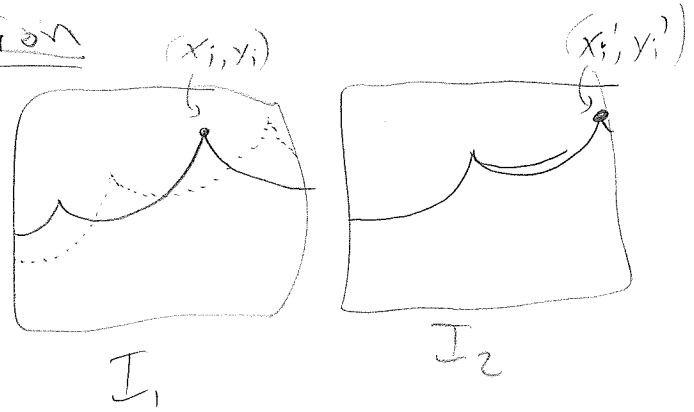


Translation

Single Match



matches

unknowns



$$x_i' = x_i + x_t$$

$$y_i' = y_i + y_t$$

$$x_t = x_i' - x_i$$

$$y_t = y_i' - y_i$$

More Matches: Average them I guess? some unknown

$$x_t = \frac{1}{n} \sum_{i=0}^n (x_i' - x_i)$$

$$y_t = \frac{1}{n} \sum_{i=0}^n (y_i' - y_i)$$

$$\begin{aligned} x_t &= x_1' - x_1 \\ x_t &= x_2' - x_2 \\ &\vdots \end{aligned}$$

More Matches: Linear Algebra Edition

$$x_t = x_i' - x_i \quad y_t = y_i' - y_i$$

⋮

2n equations
2 unknowns.

Least Squares!

Min $\|Ax = b\|^2$
↑ residuals
unknowns

$$r_{x_i}(x_t) = x_i' - (x_i + x_t)$$

$$r_{y_i}(y_t) = y_i' - (y_i + y_t)$$

A

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

t

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix}$$

b

$$\begin{bmatrix} x_1' - x_1 \\ y_1' - y_1 \\ x_2' - x_2 \\ \vdots \\ x_n' - x_n \\ y_n' - y_n \end{bmatrix}$$

Affine

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Unknowns: a, b, c, d, e, f (6)

Eqs: 2 per match

$$x'_i = ax_i + by_i + c$$

$$y'_i = dx_i + ey_i + f$$

Residuals:

$$(ax_i + by_i + c) - x'_i$$

$$(dx_i + ey_i + f) - y'_i$$

$$\text{Min} \|A \begin{matrix} \uparrow \\ \text{unk} \end{matrix} - b\|^2$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ & & \vdots & & & \\ x_n & y_n & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_n \\ y'_n \end{bmatrix}$$