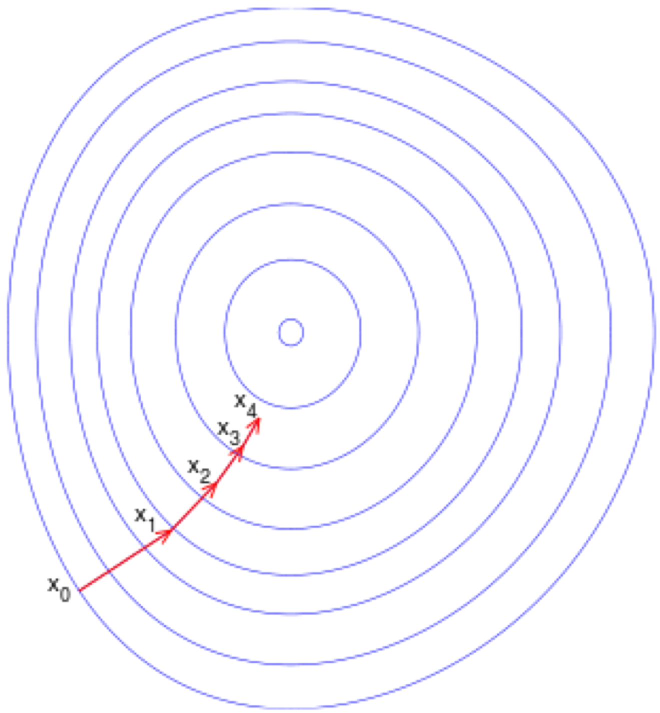
CSCI 497P/597P: Computer Vision

Scott Wehrwein

Softmax, Regularization, Gradient Descent



Reading

- http://cs231n.github.io/optimization-1/

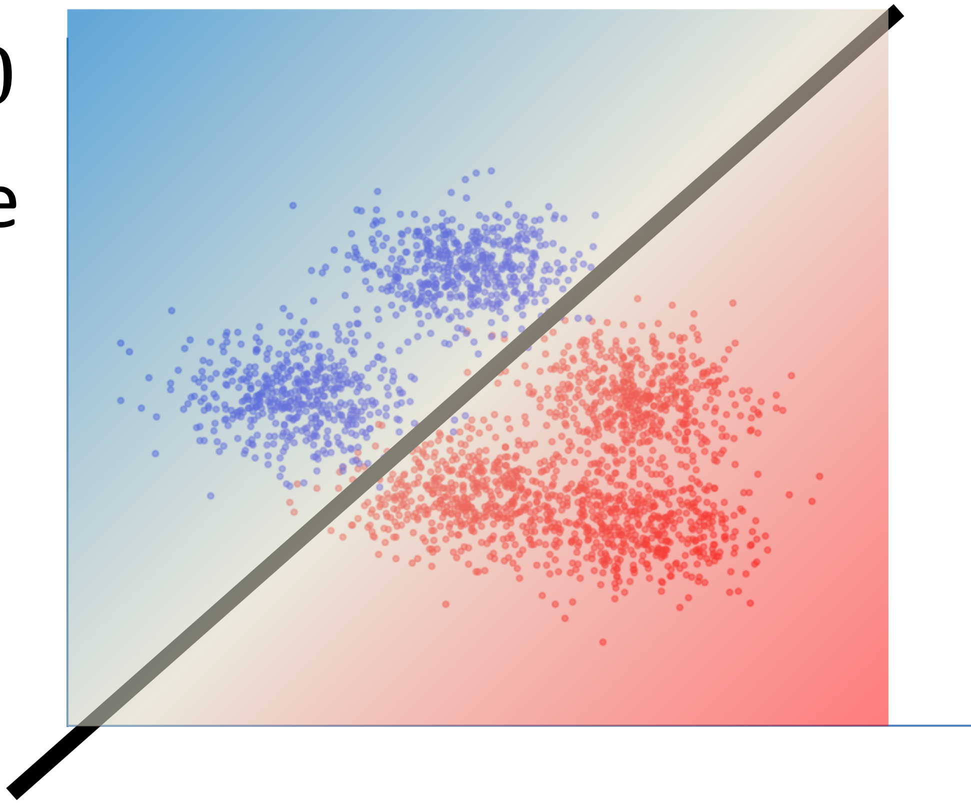
Announcements

Goals

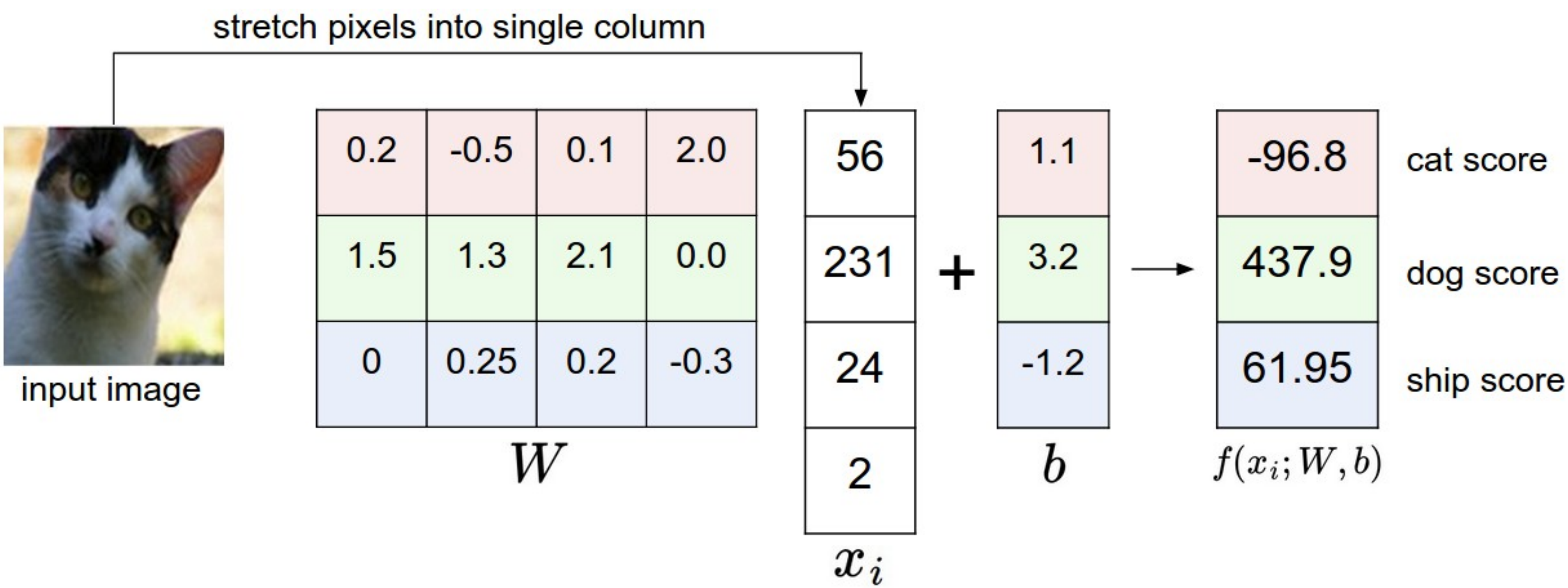
- Understand the intuition behind the softmax classifier with cross-entropy loss and its interpretation of scores as unnormalized log probabilities.
- Understand how to train a classifier by minimizing a loss function using gradient descent.
- Understand the intuition behind using Stochastic (Minibatch) Gradient Descent.

Linear classifiers

- Equation: $w^T x + b = 0$
- Points on the same side are the same class



Multiclass Linear Classifiers: Stack multiple w^T into a matrix.



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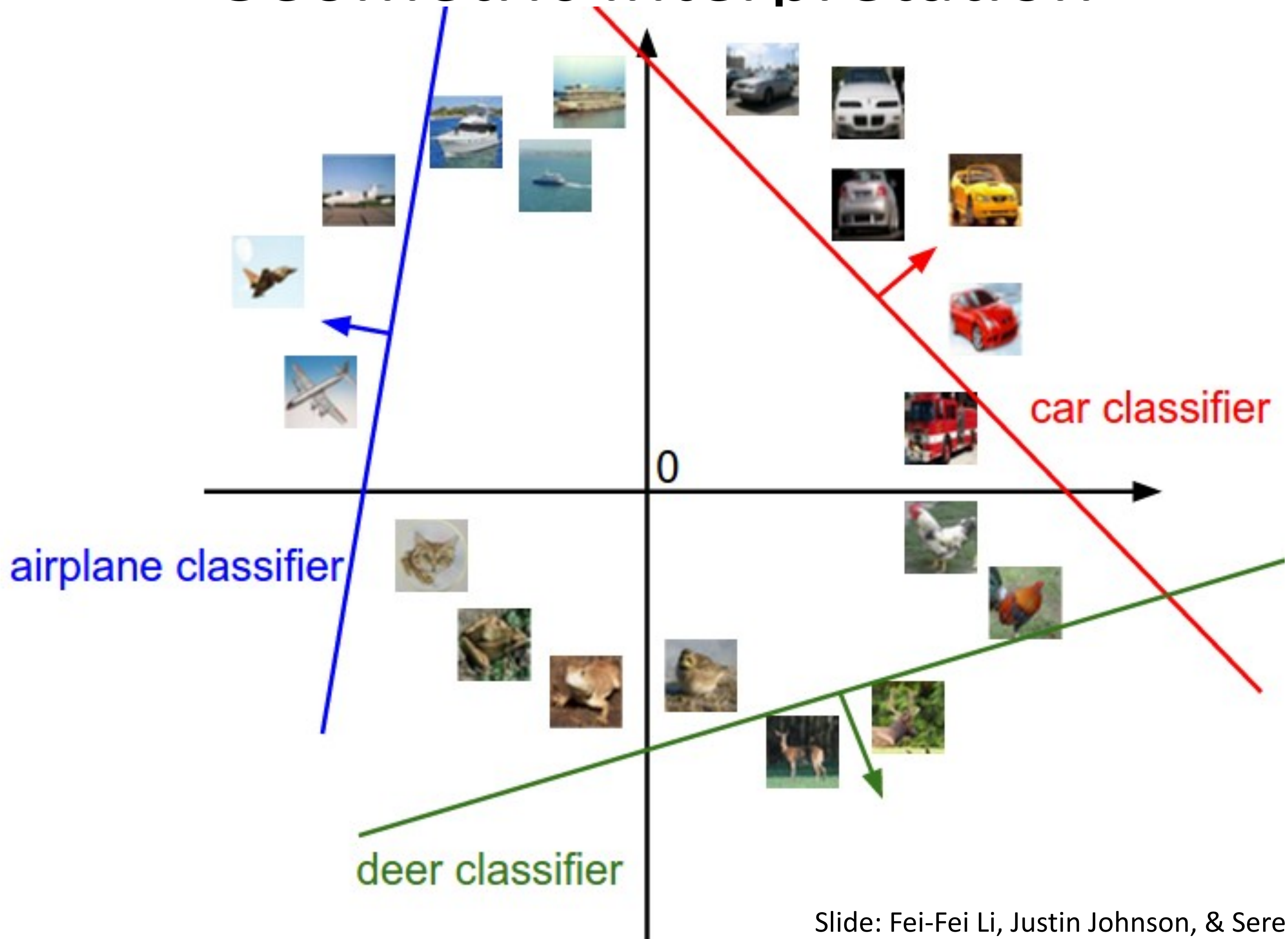
Multiclass Linear Classifier: Geometric Interpretation



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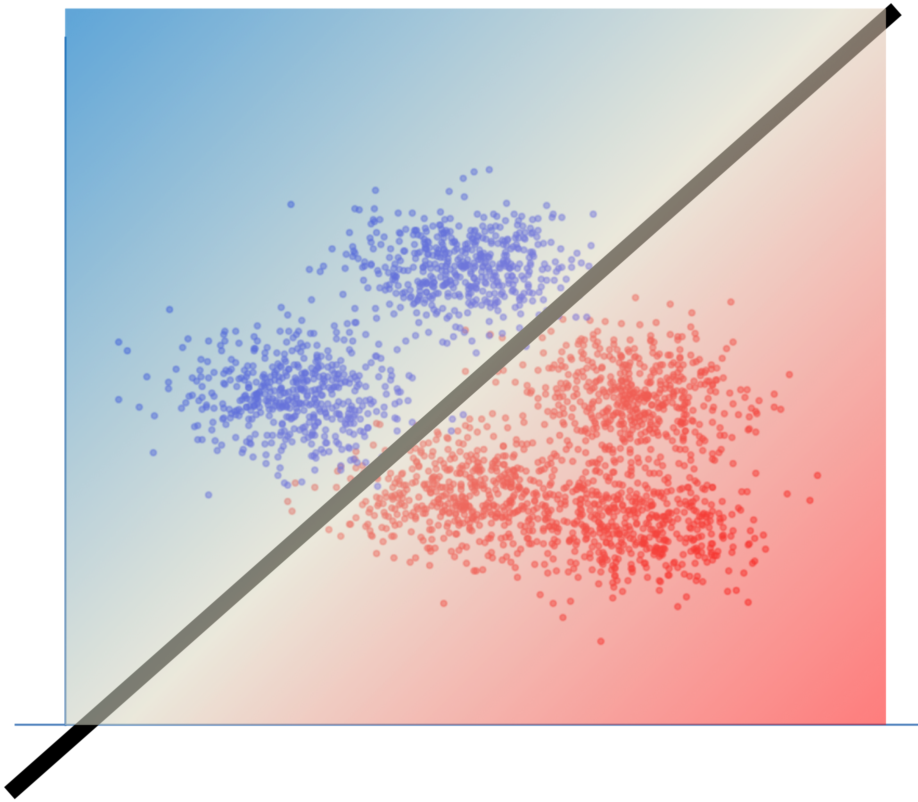
How do we find a good W, b?

- Step 1: For a given W, b, decide on a **Loss Function**: a measure of how much we dislike the line.
- Step 2: use **optimization** to find the W, b that *minimize* the loss function.



Loss Functions

- Step 1: For a given W, b, decide on a **Loss Function**: a measure of how much we dislike this classifier.
  - Last time: SVM loss (binary case)
  - Today: Softmax + cross-entropy loss
- Step 2: use **optimization** to find the W, b that *minimize* the loss function.
  - Today: gradient descent

Loss Functions

- Step 1: For a given W, b, decide on a **Loss Function**: a measure of how much we dislike this classifier.

- Loss Function intuition:
  - loss should be large if many data points are misclassified
  - loss should be small (0?) if all data is classified correctly.

Loss Functions – SVM Loss

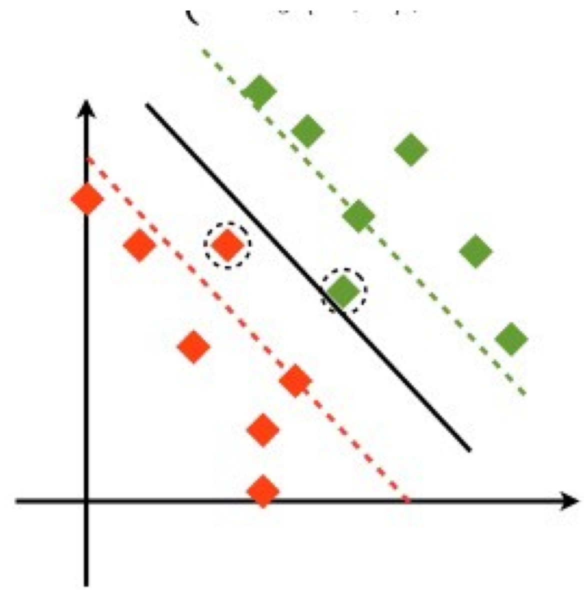
- SVM Loss:
  - Insists that data points are not just correctly classified, but a certain distance from the hyperplane:
  - $L_i = \max(0\ x_i, 1- y_i(w^T x_i + b)$

x_i = i'th data point

y_i = i'th data point's true label:

-1 if red

+1 if green



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Loss Functions – SVM Loss

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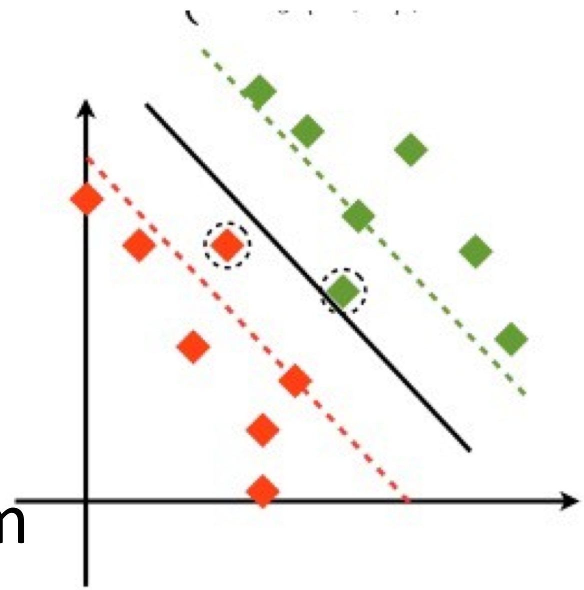
  - $L(w, b) = \Sigma_i L_i$
  - Loss for a given line is the sum of the loss for all datapoints



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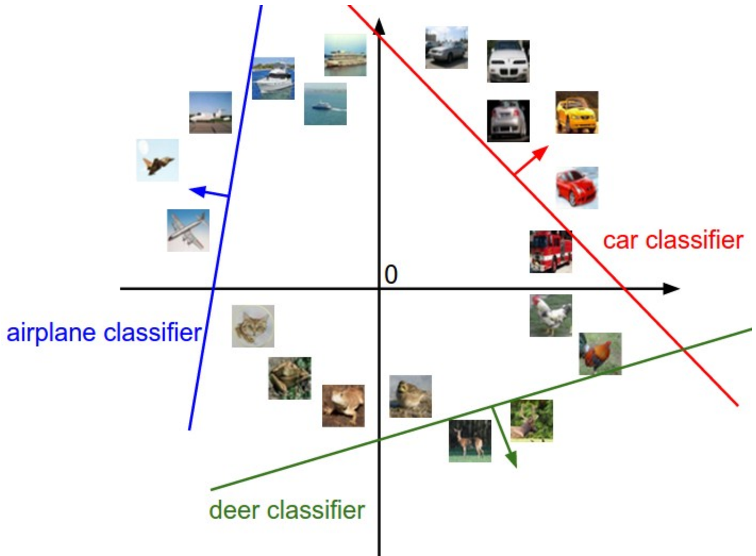
Loss Functions – SVM Loss

- SVM Loss – multiclass case:
  - Insists that data points are not just correctly classified, but correct the class score is a certain amount higher than every other class score:
  - Let f_j = the score for class j ($f_j = w_j^T x$)
  - $L_i = \Sigma_j \max(0, 1 + s_j - s_{yi})$

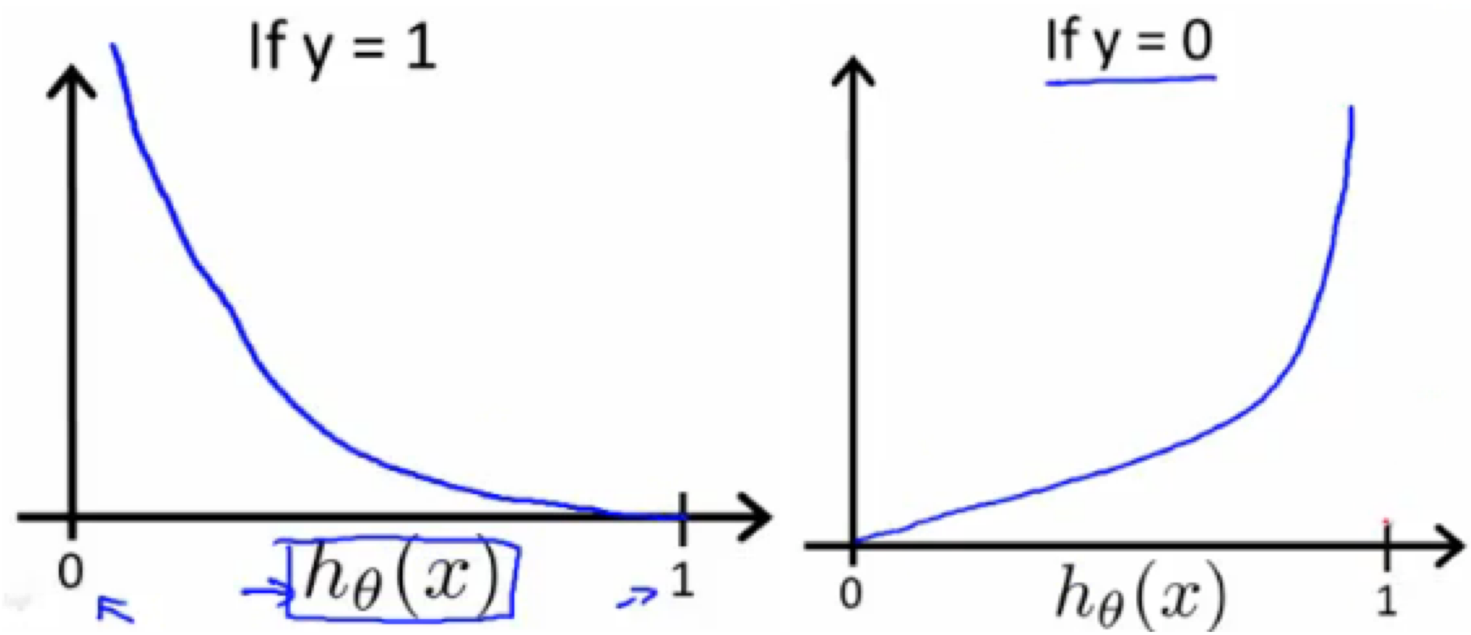


Softmax Classifier / Cross-Entropy Loss: Intuition

$W^T x + b$ gives us a vector of scores, one per class (each row of W is a classifier)

Wouldn't it be nice to interpret these as probabilities?

Binary Equivalent: Logistic Regression Loss



Softmax Classifier / Cross-Entropy Loss: Intuition

$W^T x + b$ gives us a vector of scores, one per class (each row of W is a classifier)

Wouldn’t it be nice to interpret these as probabilities?

They’re not:

- not always nonnegative
- don’t sum to 1

But we can treat them as **unnormalized log probabilities**.

Softmax Classifier / Cross-Entropy Loss: Intuition

f = W^T x gives us a vector of scores, one per class (each row of W is a classifier)

Softmax normalization: Exponentiate to get all positive values, then normalize to sum to 1:

$$p(x_i \text{ is class } k) = \frac{e^{f_k}}{\sum_j e^{f_j}}$$

Cross-entropy loss: measure *KL divergence* between the **predicted** distribution and the **true** distribution:

$$L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_j}}\right)$$

Regularization

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$

E.g. Suppose that we found a W such that L = 0.
Is this W unique?

No! 2W is also has L = 0!
Which do we prefer – W, or 2W?

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Regularization

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(f(x_i, W), y_i)$$

Data loss: Model predictions should match training data

Regularization

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(f(x_i, W), y_i) + \lambda R(W)$$

Data loss: Model predictions should match training data

Regularization: Prevent the model from doing *too* well on training data

Regularization

λ = regularization strength (hyperparameter)

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i) + \lambda R(W)$$

Data loss: Model predictions should match training data

Regularization: Prevent the model from doing *too* well on training data

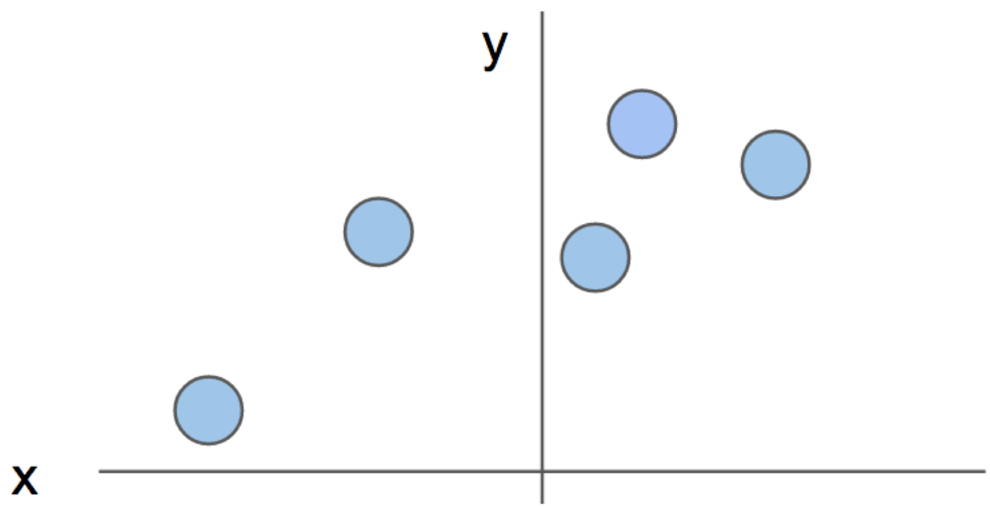
Simple examples

L2 regularization: $R(W) = \sum_k \sum_l W_{k,l}^2$

L1 regularization: $R(W) = \sum_k \sum_l |W_{k,l}|$

Elastic net (L1 + L2): $R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$

Regularization: Prefer Simpler Models



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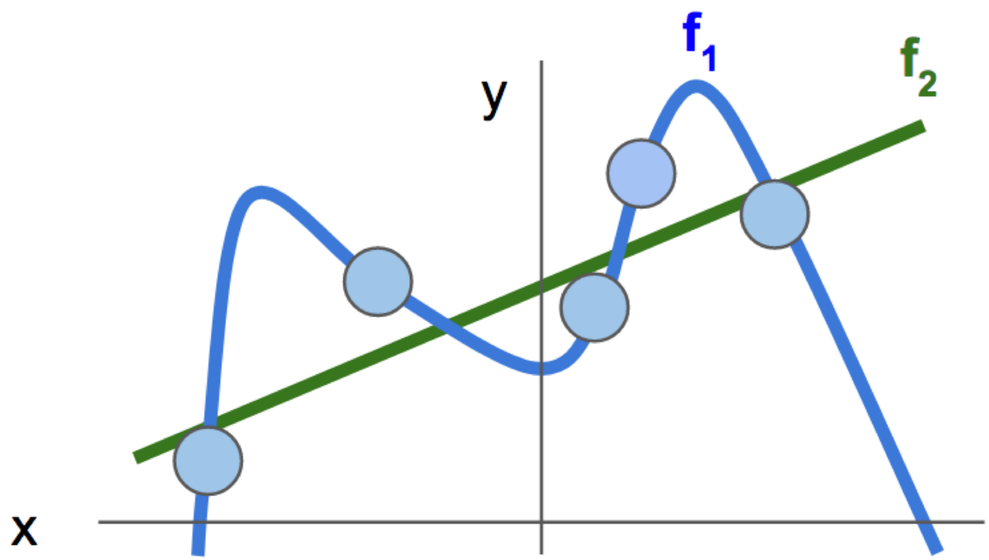
Regularization: Prefer Simpler Models



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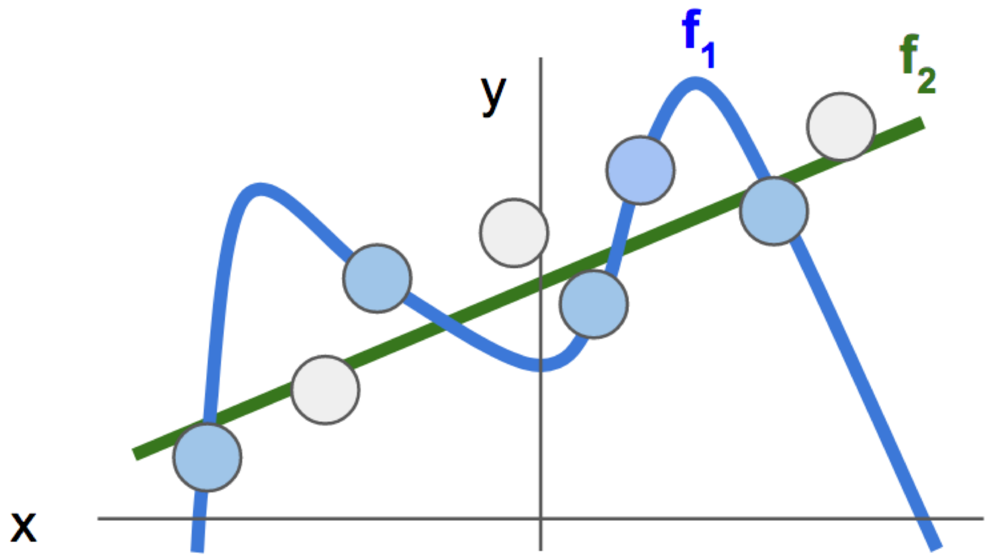
Regularization: Prefer Simpler Models



Regularization pushes against fitting the data *too* well so we don't fit noise in the data

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$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(f(x_i, W), y_i) + \lambda R(W)$$

Data loss: Model predictions should match training data

Regularization: Prevent the model from doing *too* well on training data

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Optimization

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How do we find a W that minimizes L?

- Bad idea: Random search.

```
# assume X_train is the data where each column is an example (e.g. 3073 x 50,000)
# assume Y_train are the labels (e.g. 1D array of 50,000)
# assume the function L evaluates the loss function

bestloss = float("inf") # Python assigns the highest possible float value
for num in xrange(1000):
  W = np.random.randn(10, 3073) * 0.0001 # generate random parameters
  loss = L(X_train, Y_train, W) # get the loss over the entire training set
  if loss < bestloss: # keep track of the best solution
    bestloss = loss
    bestW = W
  print 'in attempt %d the loss was %f, best %f' % (num, loss, bestloss)

# prints:
# in attempt 0 the loss was 9.401632, best 9.401632
# in attempt 1 the loss was 8.959668, best 8.959668
# in attempt 2 the loss was 9.044034, best 8.959668
# in attempt 3 the loss was 9.278948, best 8.959668
# in attempt 4 the loss was 8.857370, best 8.857370
# in attempt 5 the loss was 8.943151, best 8.857370
# in attempt 6 the loss was 8.605604, best 8.605604
# ... (truncated: continues for 1000 lines)
```

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How’d that go for you?

Lets see how well this works on the test set...

```
# Assume X_test is [3073 x 10000], Y_test [10000 x 1]
scores = Wbest.dot(Xte_cols) # 10 x 10000, the class scores for all test examples
# find the index with max score in each column (the predicted class)
Yte_predict = np.argmax(scores, axis = 0)
# and calculate accuracy (fraction of predictions that are correct)
np.mean(Yte_predict == Yte)
# returns 0.1555
```

15.5% accuracy! not bad!
(SOTA is ~95%)

Finding a W that minimizes L

- A better idea: walk downhill.

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Gradient Descent

```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

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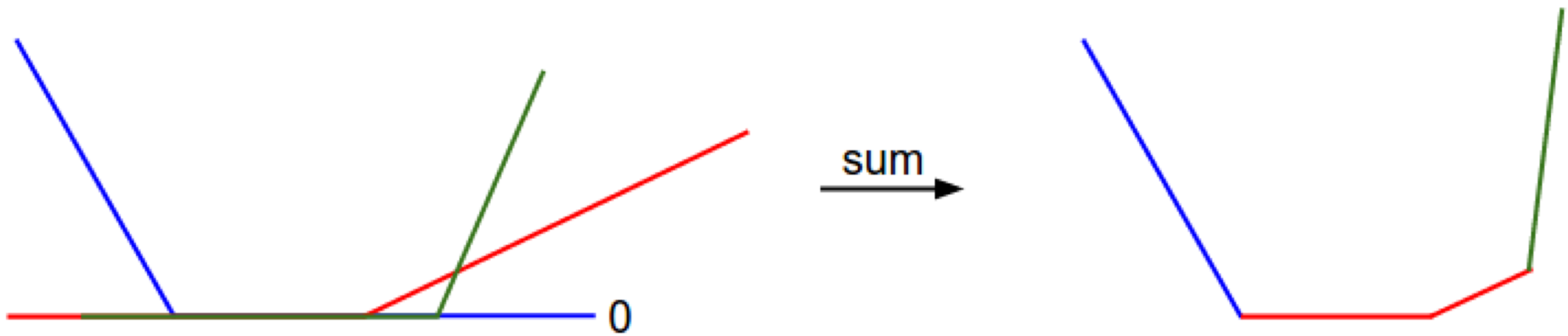
Gradient descent: SVM loss



$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + \Delta) \right]$$

$$\nabla_{w_{y_i}} L_i = -\left(\sum_{j \neq y_i} \mathbb{1}(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) \right) x_i$$

Gradient Descent

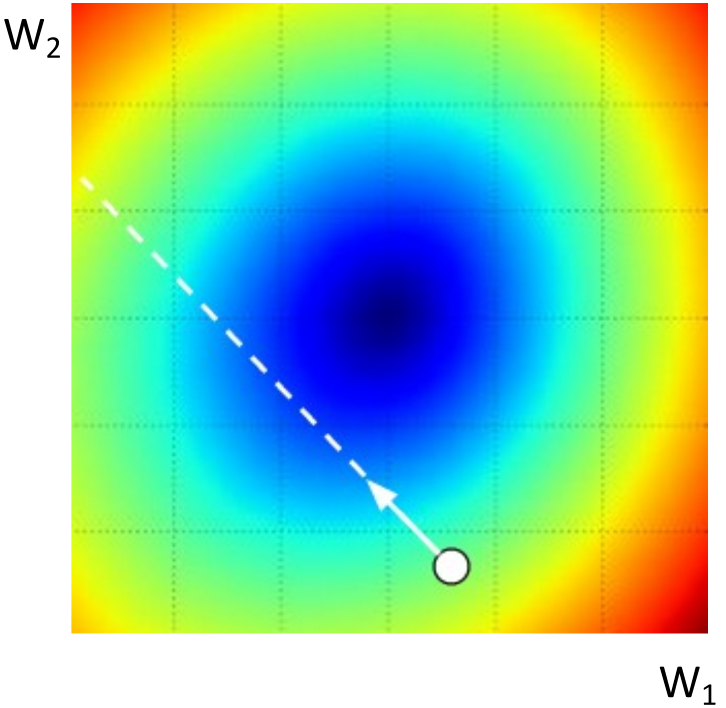
```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Gradient Descent: Generally

- Gradient of the loss function with respect to the *weights* tells us how to change the weights to improve the loss.

- L(X; W) depends on
  - All data points $x_1..x_n$
  - Very expensive to evaluate



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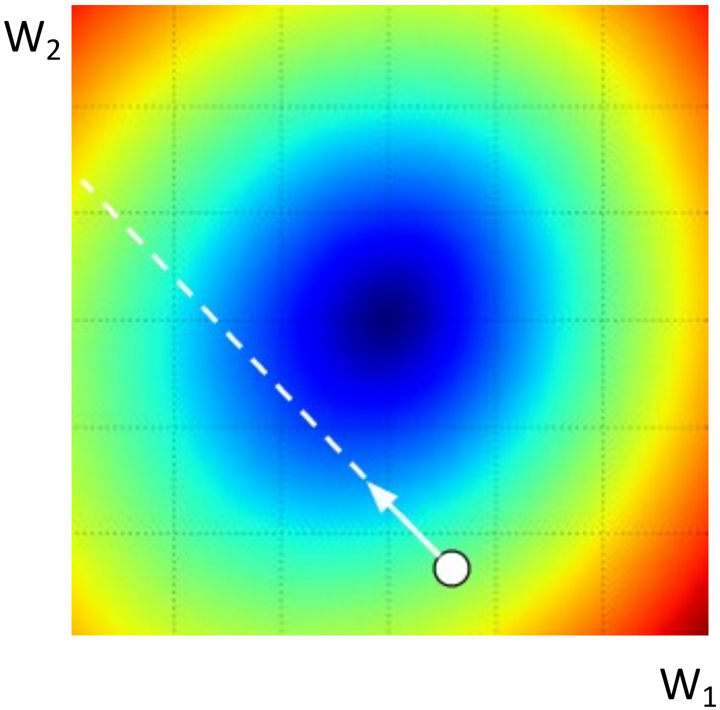

- L(X; W) depends on
  - All data points $x_1..x_n$
  - Very expensive to evaluate

Stochastic Gradient Descent

```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(f(x_i, W), y_i) + \lambda R(W)$$

- L(X; W) depends on
  - All data points $x_1..x_n$
  - Weights W
- Very expensive to evaluate if you have a lot of data.

Stochastic Gradient Descent

- Idea: consider only a few data points at a time.
- Loss is now computed using only a small batch (minibatch) of data points.
- Update weights the same way using the gradient of L wrt the weights.