RANSAC

for $i = 0..K$:

$d_i \leftarrow S$ random data points

$M_i \leftarrow$ fit_model(d_i)

inlier_count $\leftarrow \sum_{x_i, y_i \in D} \mathbb{1}(|M(x_i) - y_i| < \delta)$ #data points in agreement w/ M_i

if inlier_count < best_count:

best_count $\leftarrow$ inlier_count

best_M $\leftarrow M_i$

best_data $\leftarrow \{(x_i, y_i) : |M(x_i) - y_i| < \delta\}$ all data points in agreement w/ M_i

M_final = fit_model(best_data)

Parameters

K - # iterations (hypotheses)

S - # data points needed to fit a model

δ - inlier threshold

Choosing parameter values:

δ: based on expected inlier noise. Common case:

Assume Gaussian w/ variance σ^2

Let $\delta \approx 1$ or $2 \cdot \sigma$

S: based on specific problem:

- Linear regression - 2 x, y points
- Translation fitting - one match $(x, y) \leftrightarrow (x', y')$
- Affine - 3 matches
- Homography - 4 matches
- Ellipse (why not?) - 3 (x, y) points

Choosing Parameter Values (cont):

K (# iterations) - suppose we want to find a set of s inliers with probability $\geq P$

Assume we can estimate the inlier ratio

$$r = \frac{\#\text{ inliers}}{\#\text{ data points}}$$

In one hypothesis,

$$P(\text{choose all inliers}) = r^s$$

$$P(\text{at least one outlier}) = 1 - r^s$$

Bad Thing

Over K trials,

Bad Thing happens K times in a row

$$P(\text{at least one outlier all K trials}) = (1 - r^s)^K$$

Bad Thing doesn't happen K times in a row

$$P = P(\text{no outliers in at least one trial}) = 1 - (1 - r^s)^K$$

$\equiv$ Success

What K do I need to make $P(\text{success}) \geq P$?

$$P \geq 1 - (1 - r^s)^K$$

$$1 - P \geq (1 - r^s)^K$$

$$\log(1 - P) \geq K \log(1 - r^s)$$

$$\boxed{\frac{\log(1-P)}{\log(1-r^s)} \geq K}$$

Sanity checks:

Lower Prob. of success $\rightarrow$ fewer iterations

More points to fit a model (s) $\rightarrow$ more iterations