L30 Outline - 11/15/22

Bézier Splines solve 2 problems w/ Hermite:

1. non-intuitive tangent tails - use $p_1 - p_0$ instead of p_1
   $p_2 - p_3$ p_3
2. weak tangents - multiply by 3

(Bezier basis matrix) + Demo

Basis Matrix - why?

$$f(u) = \vec{u}^T(\underbrace{B\vec{p}}_{\vec{a},\ \text{coefficients}})$$

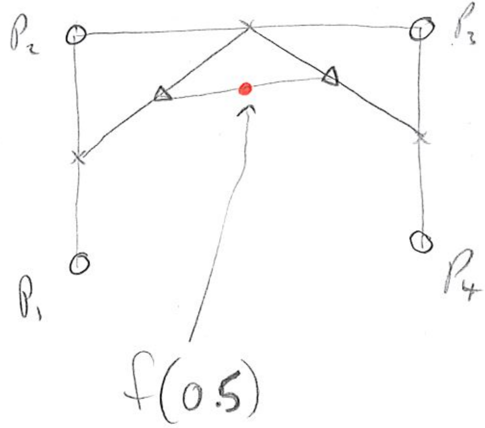
↑ "weights" on coefficients

$$f(u) = \underbrace{(u^T B)}\vec{p}$$

per-point weights - basis functions + Plot
(any order exists)

Bézier Geometry:

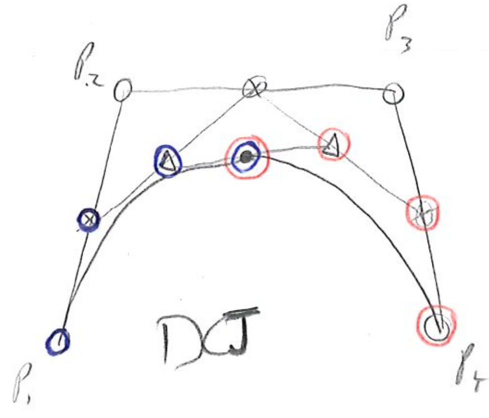
evaluation - all you need is lerp!



+ANIMATION

f(0.25)

Subdivision



Bézier Basis Matrix and Blending Functions

$$P_0 = f(0) = a_0$$

$$P_3 = f(1) = a_0 + a_1 + a_2 + a_3$$

$$3(P_1 - P_0) = f'(0) = a_1$$

$$3(P_3 - P_2) = f'(1) = a_1 + 2a_2 + 3a_3$$

$$f(u) = \vec{a} \cdot \vec{u}$$

Do some algebra...

$$\vec{a} = B\vec{p}$$

$$f(u) = \underbrace{\vec{u}^T \overbrace{B\vec{p}}^{\text{coefficients}}}_{\text{blending functions}}$$

$$B = C^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

Thought Exercise: what does $\vec{u}^T B$ ~~compute~~ represent?

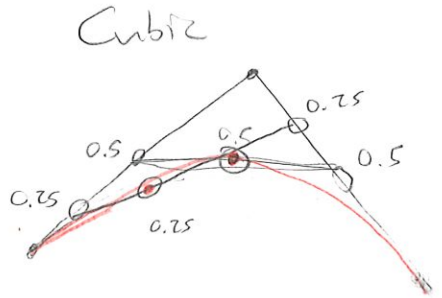
Slide: plot the blending functions

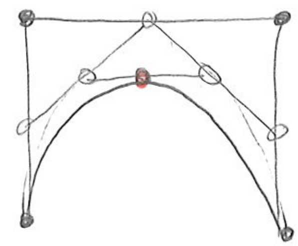
$$(\vec{u}^T B)^T = \begin{bmatrix} 1 - 3u + 3u^2 + u^3 \\ 3u - 6u^2 + 3u^3 \\ 3u^2 - 3u^3 \\ u^3 \end{bmatrix}$$

$$f(u) = \begin{bmatrix} (1-u)^3 \\ 3u(1-u)^2 \\ 3u^2(1-u) \\ u^3 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Bezier curves exist for <u>any</u> order!

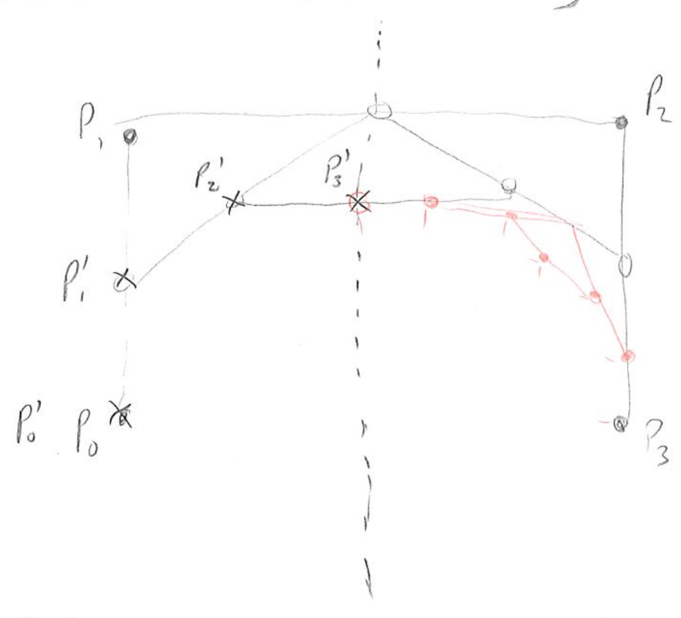
Bézier - Geometric View





Animated Demo

One more crazy cool thing:



Midpoints become new control points for half the curve!

Recursively subdivide, and you have an arbitrarily close piecewise linear approximation to the curve!

Drawing algorithm - subdivide until some error metric is small, draw line segments.

$$f(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3$$

Bezier: Algebraic Details:

$$P_0 = f(0) = a_0$$

$$P_3 = f(1) = a_0 + a_1 + a_2 + a_3$$

scale for convenience

$$3(P_1 - P_0) = f'(0) = a_1 \longrightarrow \cancel{3}P_1 = \tfrac{1}{3}a_1 + \cancel{3}P_0$$

$$3(P_3 - P_2) = f'(1) = a_1 + 2a_2 + 3a_3 \longrightarrow 3P_2 = 3P_3 - a_1 - 2a_2 - 3a_3$$

$$\cancel{3}P_2 = \cancel{3}(a_0 + a_1 + a_2 + a_3) - \tfrac{1}{3}a_1 - \tfrac{2}{3}a_2 - \tfrac{3}{3}a_3$$

$$P_2 = a_0 + a_1 + a_2 + a_3 - \tfrac{1}{3}a_1 - \tfrac{2}{3}a_2 - a_3$$