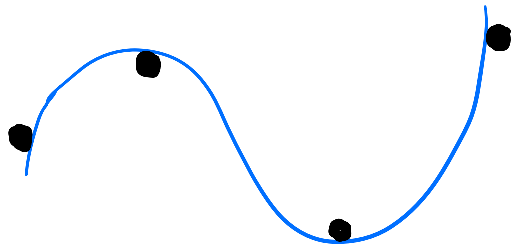
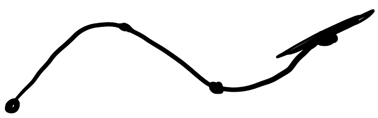
CSCI 480/580 - Lecture 28: Splines



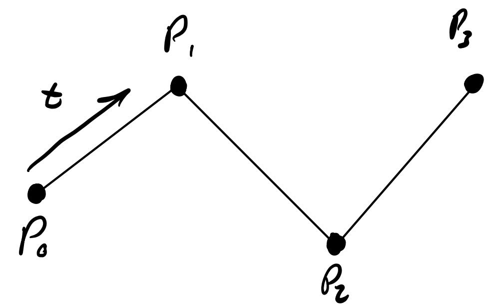
Physical world:
- pegs for control
- physics for smoothness

Math:
- points for control
- low-degree polynomials for smoothness



piecewise

Start Simple: Linear Interpolation



Reminder: 2 ways to write a line:

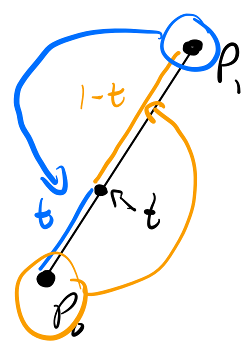
$y = mx + b$ (implicit)
or $ax + by + c = 0$

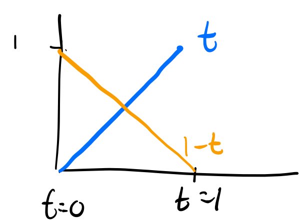
$r(t) = \vec{p} + t\vec{d}$ (parametric)

For 2 points P_0, P_1: $r(t) = P_0 + (P_1 - P_0)t$

$P_0 + P_1 t - P_0 t$

$r(t) = (1-t)\vec{P}_0 + t\vec{P}_1$





Convention: use u as a parameter that only ranges from 0 to 1.

$$f(u) = (1-u)\vec{P}_0 + u\vec{P}_1$$

$$\boxed{\begin{aligned} a_0 &= P_0 \\ a_1 &= P_1 - P_0 \end{aligned}}$$

$$f(u) = a_0 + u \cdot a_1$$

$$= u\, \underline{a_0} + u^1 \cdot \underline{a_1}$$

$$\left. f(u) = [u^0 \quad u^1] \begin{bmatrix} \underline{a_0} \\ \underline{a_1} \end{bmatrix} \right\}$$ friendly for computation (rendering)

$$P_0 = f(0) = [0^0 \quad 0^1] \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = a_0$$

$$P_1 = f(1) = [1^0 \quad 1^1] \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = a_0 + a_1$$

$$\vec{p} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \vec{a} \qquad \vec{p} = C\vec{a}$$

Control

$$\vec{a} = C^{-1}\vec{p}$$

$$\vec{a} = B\vec{p}$$

basis matrix

$$f(u) = \vec{u}^T \vec{a}$$

$$= \underline{u^T B \vec{p}}$$

Quadratic $f(u) = u^0 a_0 + u^1 a_1 + u^2 a_2$

$$P_0 = f(0) = [1 \quad 0 \quad 0] \cdot [a_0 \; a_1 \; a_2]^T = a_0$$

$$P_1 = f(0.5) = [1 \quad 0.5 \quad 0.25] \cdot \ldots = a_0 + 0.5 a_1 + 0.25 a_2$$

$$P_2 = f(1) = [1 \quad 1 \quad 1]$$

$$\vec{P} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0.5 & 0.25 \\ 1 & 1 & 1 \end{bmatrix}}_{C} \vec{a} \qquad C^{-1} = B = \begin{bmatrix} 1 & 0 & 0 \\ -3 & -4 & -1 \\ 2 & -4 & 2 \end{bmatrix}$$

$$u^T \; B \; \vec{P}$$

$P_0 = f(0.5) =$ $\qquad f(u) = u^0 a_0 + u^1 a_1 + u^2 a_2$

$P_1 = f'(0.5)$ $\qquad \rightarrow f'(u) = a_1 + 2u a_2$

$\qquad f''(u) = 2a_2$

$P_2 = f''(0.5)$

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \vec{P} \end{bmatrix} = C \begin{bmatrix} 1 & 0.5 & 0.25 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \vec{a}_0 \\ \vec{a}_1 \\ \vec{a}_2 \end{bmatrix}$$