Lecture 3: Convolution and its Properties
CSCI 497P/597P: Computer Vision

Lecture 3: Convolution and its Properties
Announcements

• HW1 is out today
  • Convolution filtering and convolution
  • Due in 1 week (10pm next Tuesday)
Goals

• Understand the distinction between cross-correlation and convolution.

• Know the properties of cross-correlation and convolution:
  • Linearity and shift-invariance (both)
  • Associativity and commutativity (convolution only)

• Understand the design of several common image filters:
  • Box blur and Gaussian blur
  • Sharpening

• Understand the limitations of linear filtering
Computing Cross-Correlation

\[ g = f \otimes w \]

output image

input image

weights, or filter, or kernel

for \( x = 0 \) to \( w \):
  for \( y = 0 \) to \( h \):
    for \( i \) in \(-k\) to \( k\):
      for \( j \) in \(-k\) to \( k\):
        \( \text{out}[x,y] += w[i,j] \times \text{in}[x+i, y+j] \)
Computing Cross-Correlation

\[ g = f \otimes w \]

output image

input image

weights, or
filter, or
kernel

for \( x = 0 \) to \( w \):
  for \( y = 0 \) to \( h \):
    for \( i \) in \( -k \) to \( k \):
      for \( j \) in \( -k \) to \( k \):
        \( \text{out}[x,y] += w[i,j] \times \text{in}[x+i, y+j] \)
A bit of practice

In groups: work on Problems #1-4

• Problems are linked from the course webpage on the Schedule table

• Write answers in your Google Doc (pinned in group Discord channels)
Questions remain

- What happens at the edges?
- What properties does this operator have?
- What can and can't this operator do?
A shift filter

Cross-correlate the image $f$ with the kernel $w$.

Use "same" output size, with zero-padding for out-of-bounds values.
Cross-correlation vs Convolution

- Cross-correlation: \[ g = f \otimes w \]
  \[ g(x, y) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i, j) f(x + i, y + j) \]

- Convolution: \[ g = f \ast w \]
  \[ g(x, y) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i, j) f(x - i, y - j) \]

These are related: \[ f \ast w = f \otimes \text{flip horz}(\text{flip vert}(w)) \]
Properties

Assume: \( f \) is an image; \( w \) and \( v \) are filters; \( s, t \) are scalars.

**Shift invariance (both)**
\[
 f(x, y) \otimes w = [f(x-s, y-t) \otimes w](x-s, y-t)
\]

**Linearity (both)**
\[
 (f \otimes w) + (f \otimes v) = f \otimes (w + v) \\
 (f \otimes sw) = s(f \otimes w)
\]

**Commutativity (conv only)**
\[
 f \ast w = w \ast f
\]

**Associativity (conv only)**
\[
 (f \ast w) \ast v = f \ast (w \ast v)
\]
What can we do with this?
What can we do with this?
What can we do with this?

Identity filter: output = input
What can we do with this?
What can we do with this?

left shift
What can we do with this?
What can we do with this?

mean filter, or
box blur
Blurring: Another example

What motivated the mean filter?

Notice: lattice-like texture
Blurring: Another example

What motivated the mean filter?
Idea: the closer the pixel, the more likely it is to be similar

Notice: lattice-like texture
Gaussian Blur

- Idea: weight closer pixels more heavily using a Gaussian kernel:

This is a bivariate (2D) Gaussian function:

\[ G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}} \]
Gaussian Blur

- Idea: weight closer pixels more heavily using a Gaussian kernel:

This is a bivariate (2D) Gaussian function:

\[
G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}
\]
Gaussian Filters

$\sigma = 1$ pixel  
$\sigma = 5$ pixels  
$\sigma = 10$ pixels  
$\sigma = 30$ pixels
Mean vs. Gaussian
Composing Filters

- Recall associativity:

\[ G_\sigma \ast G_\sigma = G_{\sqrt{2}\sigma} \]
Sharpening!?

- What gets removed when we blur?

\[ f - f \ast B = g \]
Sharpening!? 

- What gets removed when we blur?
Sharpening

Before

After
Sharpening: once more with mathing

\[ f + (f - (f * B)) = \]

\[ 2f - f * B \]

\[ (f * 2I) - (f * B) \]

\[ f * (2I - B) \]
Sharpening: once more with mathing

\[ + \left( \begin{array}{c} \text{impulse} \\ \text{Laplacian of Gaussian} \end{array} \right) = \]
Sharpening: once more with mathing

\[ \begin{pmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{pmatrix} \approx \text{Possible 3x3 Approximation} \]
Effects of Sharpening

Original

Sharpened
Limitations: What can't convolution do?

Problem #5 - discuss in groups.

- Maximum filter?
- Threshold?
- $y$ partial derivative?