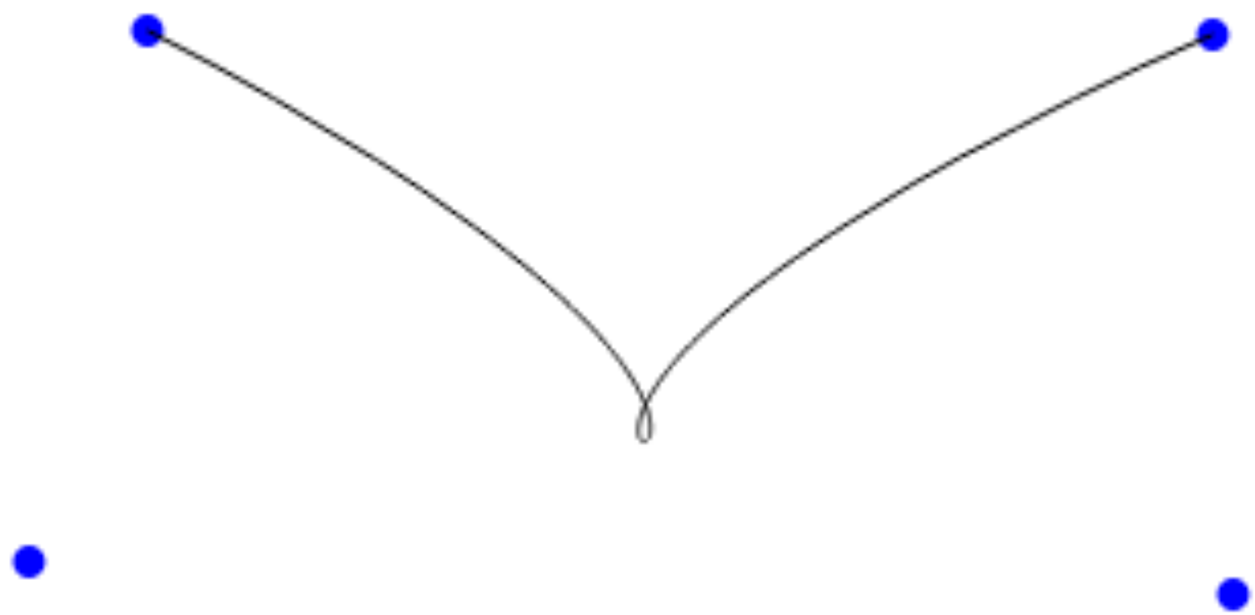


Computer Graphics

Lecture 29 - **Splines:**
Hermite Splines
Bézier Curves

Announcements

- Milestone 1 due tomorrow night
- Spline lab - starting tomorrow or Wednesday.
  - Bring a laptop if you can!
  - Javascript only, no Julia needed 😢
- Grad presentation lineup is on the Schedule table.
  - There will be a survey for attendees, worth participation points. Please plan to be there!
  - Friday 12/6 is optional, come Ask Me Anything

Goals
(for today and tomorrow combined)

- Be able to derive the control matrix for Hermite Splines and Bezier curves.
- Understand why it's called the basis matrix
- Understand some geometric properties of Bézier splines:
  - Evaluation by linear interpolation
  - Subdivision and drawing using de Casteljau's algorithm
- Know how to join multiple (e.g., Bezier) spline segments together
- Know the definition of parametric and geometric continuity.
- See a few other examples of splines, including B-splines and NURBS

Hermite Splines: Demo

https://www.desmos.com/calculator/gomvxqce4a

Hey Scott, switch to the notes

Derive Hermite Spline Matrix

Bezier Curves: Demo

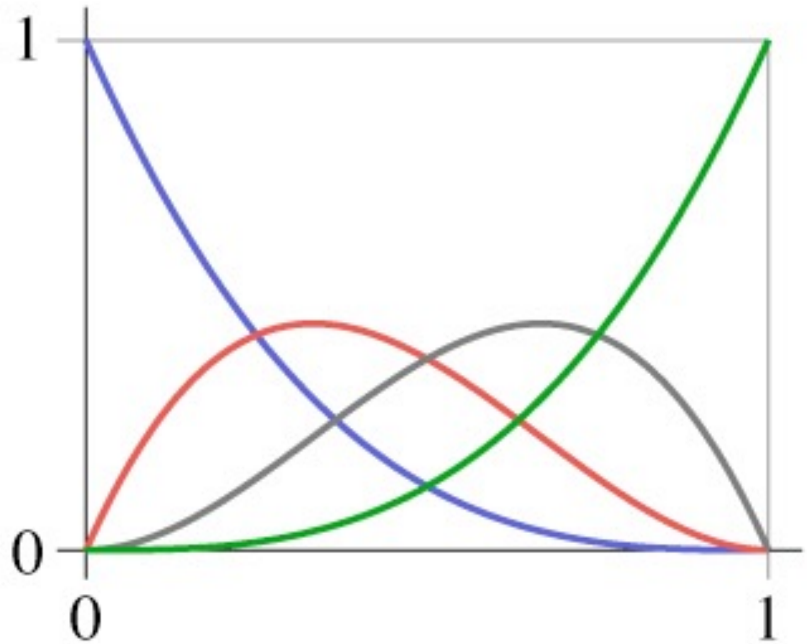
- https://celloexpressions.com/geometry/bezier-curves-splines/560.html

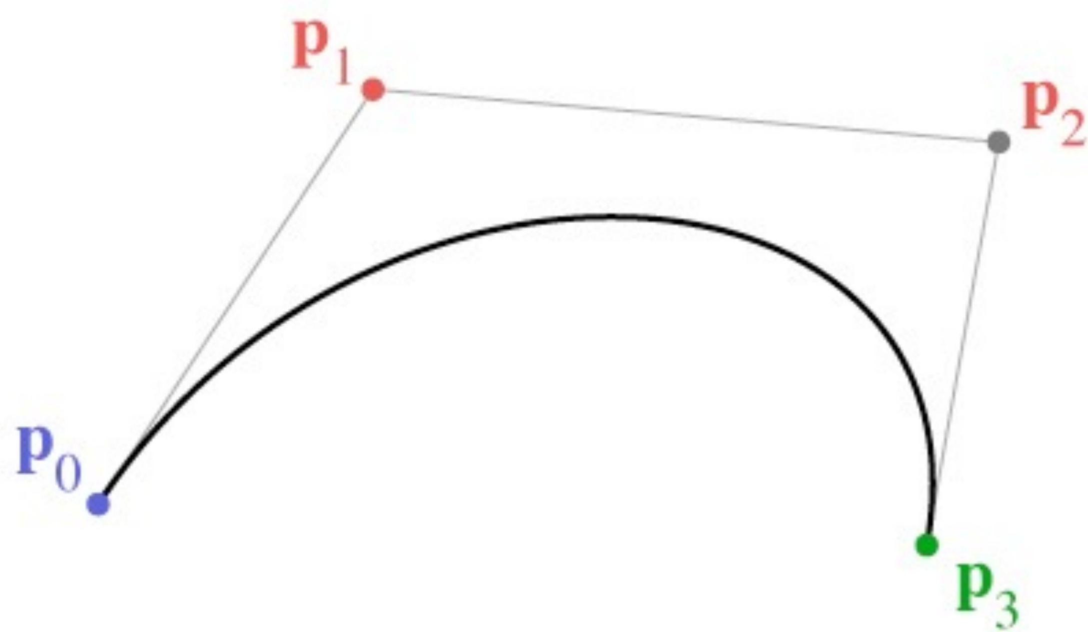
Derive Bézier Spline Matrix

Why is it called a "Basis Matrix"?

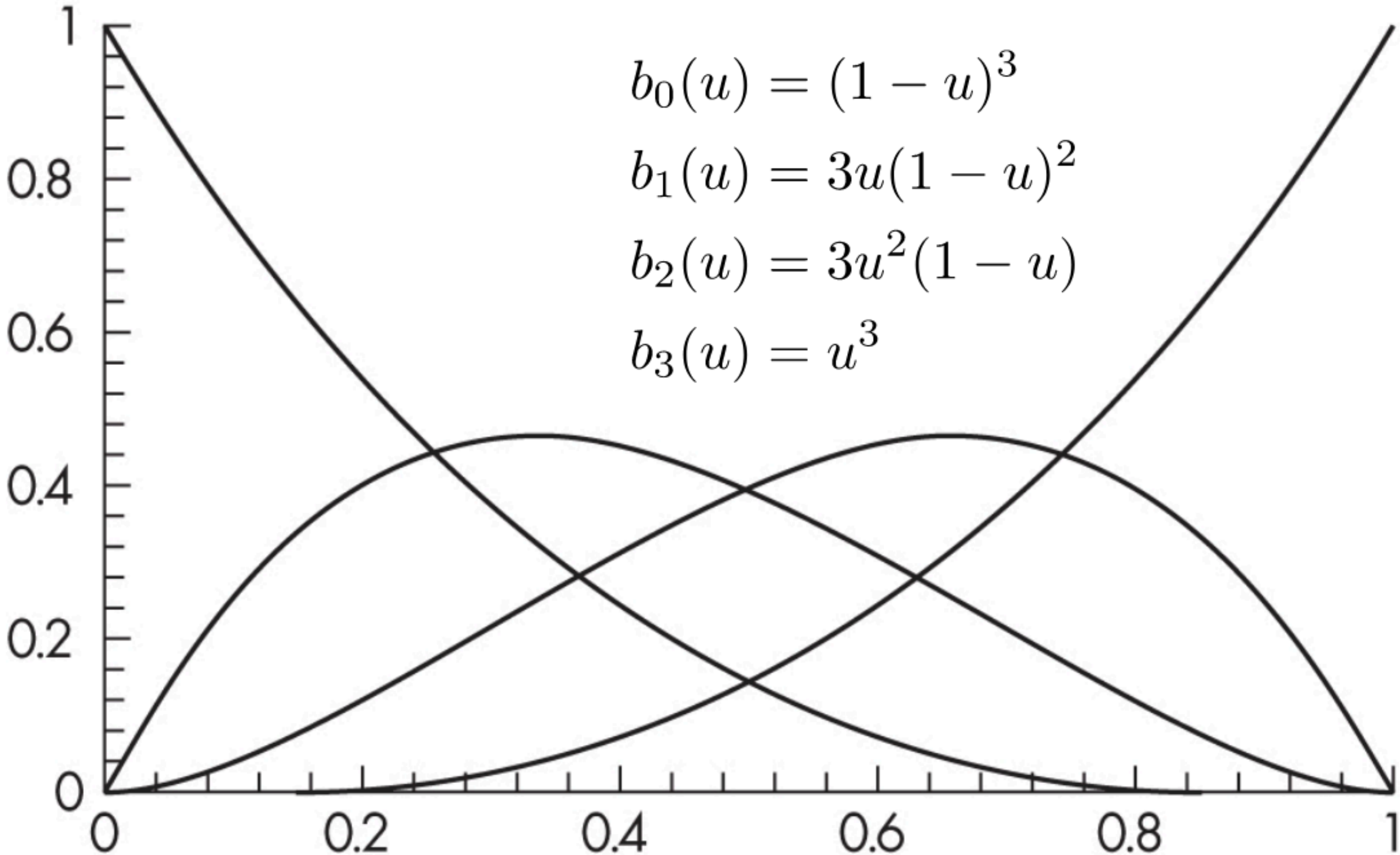
- We have: $f(u) = \mathbf{u}^T B\mathbf{p}$
- For computational purposes, we'll want to precompute Bp.
  - This is the vector of a_i's that weights each power of u
- How would we interpret $\mathbf{u}^{\mathbf{T}}B$?
  - A polynomial that specifies the weight on each control point.

Blending Functions





Cubic Bezier blending functions

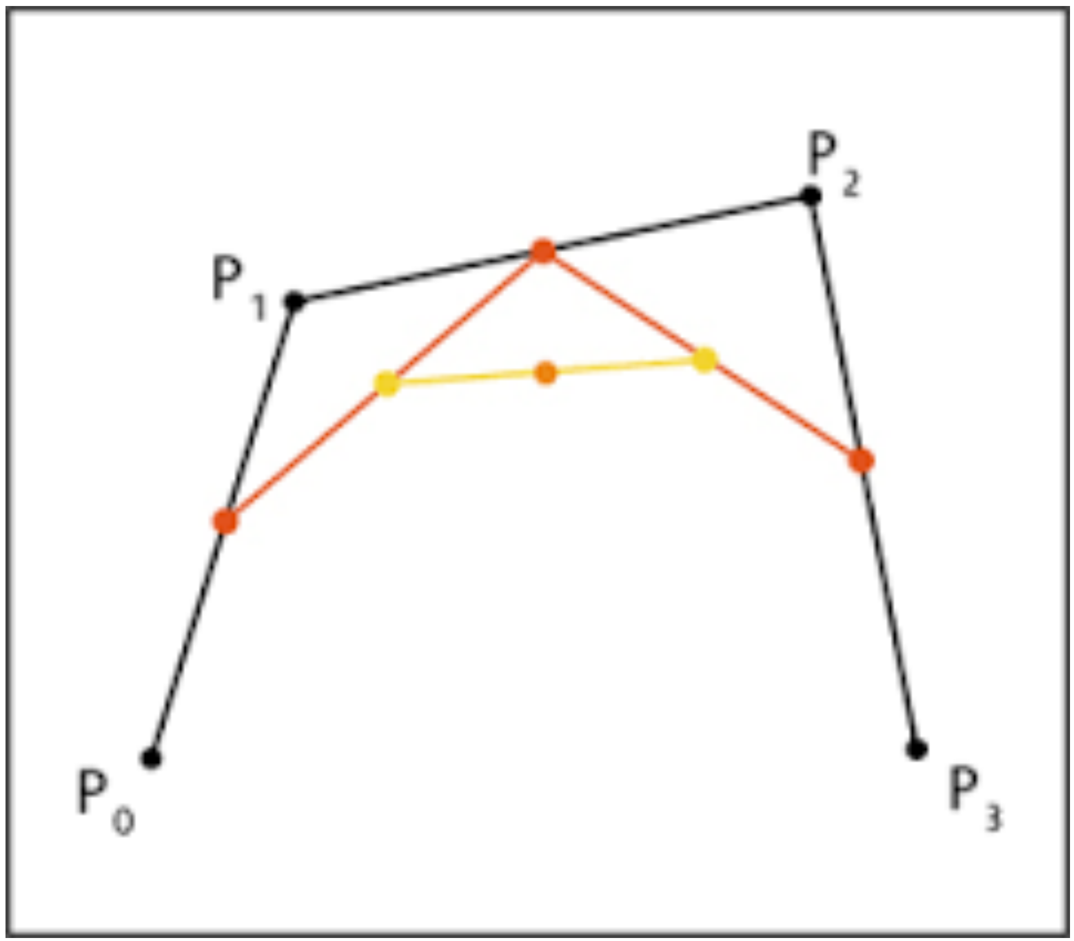


$$b_0(u) = (1-u)^3$$
$$b_1(u) = 3u(1-u)^2$$
$$b_2(u) = 3u^2(1-u)$$
$$b_3(u) = u^3$$

Coolest / most satisfying animation of the quarter

https://www.jasondavies.com/animated-bezier/

Bezier Curves: Geometry



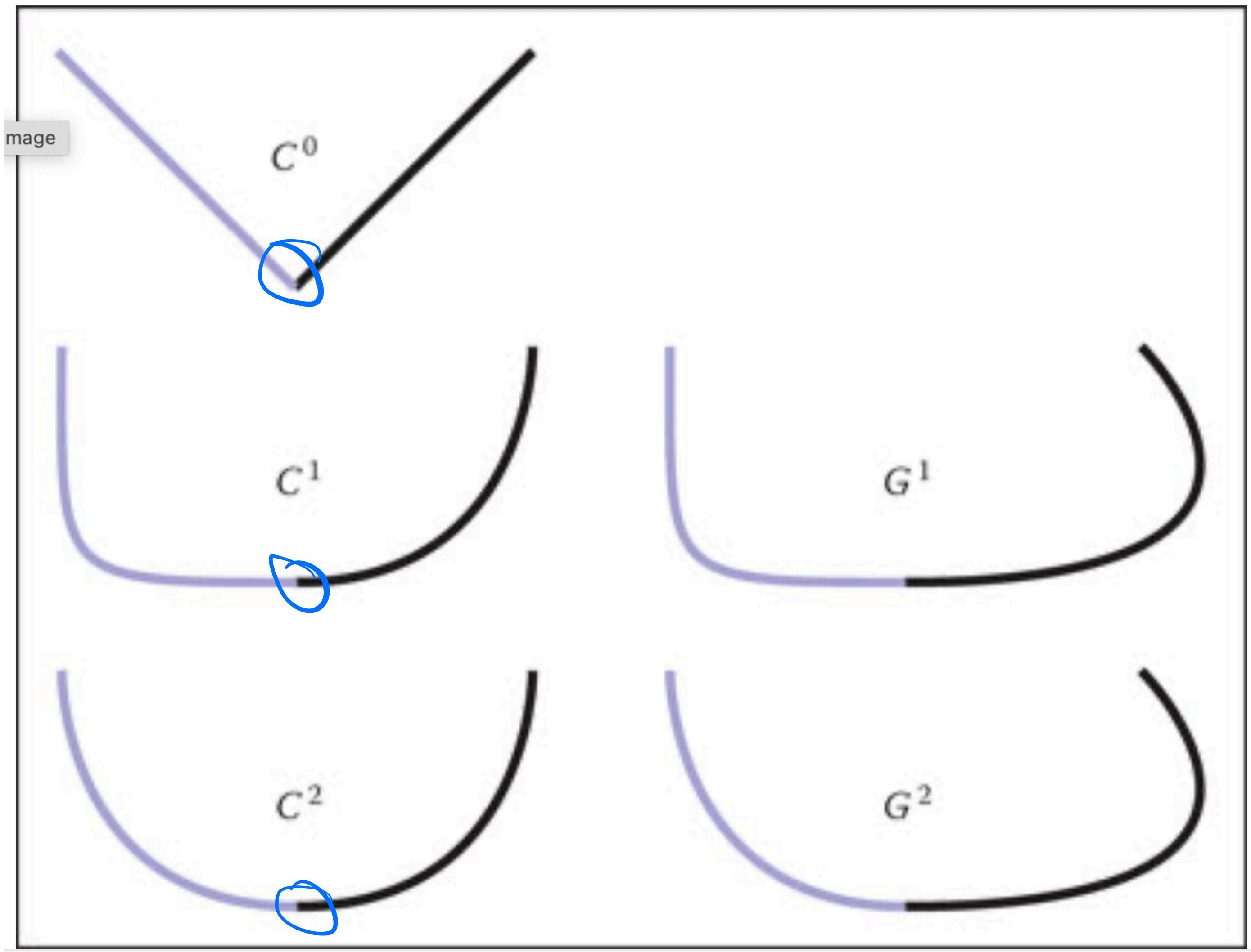
Coolest / most satisfying animation of the quarter

https://www.jasondavies.com/animated-bezier/

Joining Segments

- http://math.hws.edu/eck/cs424/notes2013/canvas/bezier.html

Curve Properties: Continuity

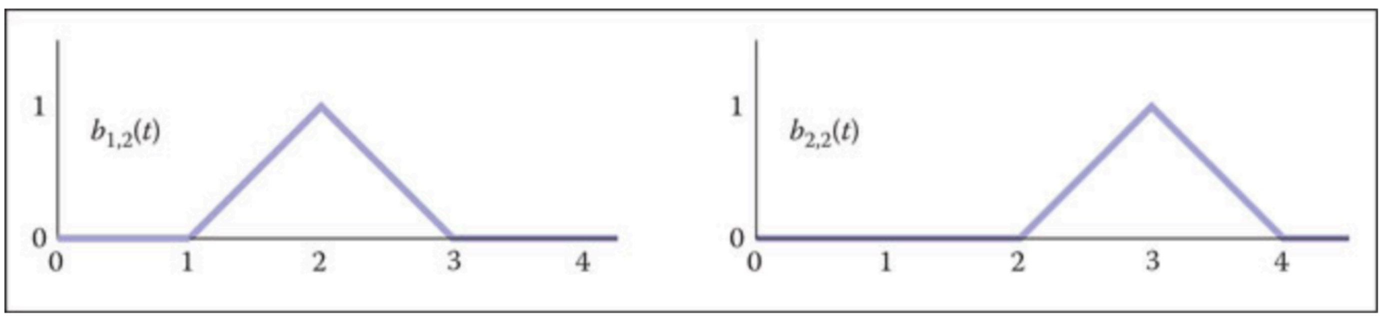


$C^1 : \mathbf{f}_1'(1) = \mathbf{f}_2'(0)$ $G^1 : \mathbf{f}_1'(1) = k\mathbf{f}_2'(0)$

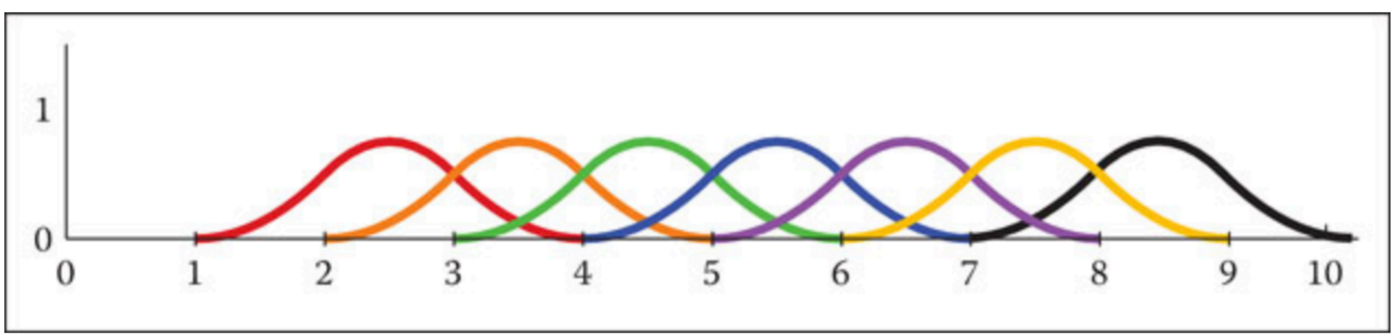
B-Splines

- Offer arbitrary continuity
- The basis polynomials are splines themselves!

k: polynomial order of "bump"

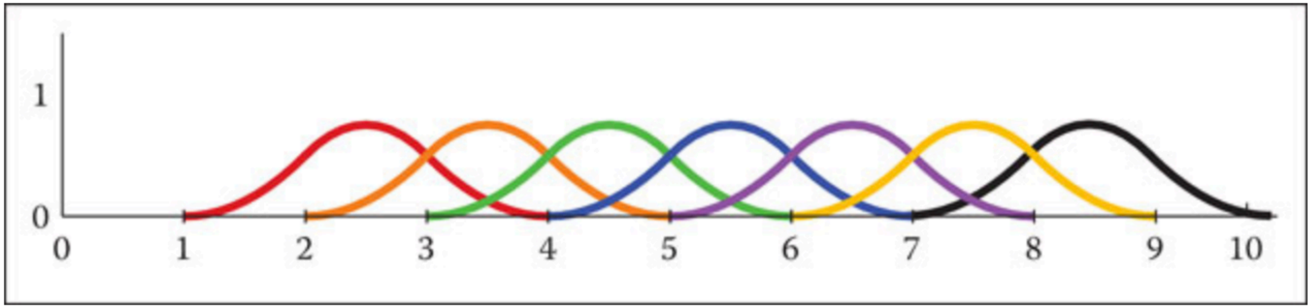


k = 1



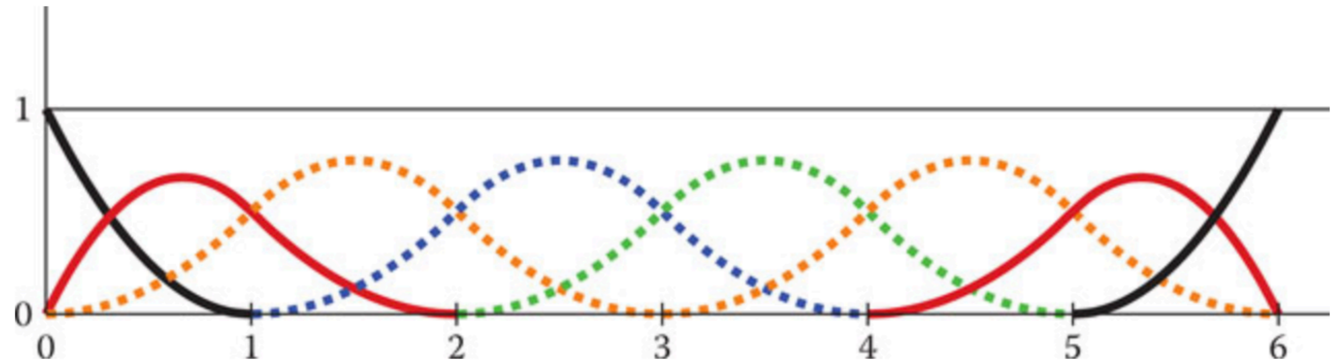
k = 3

Uniform B-Splines

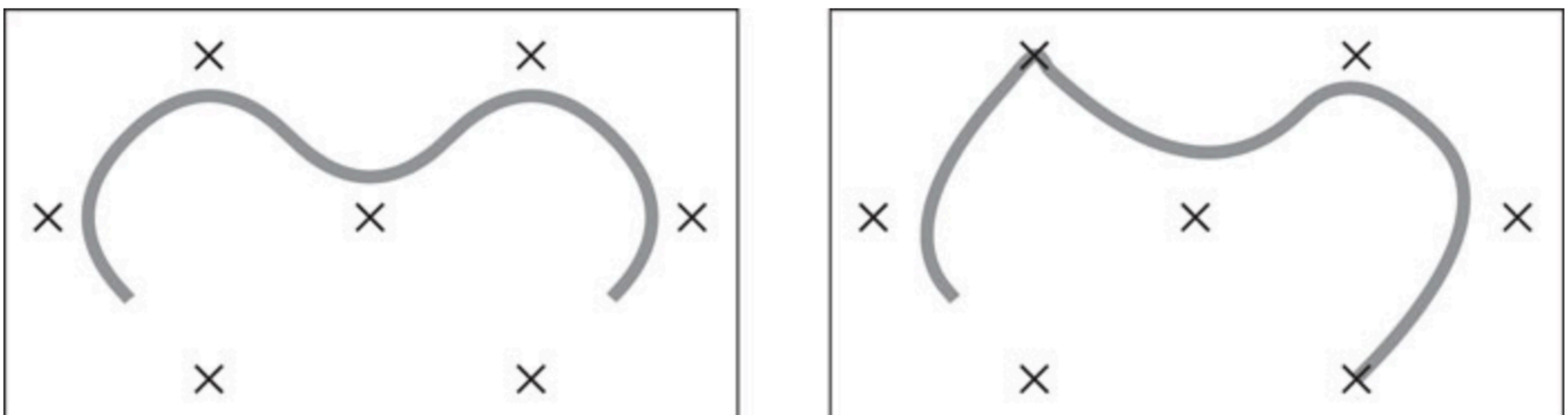


uniform B-spline: "bumps" (knots) evenly spaced

Non-Uniform B-Splines



non-uniform B-spline: "bumps" (knots) are not evenly spaced



(a) Uniform knots

(b) Nonuniform knots

Non-Uniform **Rational** B-Splines: NURBS

- B-spline bases are polynomials - can't represent conic sections e.g., a circle:
  - $x^2 + y^2 = 0$
- Rational B-splines - use a **ratio** of two polynomials.
  - Numerator and denominator are both B-splines

Curves are great, but.

https://youtu.be/AcFwH161XtM?t=68

https://youtu.be/Zkx1aKv2z8o?t=1080