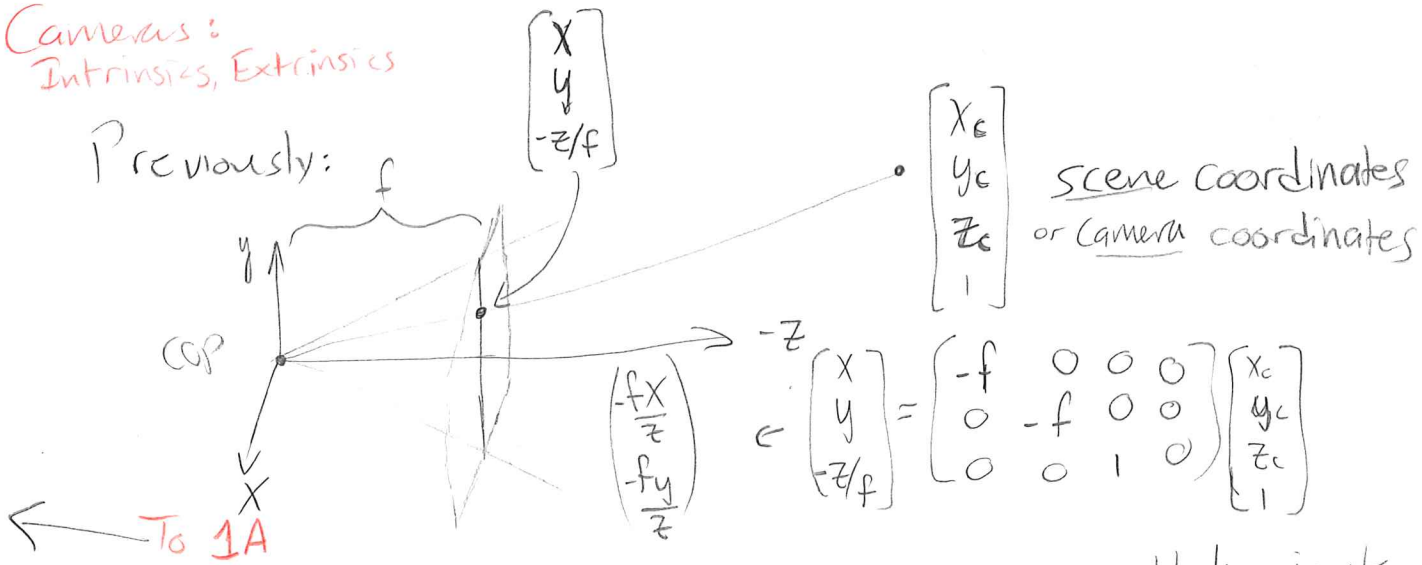
Cameras: Intrinsics, Extrinsics

Previously:



To 1A

Extrinsics

What if we have multiple cameras, or the world lives in its own coordinate system?

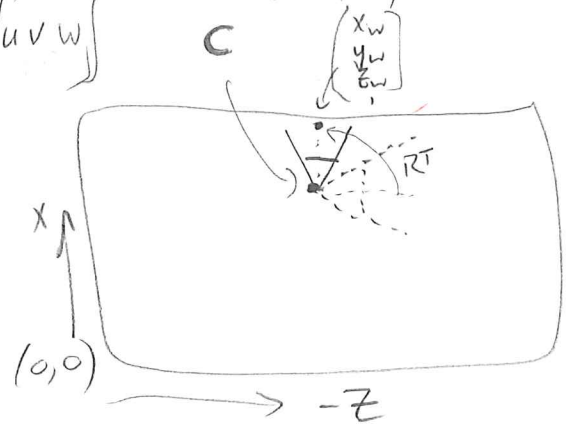
world coordinates

$$\begin{bmatrix} X_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} & ? & \end{bmatrix} \begin{bmatrix} X_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

What goes here? Transform world points to camera points

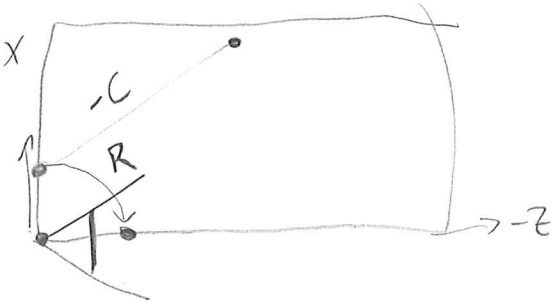
Suppose camera is centered at world coordinates C. $\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$

can be rotated to its world orientation by a 3x3 matrix $R^T = \begin{bmatrix} u & v & w \end{bmatrix}$



$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R_{3\times3} & 0 \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_{3\times3} & -C \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

translate ↙ world



$$\begin{bmatrix} x \\ y \\ z_f \end{bmatrix} = \begin{bmatrix} -f s_x & 0 & x_c' \\ 0 & -f s_y & y_c' \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & 0 \\ & 0 \\ & 0 \\ 0 & 0 \; 0 \; 1 \end{bmatrix} \begin{bmatrix} I_{3\times 3} & -C \\ 0 \; 0 \; 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \quad (2)$$

pixel coords

Intrinsics

project

camera coords

rotate

translate

extrinsics

Π

1A

$$\begin{bmatrix} x \\ y \\ -z/f \end{bmatrix} = \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

intrinsics ↙ ↘ projection

$$K: \begin{bmatrix} f & 0 & x'_c \\ 0 & -\alpha f & y'_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- f is focal length
- α = aspect ratio
- x'_c, y'_c = principal point

Because K is scale invariant, important is α = aspect ratio.