# CSCI 497P/597P: Computer Vision

Scott Wehrwein

## Linear Classifiers

# Reading

- http://cs231n.github.io/classification/

# Announcements

- P4 out today. For real.

# Goals

- Understand the benefits and limitations of linear classifiers over KNN.
- Understand the tradeoff between complexity in the feature extractor vs. complexity in the classifier.
- Understand the mathematical formulation of a binary and multiclass linear classifier.
- Know the definition and purpose of a loss function
- Understand the intuition behind linear classifiers with:
  - SVM loss
  - Softmax loss

# Image classification - Multiclass classification

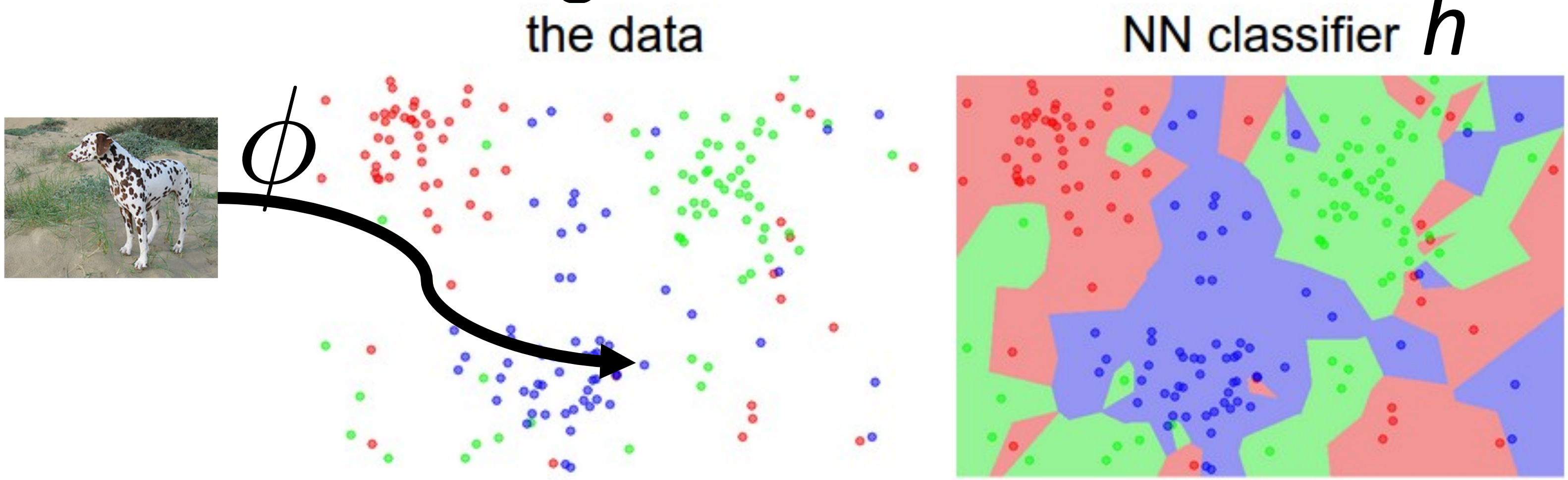
Which of these is it:
dog, cat or zebra?
Dog

# KNN: Bottom Line

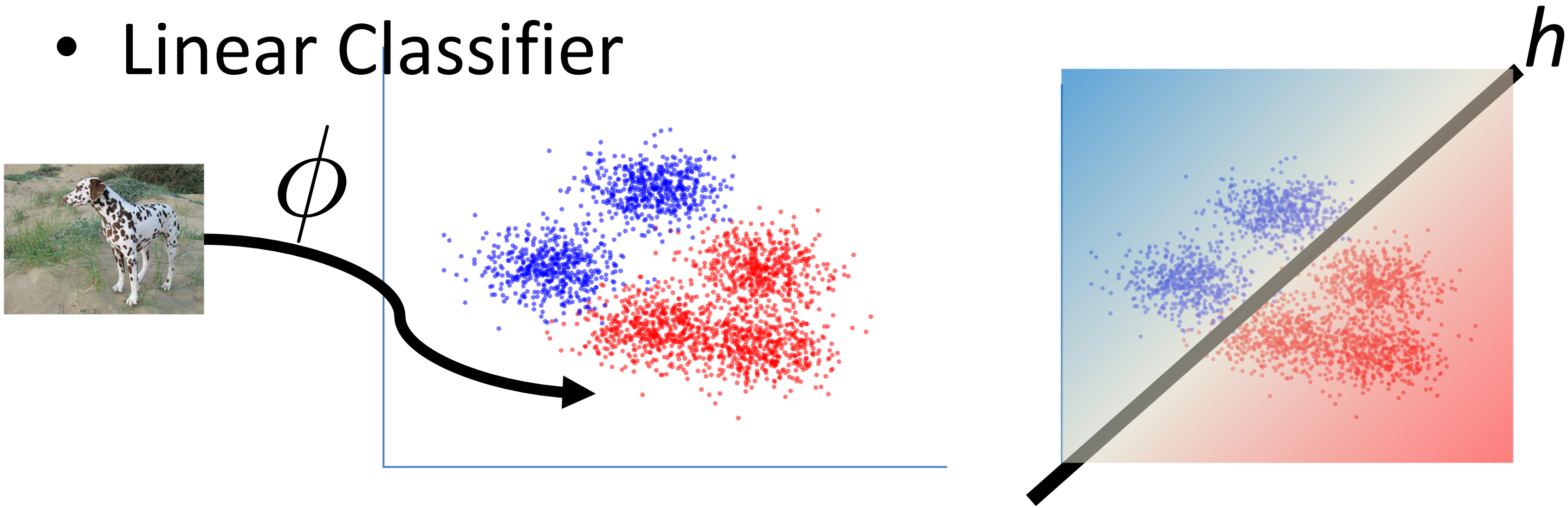
- Fast to train but slow to predict
- Distance metrics don’t behave well for high-dimensional image vectors

# Classifying Images

- Nearest Neighbor Classifier

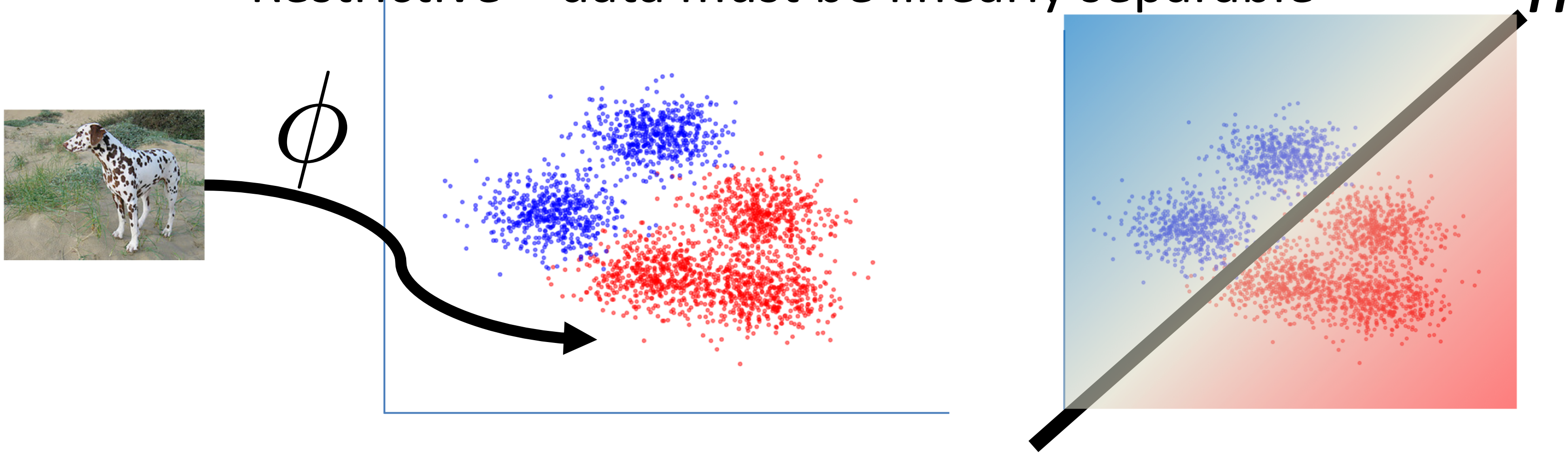



- Linear Classifier



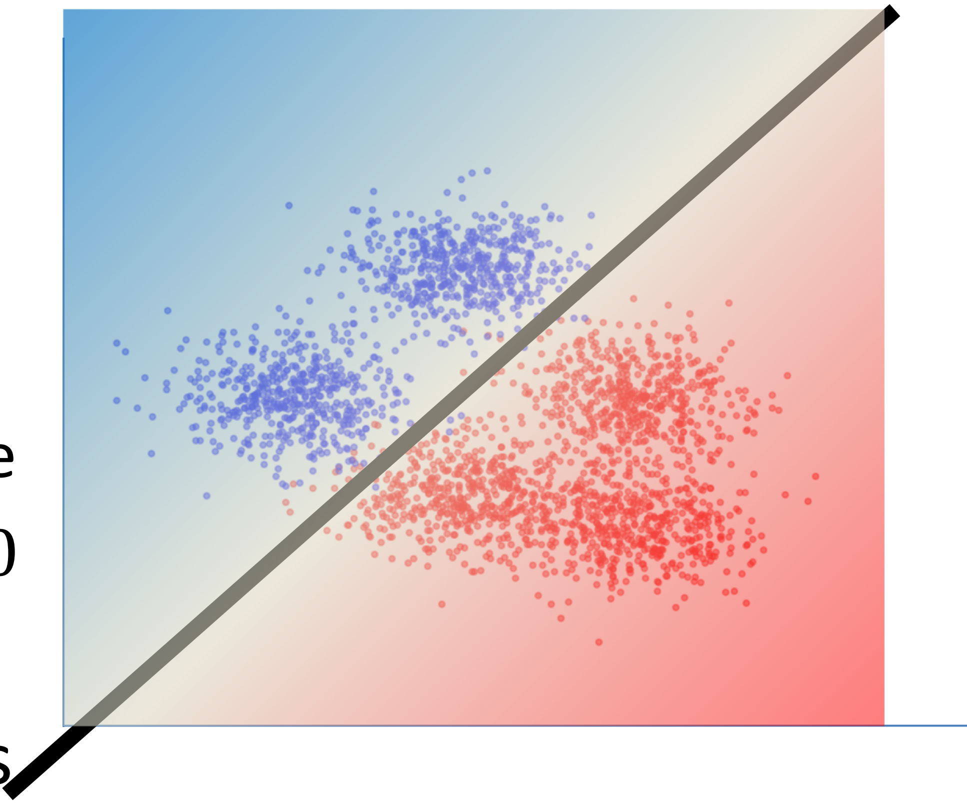
# Linear classifiers

- Finding nearest neighbor is slow.
- Basic idea:
  - Training time: find a line that separates the data
  - Testing time: which side of the line is $\phi$(x) on?
    - +Fast to compute
    - -Restrictive – data must be linearly separable



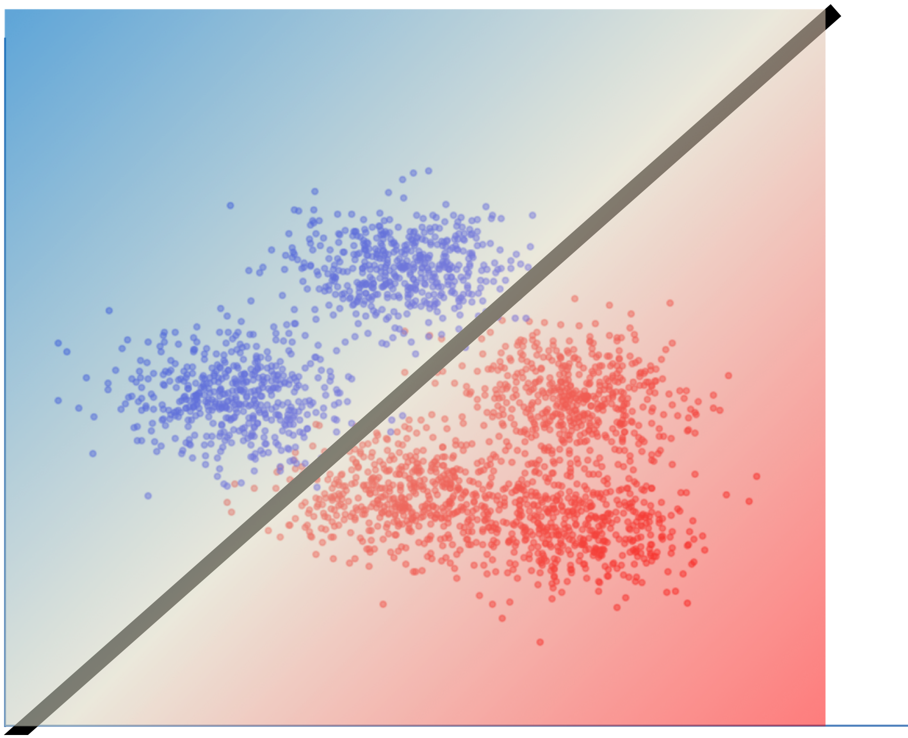
# Linear classifiers

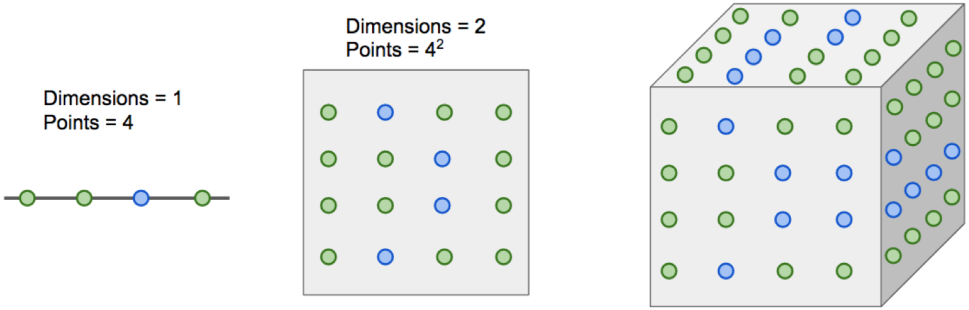
- A linear classifier corresponds to a hyperplane
  - Equivalent of a line in high-dimensional space
  - Equation: $w^T x + b = 0$
- Points on the same side are the same class



# Does this ever work?

- It's easier to be linearly separable in high-dimensional space.
- But simple linear classifiers still don't work on most interesting data.





# Some history from the Ante**deep**luvian Era

- Example pipeline from days of yore:
  - Detect corners and extract SIFT features
  - Collect features into a “bag of features”
  - (if you’re feeling fancy) maintain some spatial information
  - Somehow convert feature bag to fixed size
  - Apply **linear** classifier

- Key idea: $\phi$ is designed by hand, while *h* is learned from data.

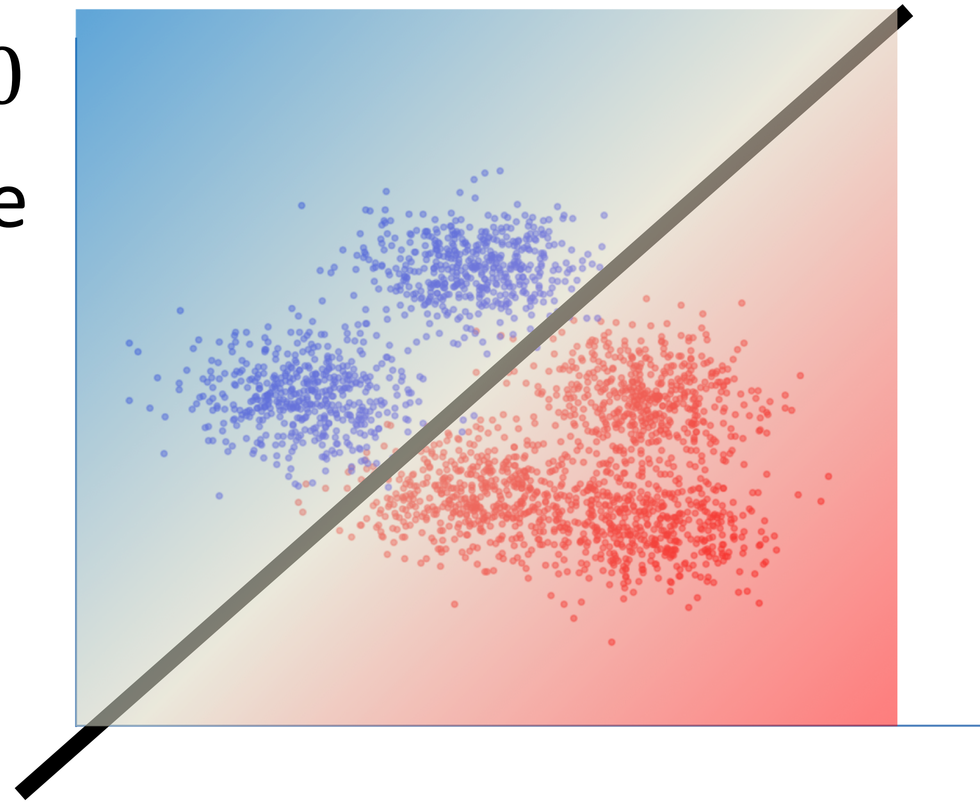
# Some history of the Ante**deep**luvian Era

- Key idea: $\phi$ is designed by hand, while *h* is learned from data.

- Nowadays: learn both from data - "end-to-end": image goes in, label comes out.
  - Enabled only recently by bigger
    - labeled datasets
    - compute power (GPUs)

# Linear classifiers

- Equation: $w^T x + b = 0$
- Points on the same side are the same class

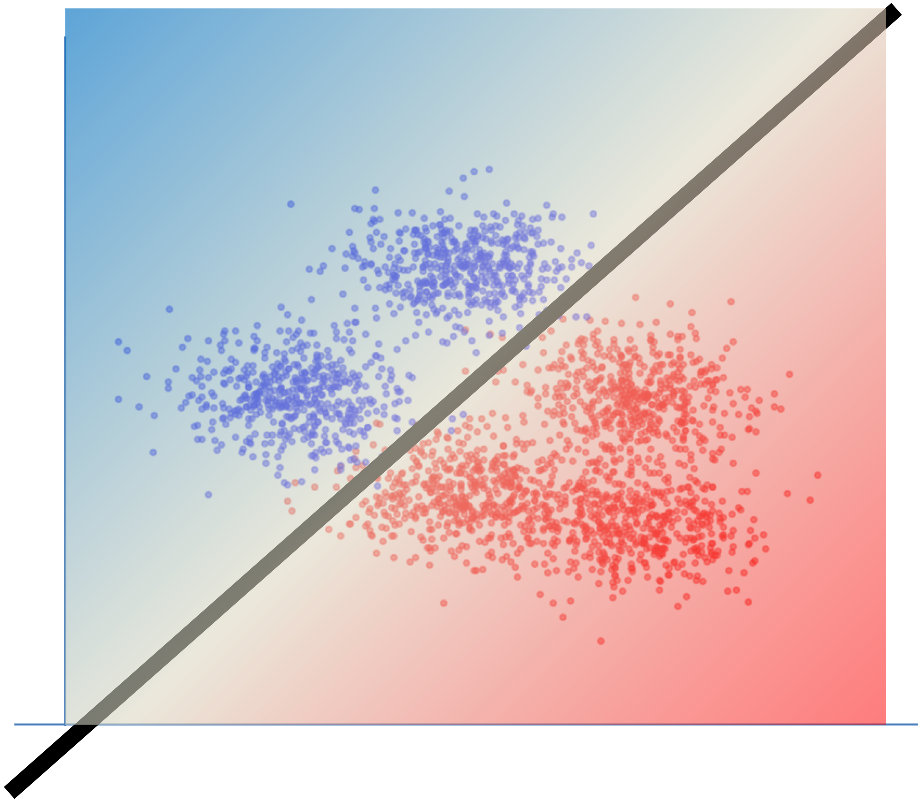


# We have a classifier

- $h(x) = w^T x + b$ gives a *score*

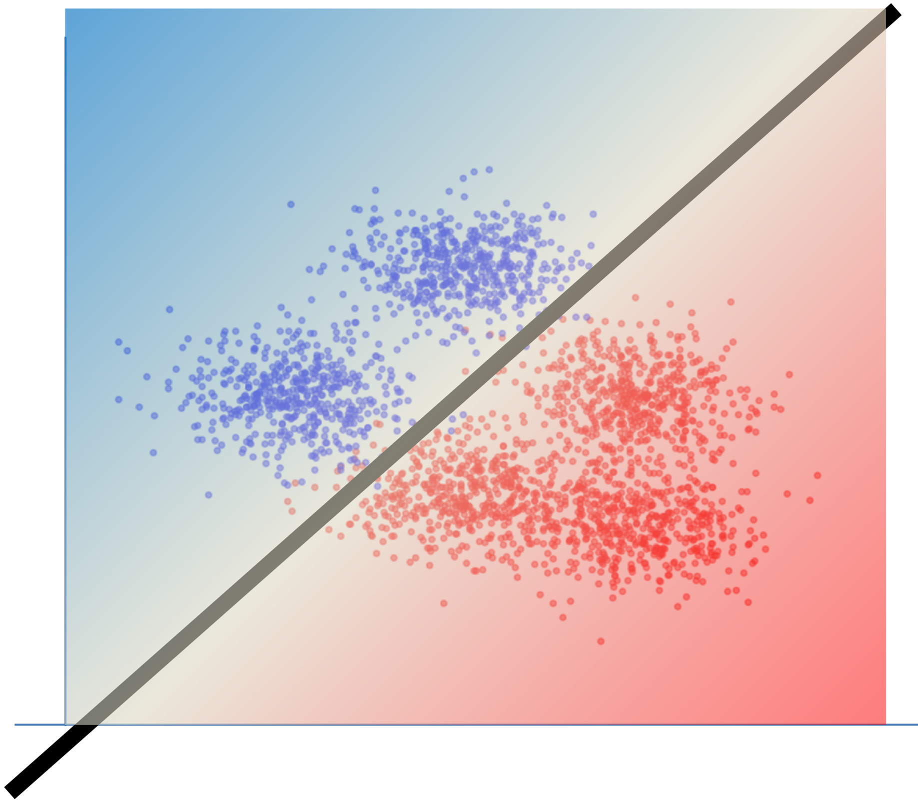
- Score negative: red
- Score positive: blue

- Does it solve the runtime issues of KNN?



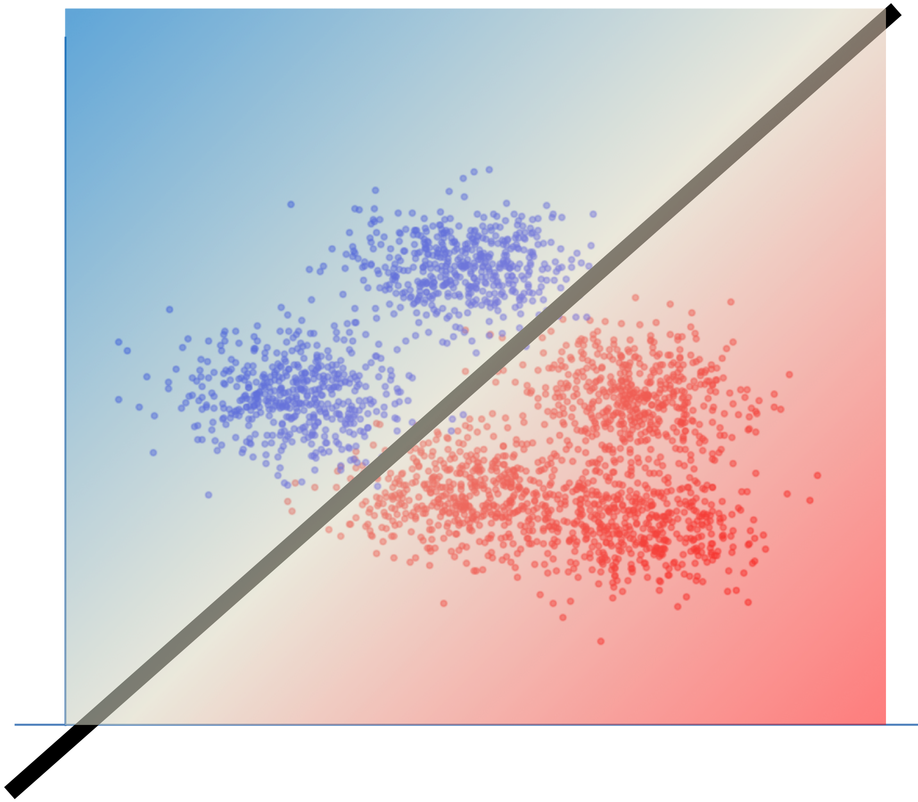
# We have a classifier

- $h(x) = w^T x + b$ gives a *score*
- Score negative: red
- Score positive: blue
- Where do W and b come from?



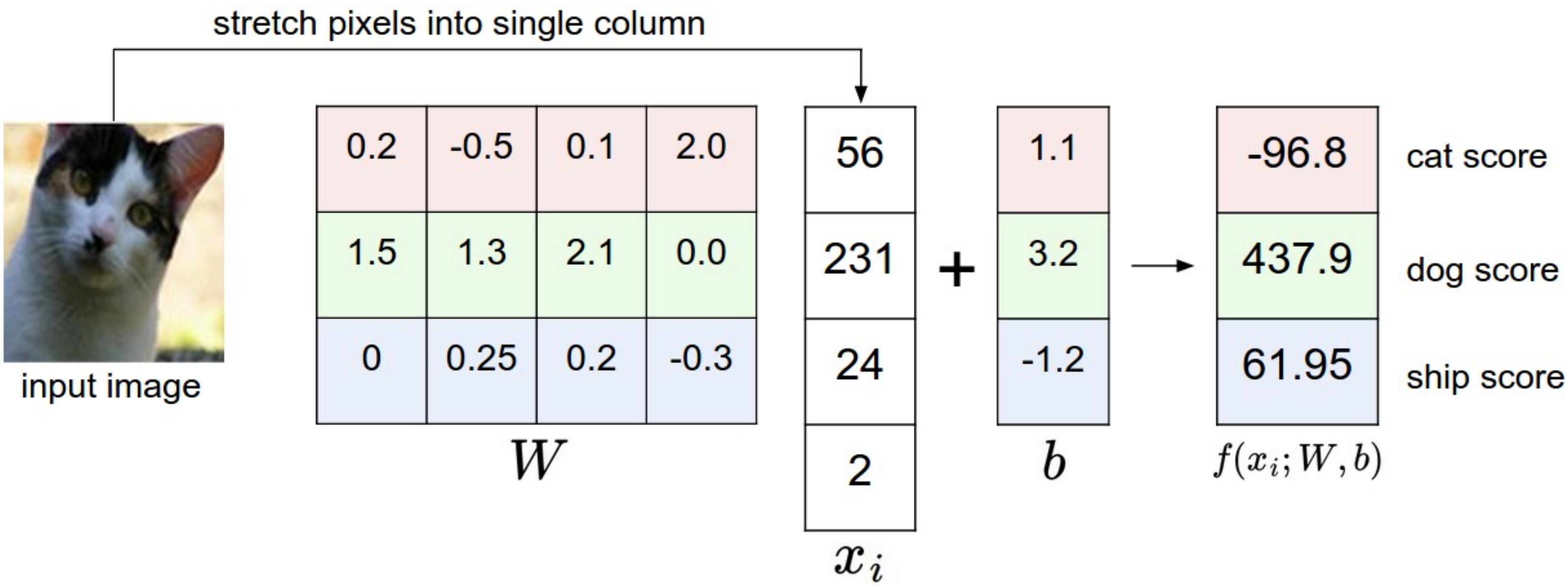
# How do we find a good W, b?

- Step 1: For a given W, b, decide on a **Loss Function**: a measure of how much we dislike the line.
- Step 2: use **optimization** to find the W, b that *minimize* the loss function.

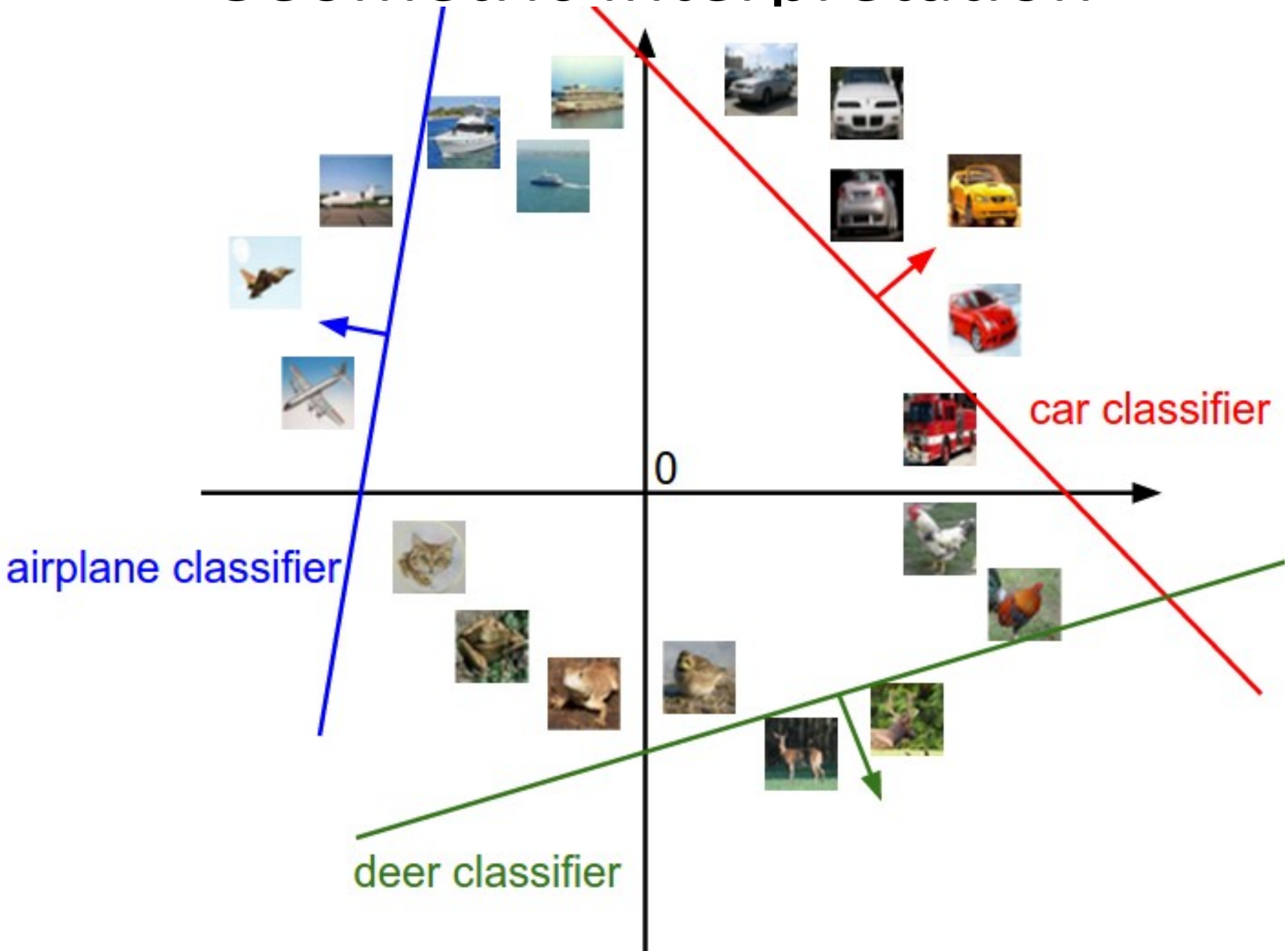


# Questions?

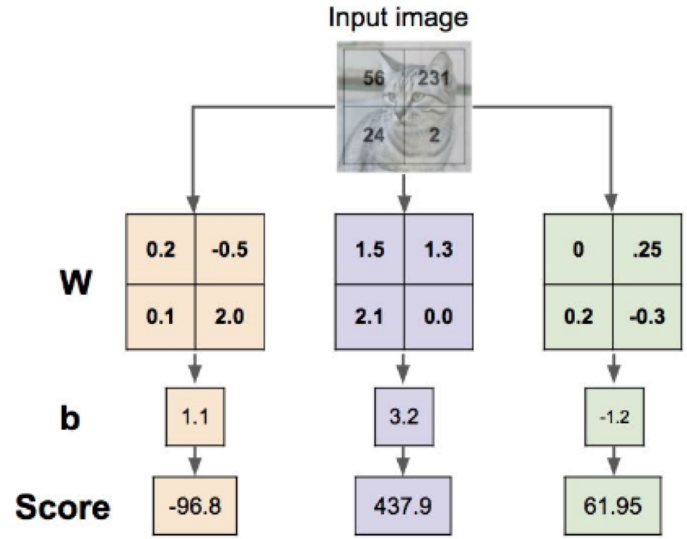
# Multiclass Linear Classifiers: Stack multiple $w^T$ into a matrix.



# Multiclass Linear Classifier: Geometric Interpretation



# Interpreting a Linear Classifier: Visual Viewpoint



Slide: Fei-Fei Li, Justin Johnson, & Serena Yeung

# Loss Functions

- Step 1: For a given W, b, decide on a **Loss Function**: a measure of how much we dislike this classifier.
- Step 2: use **optimization** to find the W, b that *minimize* the loss function.

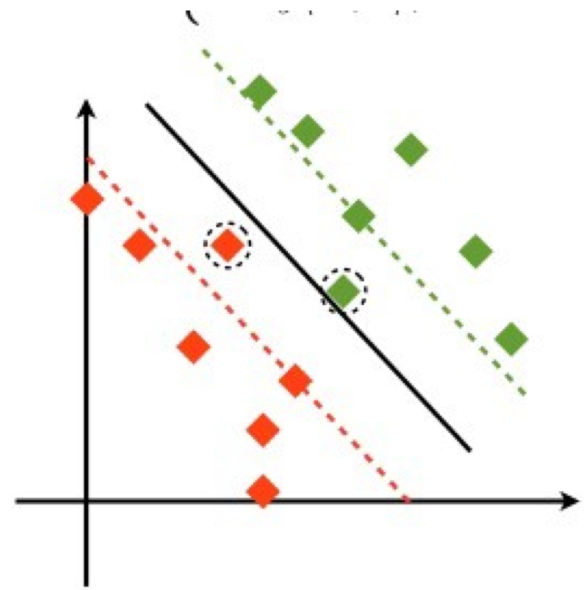
# Loss Functions

- Step 1: For a given W, b, decide on a **Loss Function**: a measure of how much we dislike this classifier.

- Loss Function intuition:
  - loss should be large if many data points are misclassified
  - loss should be small (0?) if all data is classified correctly.

# Loss Functions – SVM Loss

- SVM Loss:
  - Insists that data points are not just correctly classified, but a certain distance from the hyperplane:
  - $L_i = \max(0\ x_i, 1 - y_i(w^T x_i + b)$



Andrea Vedaldi and Brian Fulkerson

$x_i$ = i'th data point

$y_i$ = i'th data point's true label:

-1 if red

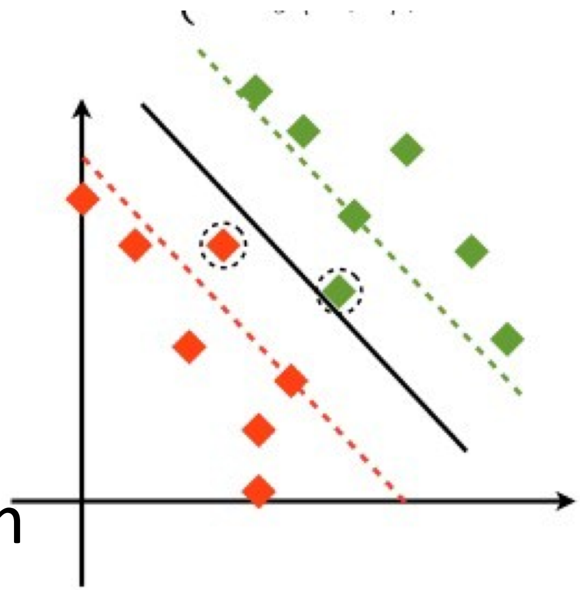
+1 if green

# Loss Functions – SVM Loss

- SVM Loss:
  - Insists that data points are not just correctly classified, but a certain distance from the hyperplane:
  - $L_i = \max(0\ x_i, 1- y_i(w^T x_i + b)$

    $x_i$ = i'th data point
    $y_i$ = i'th data point's true label:
    -1 if red
    +1 if green

  - $L(w, b) = \Sigma_i L_i$
  - Loss for a given line is the sum of the loss for all datapoints



Andrea Vedaldi and Brian Fulkerson

# The Bias Trick

# The Bias Trick

- Fold b into an additional dimension of w
- Add a fixed 1 to all feature vectors.

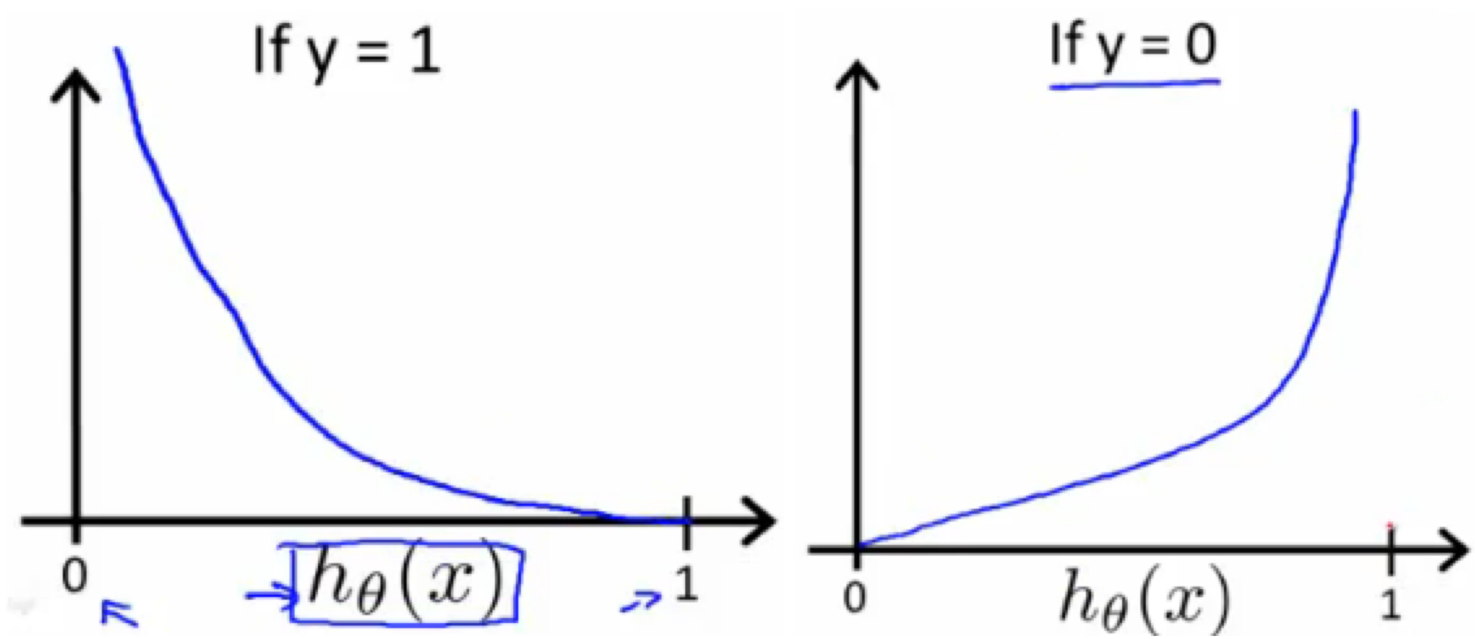
- Now, $h(x) = w^T x$

# Softmax Classifier / Cross-Entropy Loss: Intuition

$W^T x + b$ gives us a vector of scores, one per class (each row of W is a classifier)

Wouldn’t it be nice to interpret these as probabilities?

# Binary Equivalent: Logistic Regression Loss



# Softmax Classifier / Cross-Entropy Loss: Intuition

$W^T x + b$ gives us a vector of scores, one per class (each row of W is a classifier)

Wouldn’t it be nice to interpret these as probabilities?

They’re not:

- not always nonnegative
- don’t sum to 1

But we can treat them as **unnormalized log probabilities**.

# Softmax Classifier / Cross-Entropy Loss: Intuition

But we can treat scores as **unnormalized log probabilities**.

# Cross-Entropy Loss: Intuition

$W^T x + b$ gives us a vector of scores, one per class (each row of W is a classifier)

Cross-Entropy loss: Apply a sigmoid to get values between 0 and 1, then normalize them to sum to 1.

Then, compute the difference between the **predicted** distribution and the **true** distribution.