Homography

$$\begin{bmatrix} x_h \\ y_h \\ w_h \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Make this h_{22}
then rewrite

$$x' = \frac{\overbrace{h_{00}x + h_{01}y + h_{02}}^{x_h}}{\underbrace{h_{20}x + h_{21}y + 1}_{w_h}}$$

$$y' = \frac{h_{10}x + h_{11}y + h_{12}}{w_h}$$

Not linear!

$$\begin{bmatrix} x_h \\ y_h \\ w_h \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\frac{h_{00}x + h_{01}y + h_{02}}{\boxed{h_{20}x + h_{21}y + h_{22}}} = x'$$

$$h_{00}x + h_{01}y + h_{02} = x'(h_{20}x + h_{21}y + h_{22})$$

X Residual: $(h_{00}x + h_{01}y + h_{02}) - x'(h_{20}x + h_{21}y + h_{22})$

$$= (h_{00}x + h_{01}y + h_{02}) - (h_{20}xx' + h_{21}yx' + h_{22}x')$$

$$\frac{h_{10}x + h_{11}y + h_{12}}{h_{20}x + h_{21}y + h_{22}} = y'$$

y residual: $h_{10}x + h_{11}y + h_{12} - (h_{20}xy' + h_{21}yy' + h_{22}y')$

We had: $x' = \dfrac{x_h}{w_h}$ now $x_h = x'w_h$

$y' = \dfrac{y_h}{w_h}$ now $y_h = y'w_h$

Residuals

$$x: \overbrace{h_{00}x + h_{01}y + h_{02}}^{x_h} \overbrace{- h_{20}xx' - h_{21}x'y - h_{22}x'}^{x'w_h}$$

$$y: h_{10}x + h_{11}y + h_{12} - h_{20}xy' - h_{21}yy' - h_{22}y'$$

$$\min \|Ah - b\|^2$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & xx_1' & x_1'y_1 & x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & xy_1' & yy_1' & y_1' \\ & & & & \vdots & & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & x_n x_n' & x_n' y_n & x_n' \\ 0 & 0 & 0 & x_n & y_n & 1 & x_n y_n' & y_n y_n' & y_n' \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

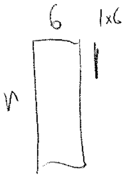
Minimizing $\|Ah - 0\|^2$

Trivial: $h = \vec{0}$

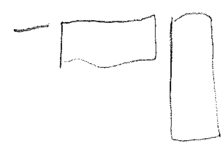
Solution: constrain $\|h\| = 1$

$$\min \|Ah - 0\|^2 \text{ S.T. } \|h\| = 1$$

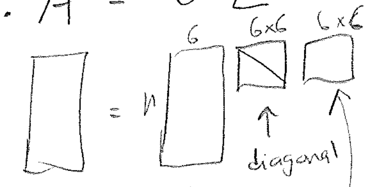
$$\min \|Ah\|^2 \text{ S.T. } \|h\| = 1$$



$$\|Ah\|^2 = (Ah)^T (Ah) = h^T A^T A h$$

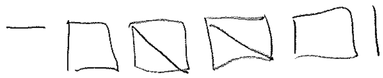


Singular value decomposition: $A = U \Sigma V^T$



orthogonal, unitary : $u_i^T \cdot u_j = 0$
$u_i^T u_i = 1$

$$h^T A^T A h = h^T V \Sigma U^T U \Sigma V^T h$$
$$= h^T V \Sigma \Sigma V^T h$$



$$\Sigma V^T h : \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} \text{---} \; v_1 \; \text{---} \\ \text{---} \; v_2 \; \text{---} \\ \text{---} \; v_3 \; \text{---} \\ \vdots \end{bmatrix} \begin{bmatrix} \\ h \\ \\ \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} v_1 h \\ v_2 h \\ v_3 h \\ \vdots \end{bmatrix} = \begin{bmatrix} \sigma_1 v_1 h \\ \sigma_2 v_2 h \\ \sigma_3 v_3 h \\ \vdots \end{bmatrix}$$

$\|h\| = 1$

$\|v_i\| = 1$

$v_i^T \cdot v_j = 1$

$h = v_3 \rightarrow$

$\sigma_1 v_1 v_3 \rightarrow 0$

$\sigma_2 v_2 v_3 \rightarrow 0$

$\sigma_3 v_3 v_3 \rightarrow \sigma_3$

$\vdots$

0

In Practice:

1. Compute SVD of A.
2. Find index of smallest σ_i in Σ
3. Take the ith column of V (ith row of V^T) as solution h.