Lecture 9:
Image Features: Harris Corner Detection
Announcements

• How's P1 going?
Goals

• Understand the derivation of the Harris corner detector.

• Know how to efficiently compute the Harris score at every pixel in an image.

• Understand the concept of invariance as pertains to the Harris corner detectors.
if you **nudge** the patch, how much does its appearance change?

Not at all in any direction?  
A lot in some directions?  
A lot in **all** directions?

If the **smallest** \( E(u,v) \) is large,  
then **W** is a corner.  
**Corner score** = *minimum* \( E(u,v) \)
Mathily:

Start here: how different are W and W'?

\[ E(u, v) = \sum_{(x, y) \in W} \left[ I(x + u, y + v) - I(x, y) \right]^2 \]
Efficiently:
assume the image function is **locally linear**.

\[
I(x + u, y + v) \approx I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \text{H.O.T}
\]
Locally Linear Approximation

- $E(u, v)$ tells us how much the window changes in the direction $(u, v)$.

- Because of our approximations, this is a **linear** model in intensity, and a **quadratic model** in the error function.

1D intuition:

$$I(x+u)-I(x) = u \cdot \text{slope}$$
What does this mean?

• $E(u, v)$ tells us how much the window changes in the direction $(u, v)$.
  
  • Because of our approximations, this is a linear model in intensity, and a quadratic model in the error function.

(Demo)
Plug the approximation into $E$...

...and do some good ol' algebraic manipulation

Error function:

$$E(u, v) = \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

Linear image approximation:

$$I(x + u, y + v) \approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{H.O.T}$$
\[ E(u,v) = \sum_{x,y \in W} \begin{bmatrix} u \\ v \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \]

\[ I_x = \frac{\partial I}{\partial x} \]

\[ I_y = \frac{\partial I}{\partial y} \]
...rewrite in matrix form:

\[ E(u, v) = (u, v) \begin{bmatrix} \epsilon \iota_{xx} & \epsilon \iota_{xy} \\ \iota_{yx} & \epsilon \iota_{yy} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \]
What is this matrix thing

The four numbers that determine the shape of our (approximate) quadratic error surface.
\[ E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \]

One scenario:

\[ A = \sum_{(x,y) \in W} I_x^2 \]
\[ B = \sum_{(x,y) \in W} I_x I_y \]
\[ C = \sum_{(x,y) \in W} I_y^2 \]

\[ H = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix} \]
One scenario:

\[ E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \]

Structure tensor

\[ \sqrt{\ \ } \]

\[ A = \sum_{(x, y) \in W} I_x^2 \]

\[ B = \sum_{(x, y) \in W} I_x I_y \]

\[ C = \sum_{(x, y) \in W} I_y^2 \]

\[ H = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \]
In general...

Direction of the fastest change: $\lambda_{\text{max}}^{-1/2}$

Direction of the slowest change: $\lambda_{\text{min}}^{-1/2}$
Eigenvectors!?

\[ Ax = \lambda x \]

\[ \min_x \| x^T A x \| \]

Scalar

\( (x, \lambda) \)

\[ \text{eigvec} \quad \text{eigval} \]
Corner Detection: Upshot

- The **smaller** eigenvalue of $H$ is **large** when the patch is centered on a corner.
Smallest eigenvalue
Thresholded
Keep only Local Maxima
Resulting Corners
Harris Corners: TL;DM

- **Goal**: Find "unique" patches.
  Proxy for uniqueness: not like neighboring patches

- **Approach**: Eigenanalysis of SSD error on locally-linearized image windows.

- **Upshot**: The smaller eigenvalue of this 2x2 matrix indicates cornerishness:

\[
H = \begin{bmatrix}
\sum_{(x,y) \in W} I_x^2 & \sum_{(x,y) \in W} I_x I_y \\
\sum_{(x,y) \in W} I_x I_y & \sum_{(x,y) \in W} I_y^2
\end{bmatrix}
\]
Harris Corners: TL;DM

Algorithm:

1. **Use Sobel filter to estimate gradients** $I_x, I_y$

2. **Compute** $I_x^2, I_y^2, I_x I_y$

3. **Filter each with a K x K mean filter**

4. **Approximate smallest eigenvalue as:**

   $$\frac{\det(H)}{\text{tr}(H)}$$

   $\det(H) = A*C - B*B$

   $\text{tr}(H) = A+C$
Corner Detection: Upshot

- The **smaller** eigenvalue of $H$ is **large** when the patch is centered on a corner.
Harris Corners: TL;DM

Algorithm:

1. Use Sobel filter to estimate gradients $I_x, I_y$
2. Compute $I_x^2, I_y^2, I_xI_y$
3. Filter each with a K x K mean filter
4. Approximate smallest eigenvalue as: $\frac{\text{det}(H)}{\text{tr}(H)}$ (Harris score)
5. Threshold
6. Maximum filter
Two desirable properties:

- **Uniqueness**: features *shouldn't* match if they're from different points in the scene.

- **Invariance**: features *should* match if they do come from the same point in the scene.
Invariance
Invariance: Hard mode
Invariance: Mars Mode
Invariance

• Suppose we're comparing two images of the same scene. What kinds of transformations could relate the two images if:
  
  • They are part of a panorama sequence?
  
  • They were taken at different times of day?
  
  • They were taken by different cameras?
  
  • They were taken from different viewpoints?