affine

Which of these **can** be done by a **2D linear** transformation?



Which of these **can't** be done by a **2D affine** transformation?



Transformations: Properties

- Anything you get from matrix multiplication comes for free!
  - Associative! Composable!
  - **Not** commutative.

Linear Transformations: Properties

- Linear transformations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & 0 \\ d & e & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties:
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios of lengths along lines are preserved
  - Closed under composition
  - **Origin maps to origin**

linear

Affine Transformations: Properties

- Affine transformations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

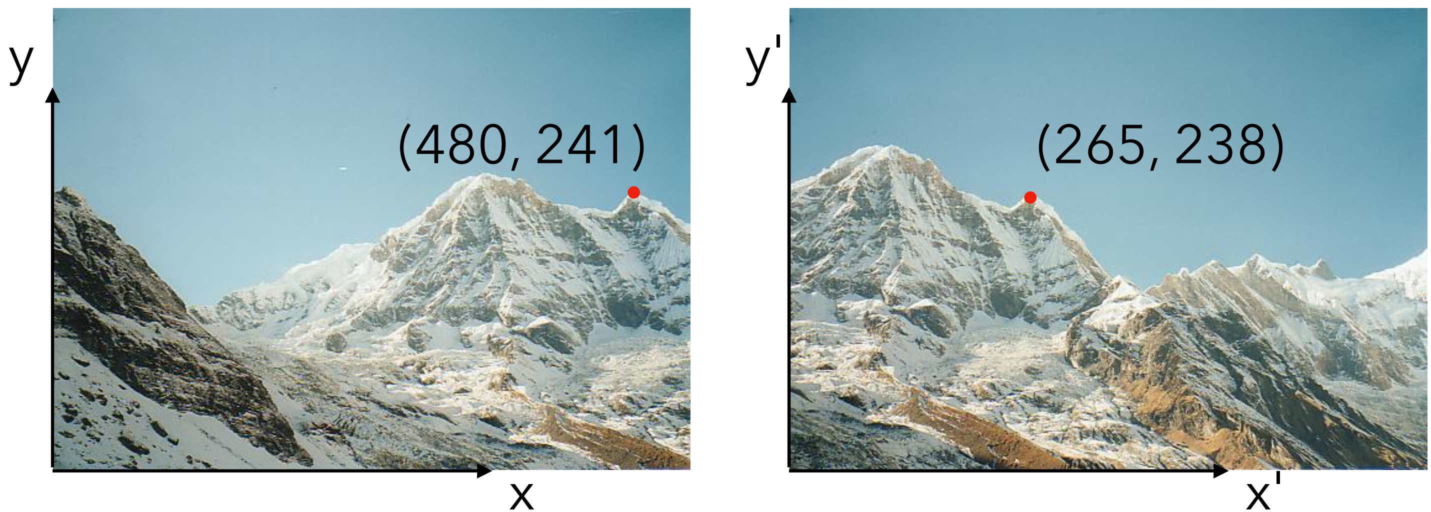
- Properties:
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios of lengths along lines are preserved
  - Closed under composition

  linear

  - Origin **does not** necessarily map to origin

  affine

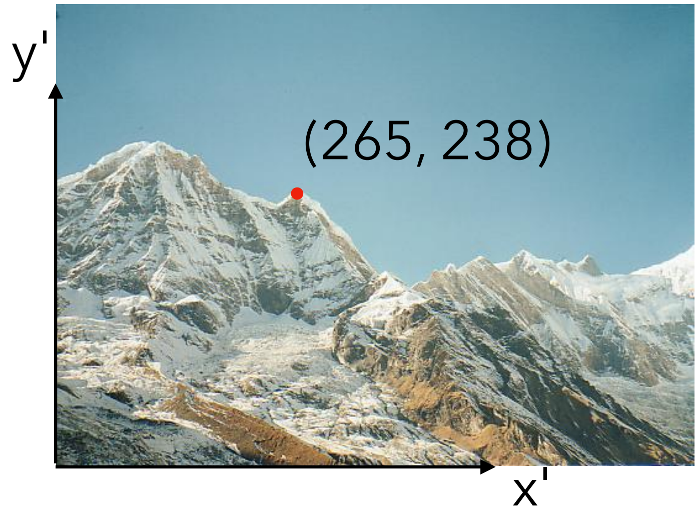
Warping



We've found correspondence. What's the transformation?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

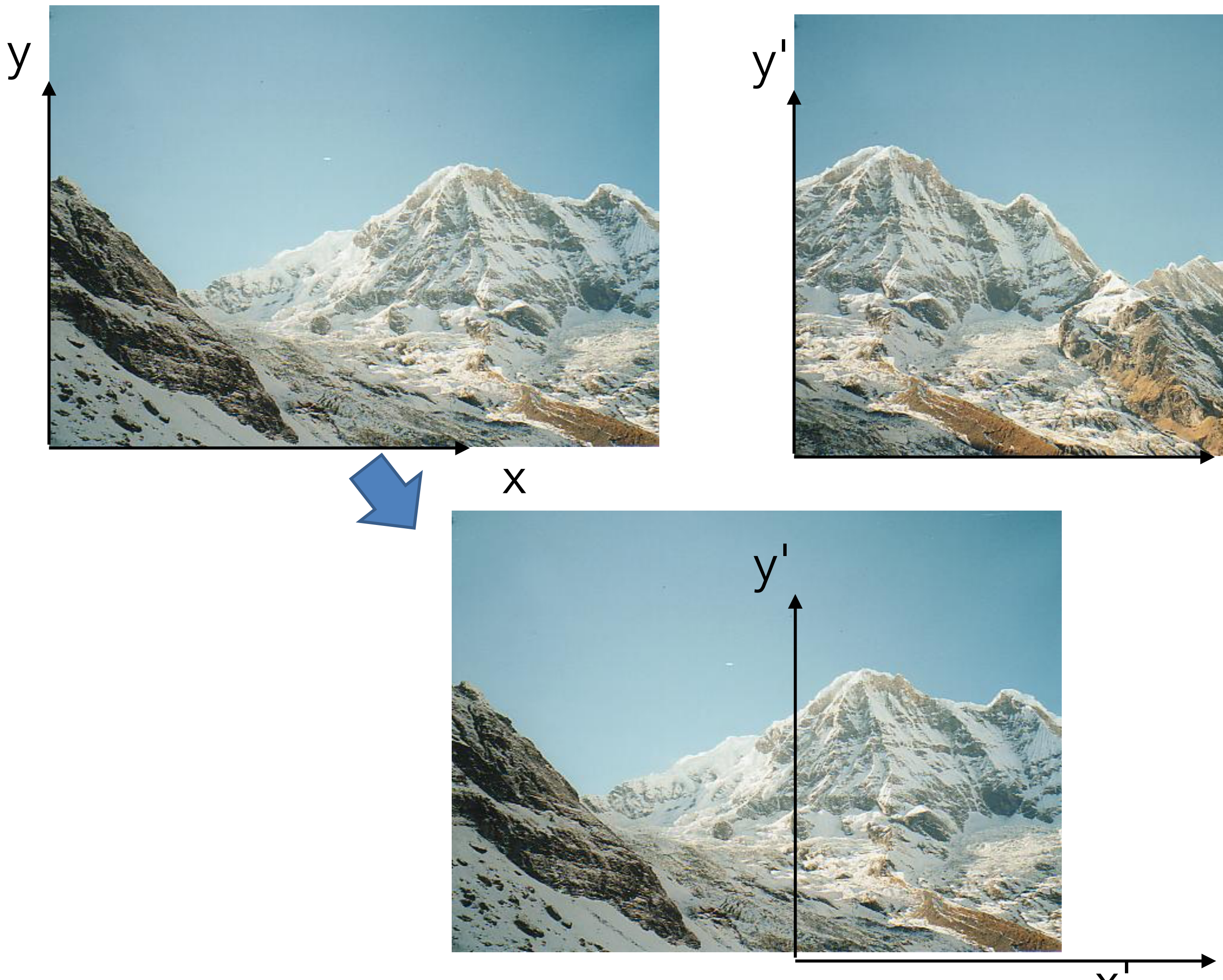
Warping



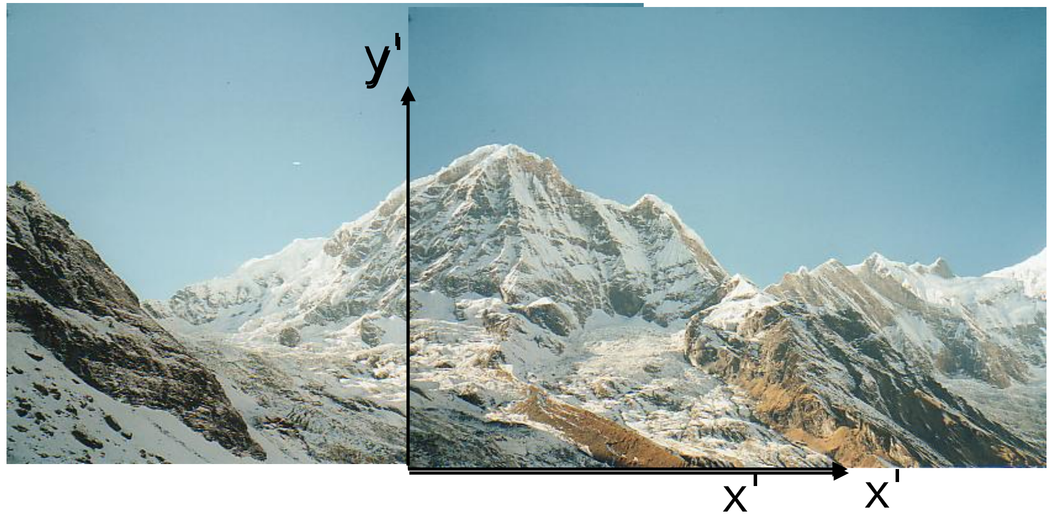
We've found the transformation. How do we warp the image?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -115 \\ 0 & 1 & -3 \\ 0 & 9 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

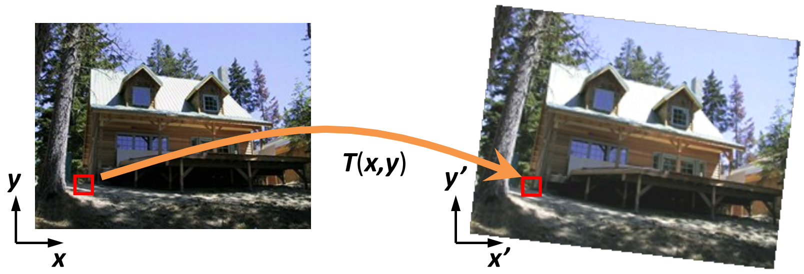
Warping



Warping



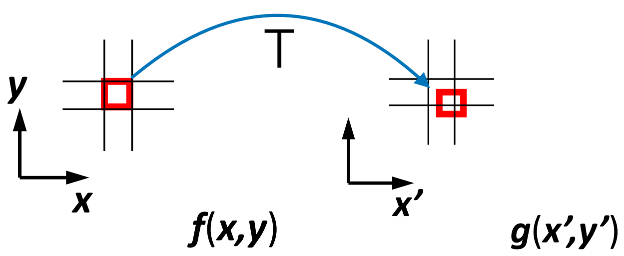
Forward Warping



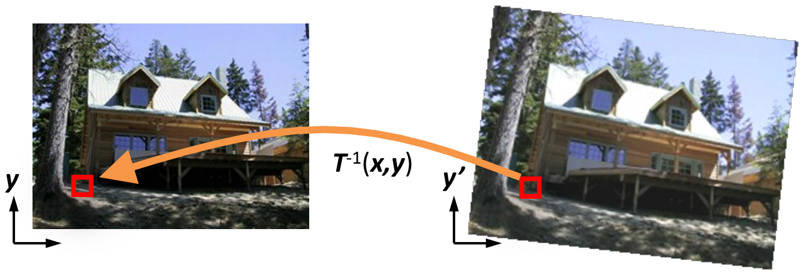
Warping

We've found the transformation. How do we warp the image?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -114.8 \\ 0 & 1 & -2.9 \\ 0 & 9 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



Inverse Warping



Bilinear Interpolation