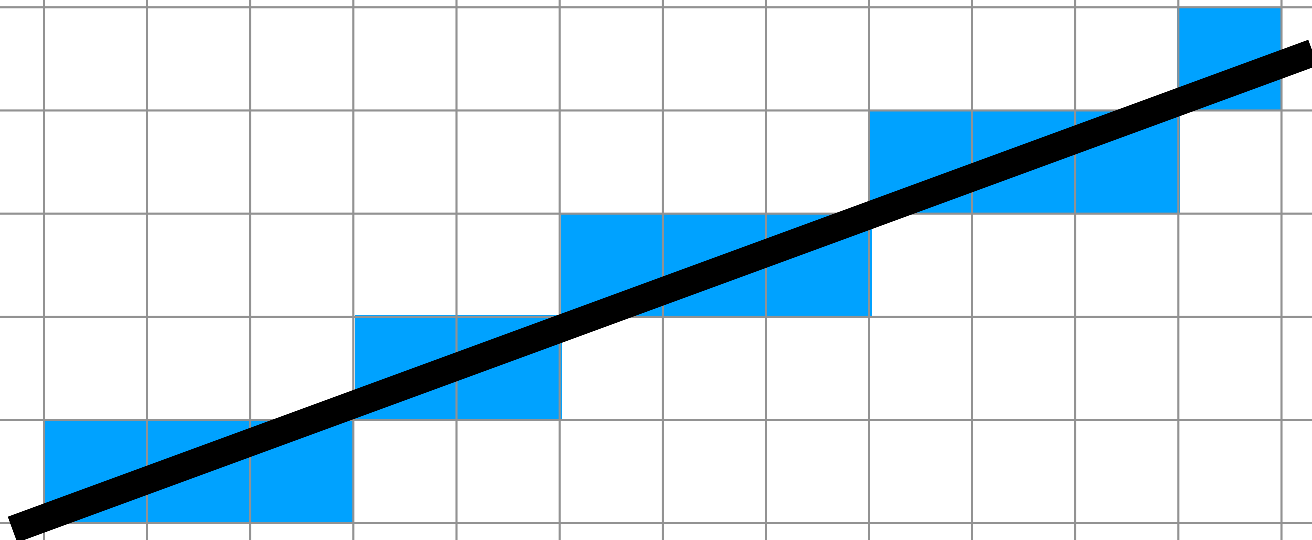


Computer Graphics

Lecture 25

Rasterizing Lines

Logistics: Final Project

- Project proposals due tonight
  - Ideally you will have done enough investigation to conclude that your plan is achievable.
  - Err on the side of ambition
- Slip days can't be used on any final project deadline
- *"...although the intermediate deadlines have only small point values associated with them on Canvas, your Final Project Report grade will take the quality, timeliness, etc. of intermediate deliverables into consideration."*

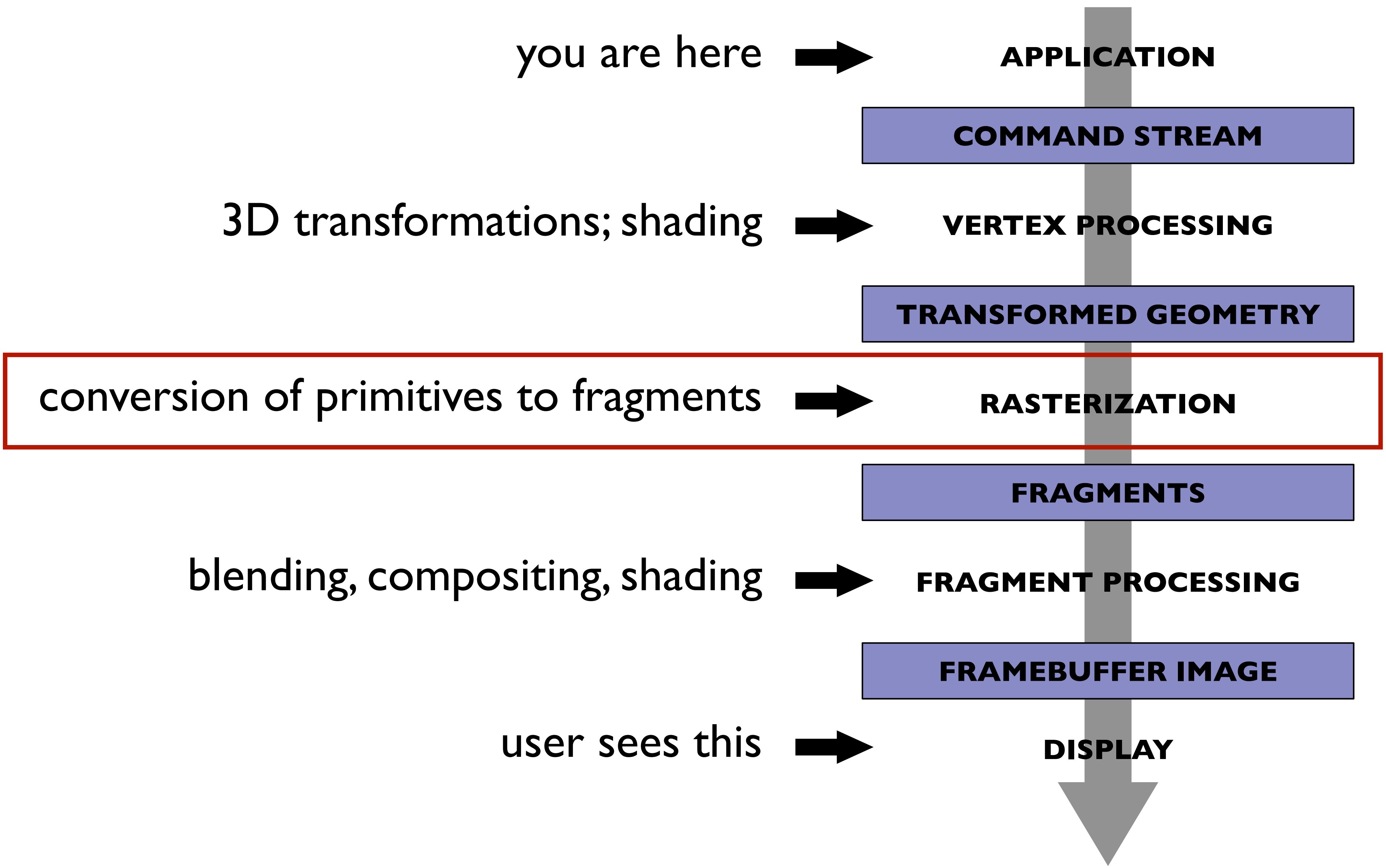
Logistics: Exam

- Exam out today
  - Do not discuss with anyone; do not use resources other than those linked from the course webpage

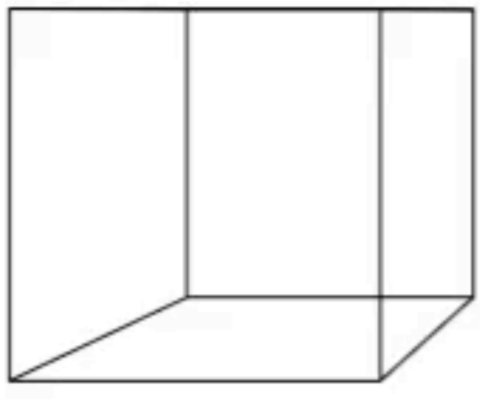
Goals

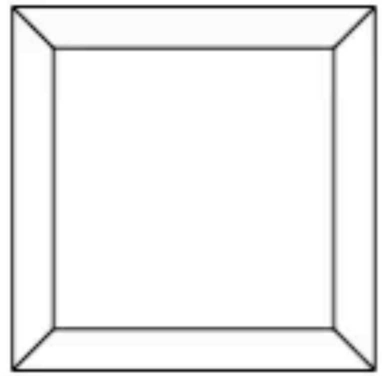
- Know how to draw lines using point sampling, and why this causes variable apparent line widths.
- Know how to draw lines with slope between 0 and 1 using the midpoint algorithm.
- Know how to draw lines with any slope by adjusting the inputs to the midpoint algorithm.
- Know how to interpolate arbitrary quantities across a line drawn using the midpoint algorithm.

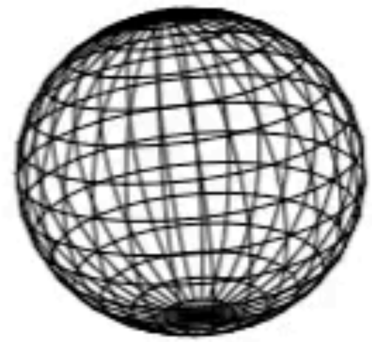
Graphics Pipeline: Overview



Remember Wireframe?







$M = M_{vp}\, M_{proj}\, M_{view}\, M_{model}$

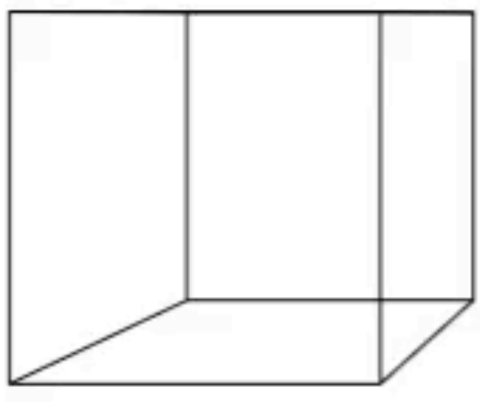
for each line segment $\mathbf{a}_i$, $\mathbf{b}_i$

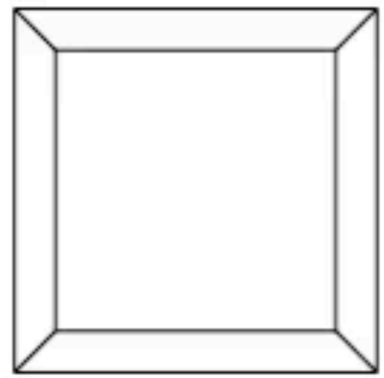
  $\mathbf{p} = \mathbf{M}\ \mathbf{a}_i$

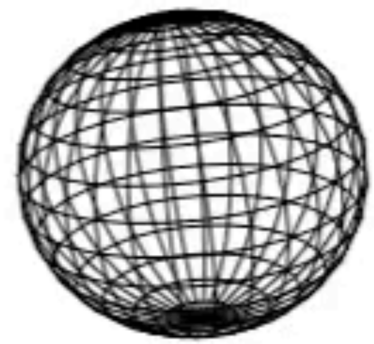
  $\mathbf{q} = \mathbf{M}\ \mathbf{b}_i$

  draw_line($\mathbf{p}$, $\mathbf{q}$)

Remember Wireframe?







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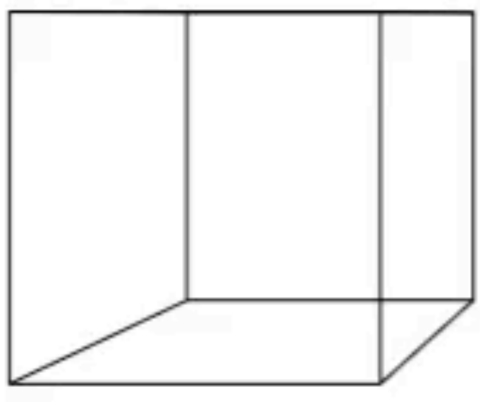
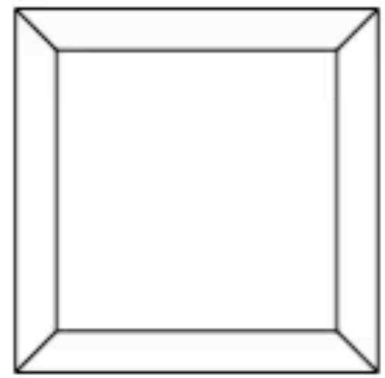
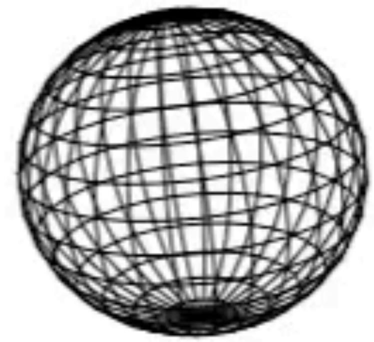
  draw_line(**p**, **q**)   How do we do this?

Remember Wireframe?







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  draw_line(**p**, **q**)

How do we do this?

Line Drawing

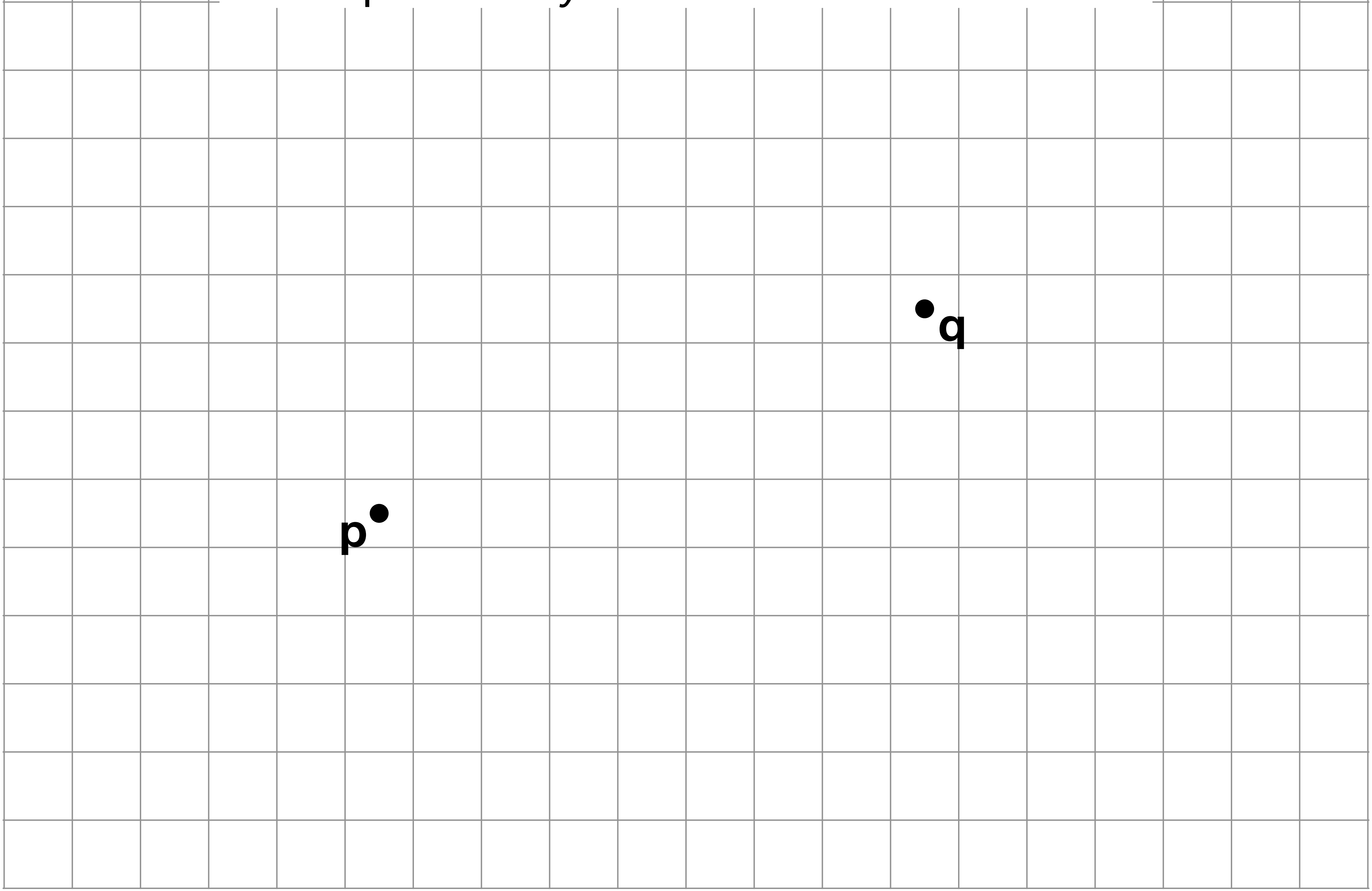
This is a **rasterization** problem:
given a primitive (line segment),
generate fragments (aspiring pixels)

```
M = Mvp Mproj Mview Mmodel
for each line segment ai, bi
  p = M ai
  q = M bi
  draw_line(p, q)
```

How do we do this?

Problem 1: **Artisanal Line Drawing**

Which pixels do *you* think should be filled in?



Problem 1: **Artisanal Line Drawing**

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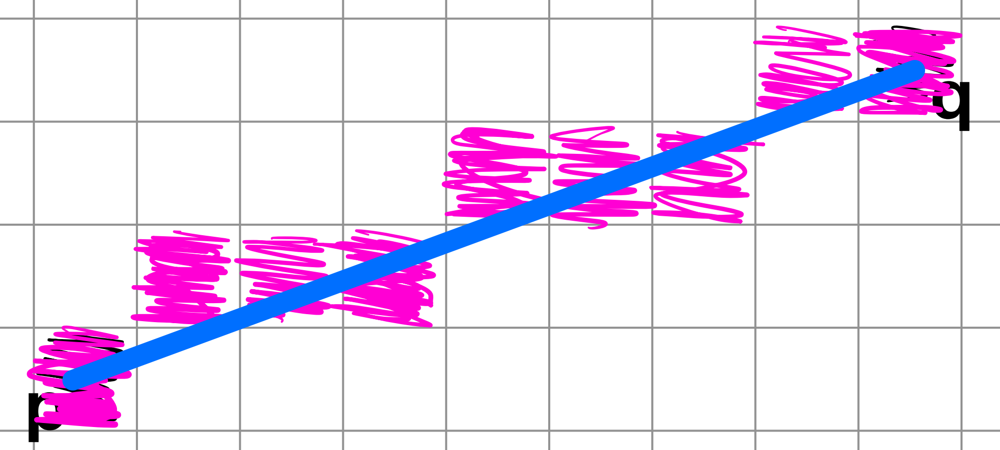
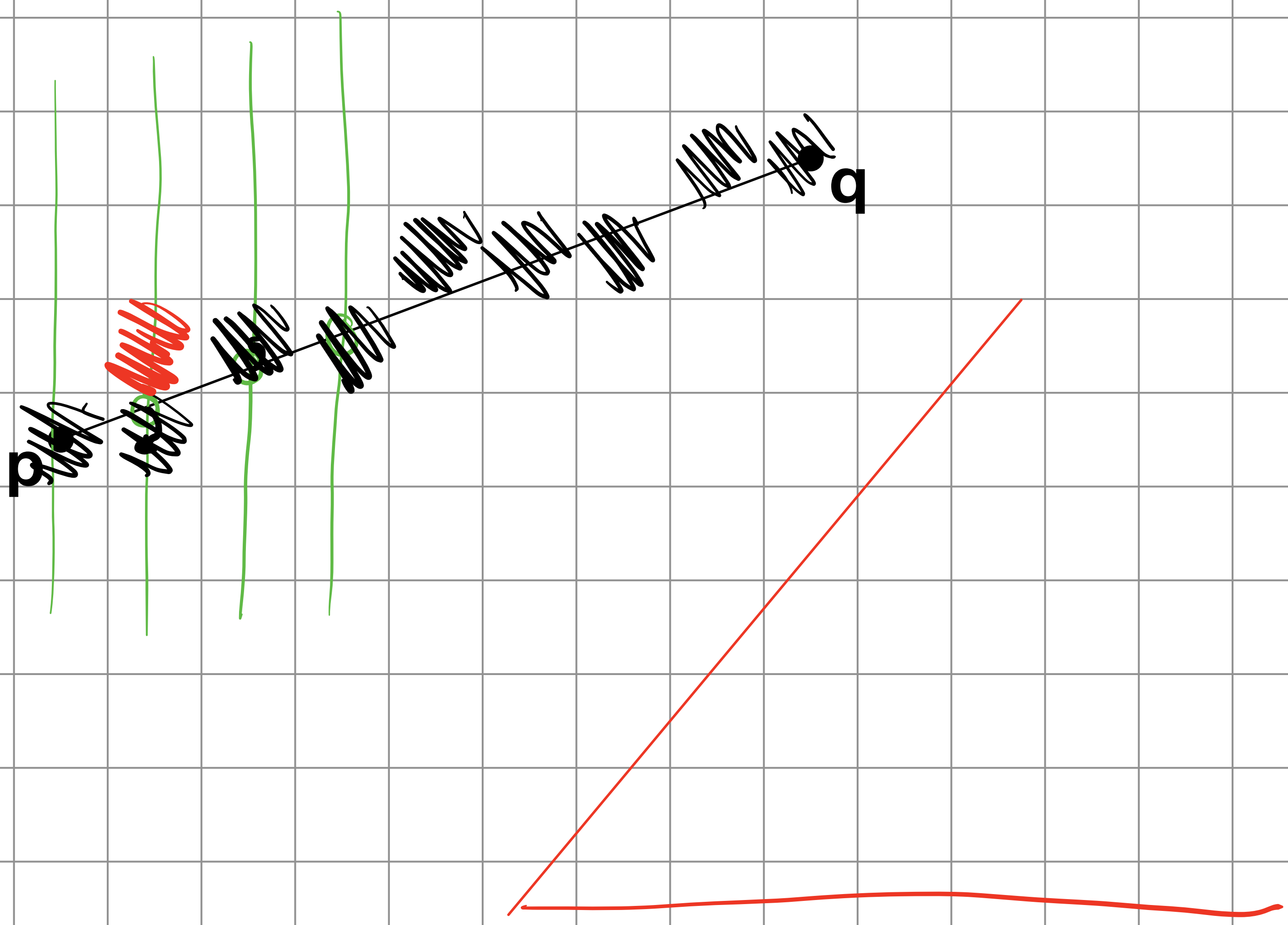


Compare your line to a neighboring group's line. Did you pick the same pixels?

Exercise 2:
Algorithm to draw a line?



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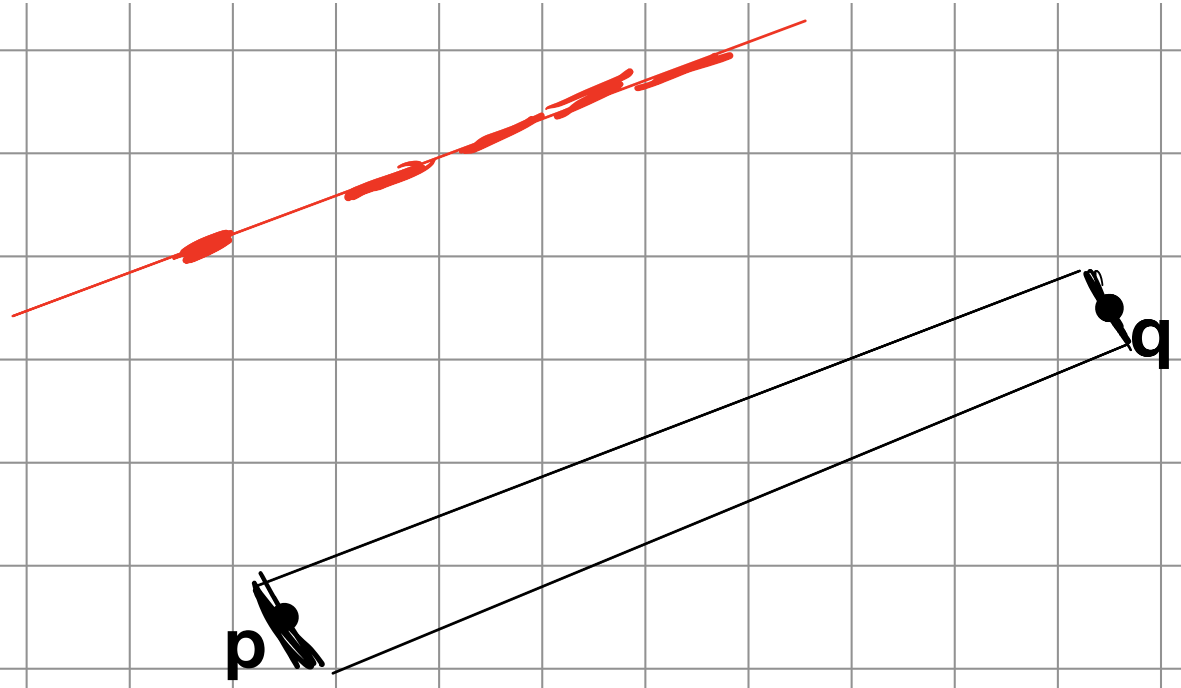
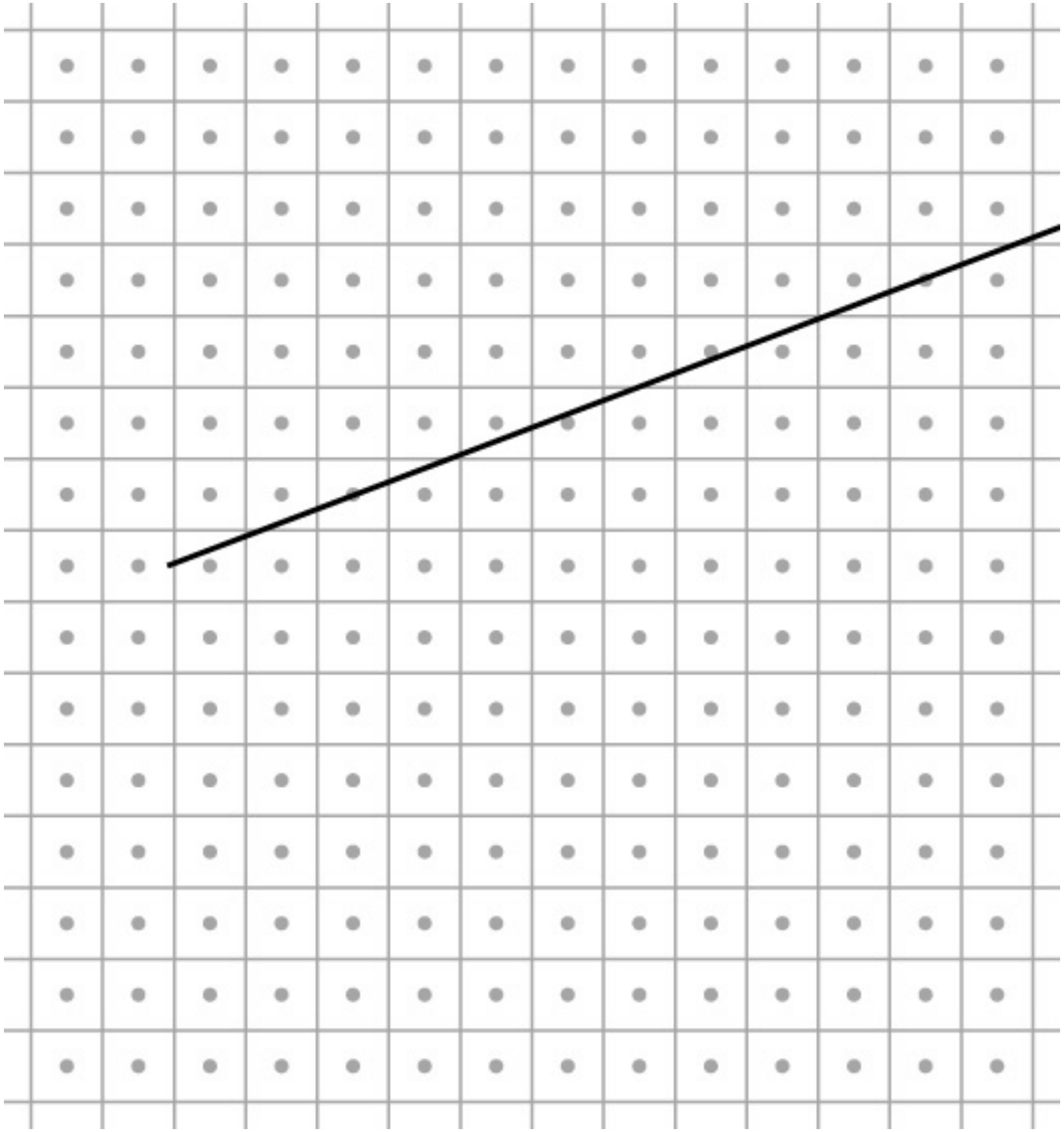


What makes a line good?

- uniform width / intensity
  - ↳ no 2 y's per x

Rasterizing lines - possible definition

- Define line as a rectangle
- Specify by two endpoints
- Ideal image: black inside, white outside



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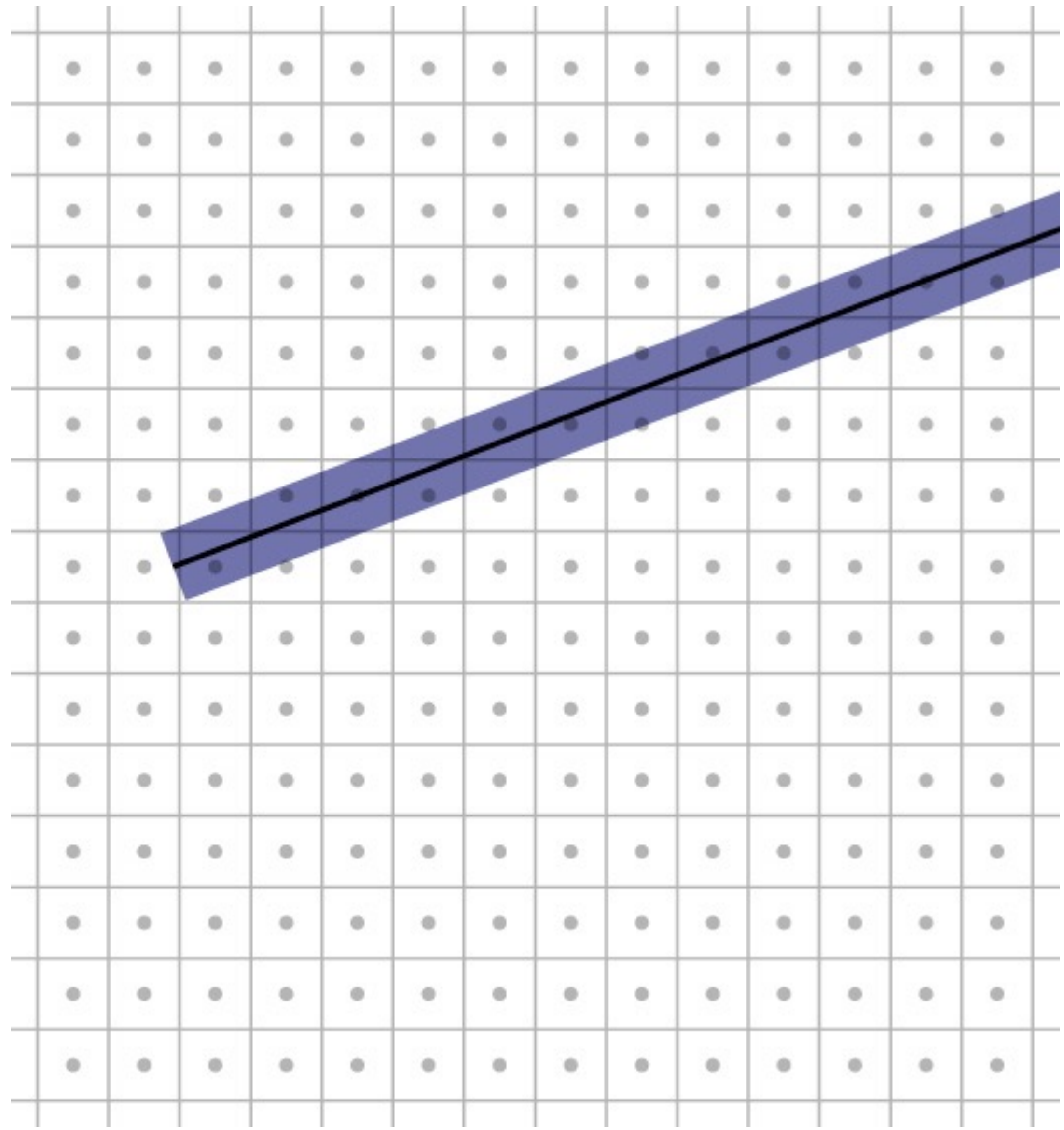
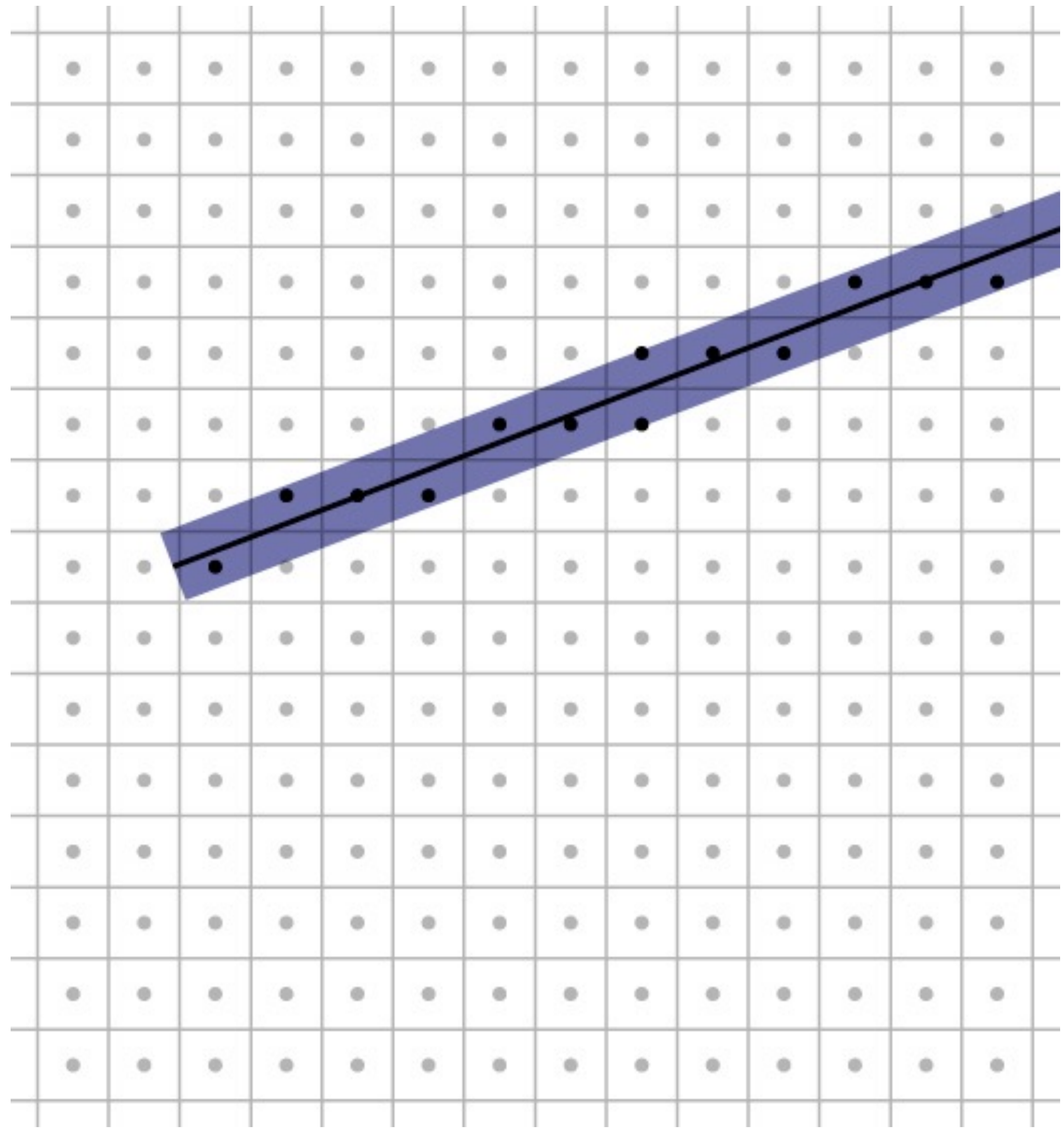


Point sampling

- Approximate rectangle by drawing all pixels whose centers fall within the line
- Problem: sometimes turns on adjacent pixels



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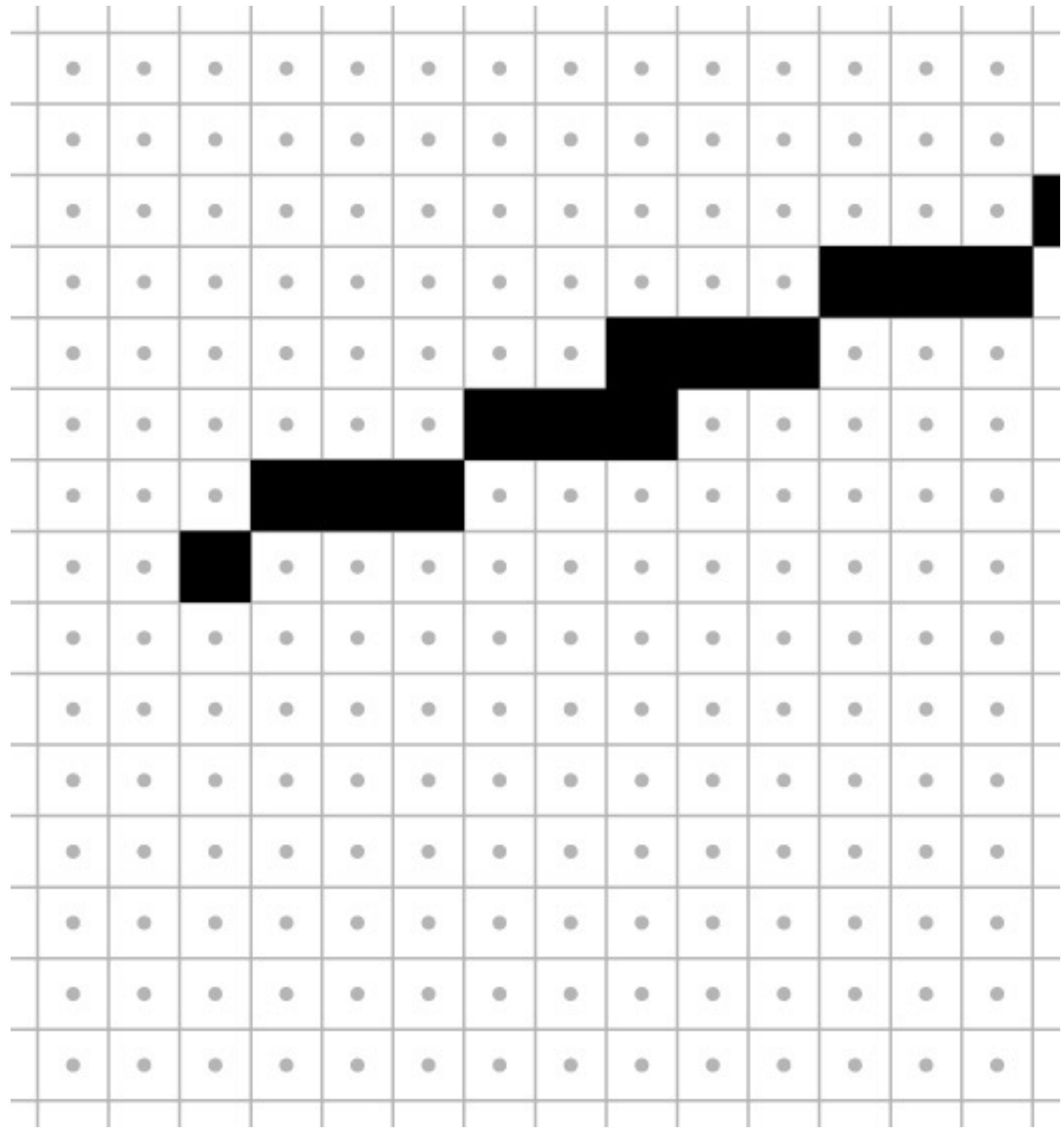
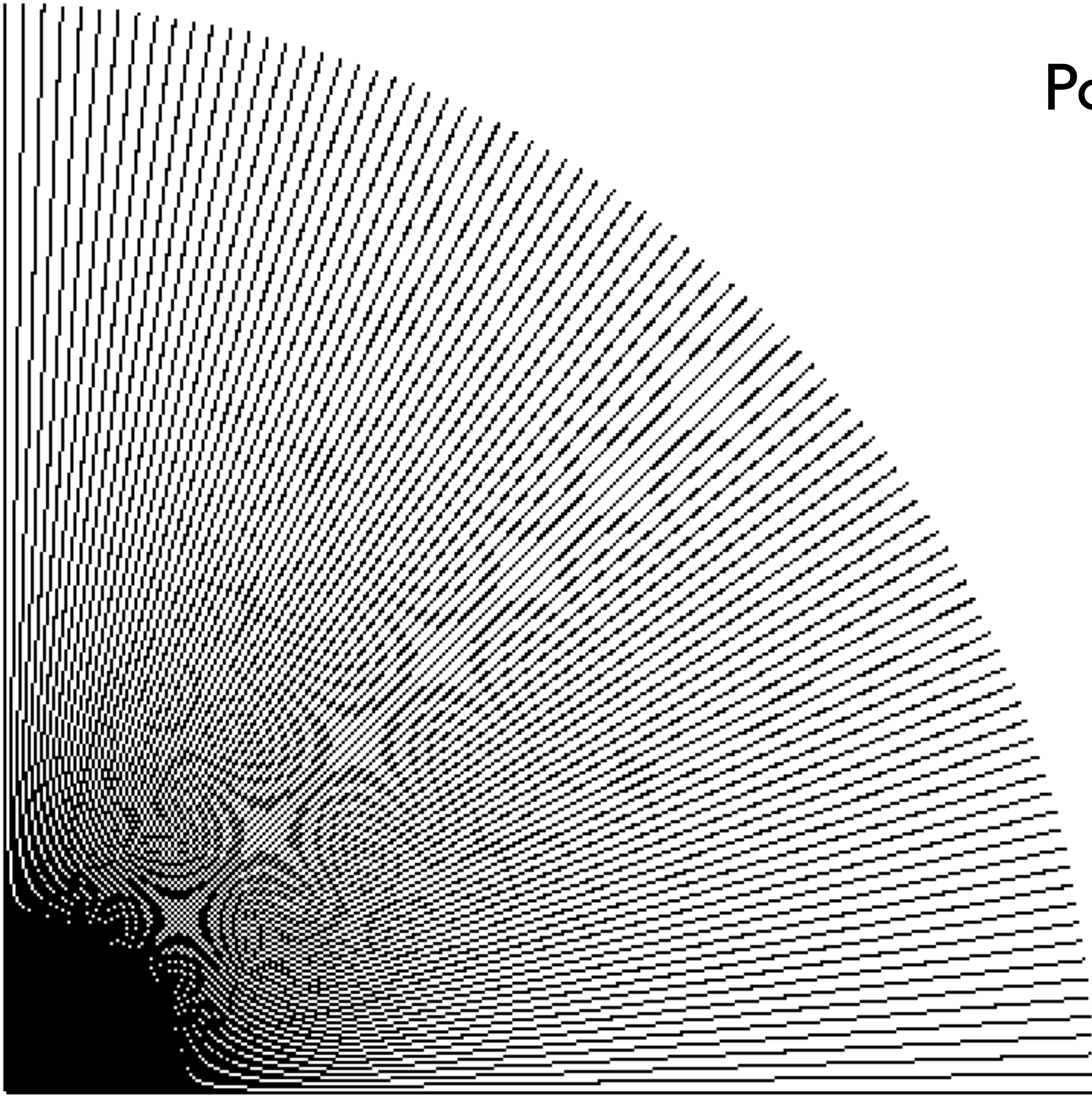
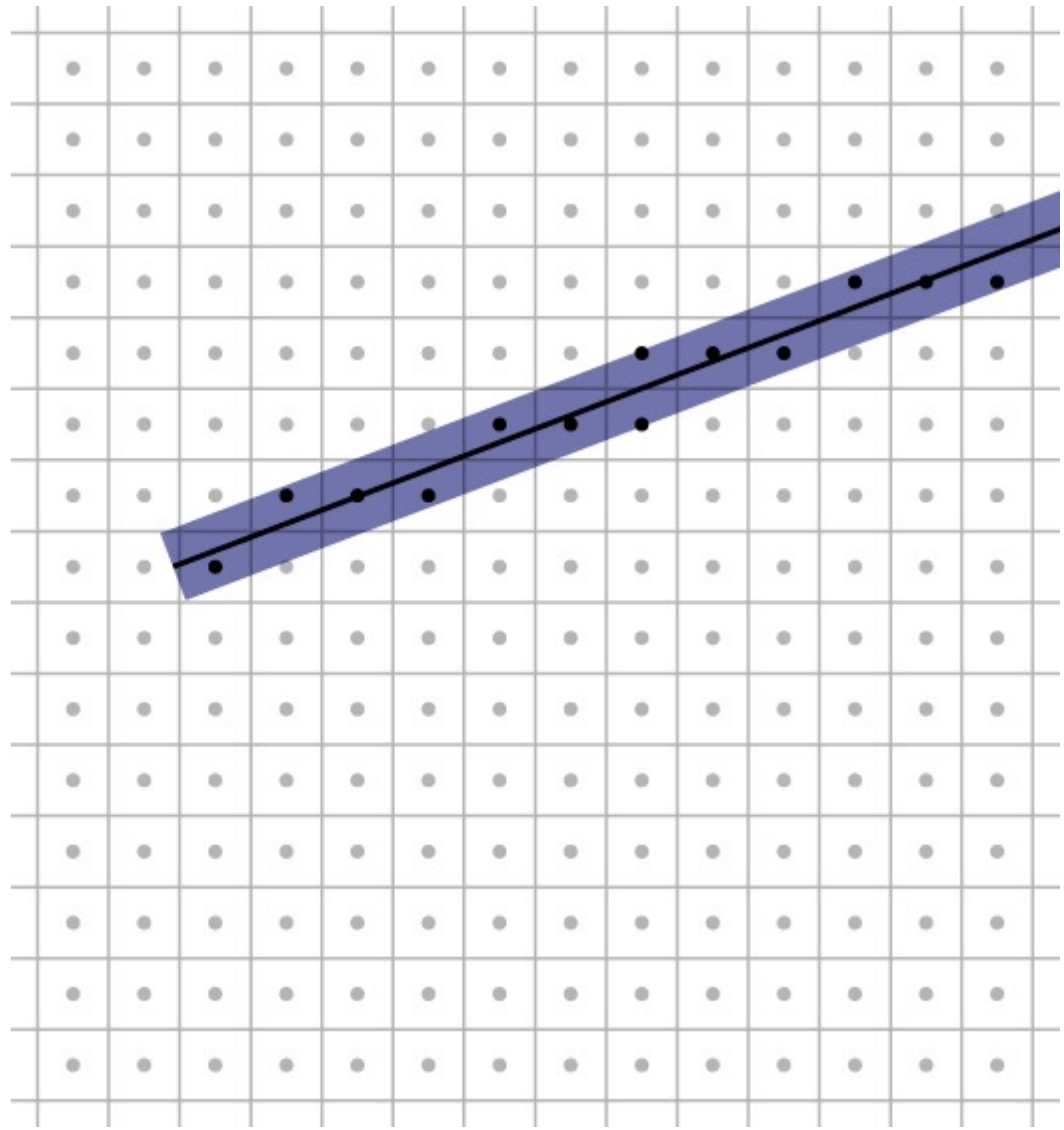




Point sampling in action

Bresenham lines (midpoint alg.)

- Point sampling unit width rectangle leads to uneven line width
- Define line width parallel to pixel grid
- That is, turn on the single nearest pixel in each column
- Note that 45° lines are now thinner



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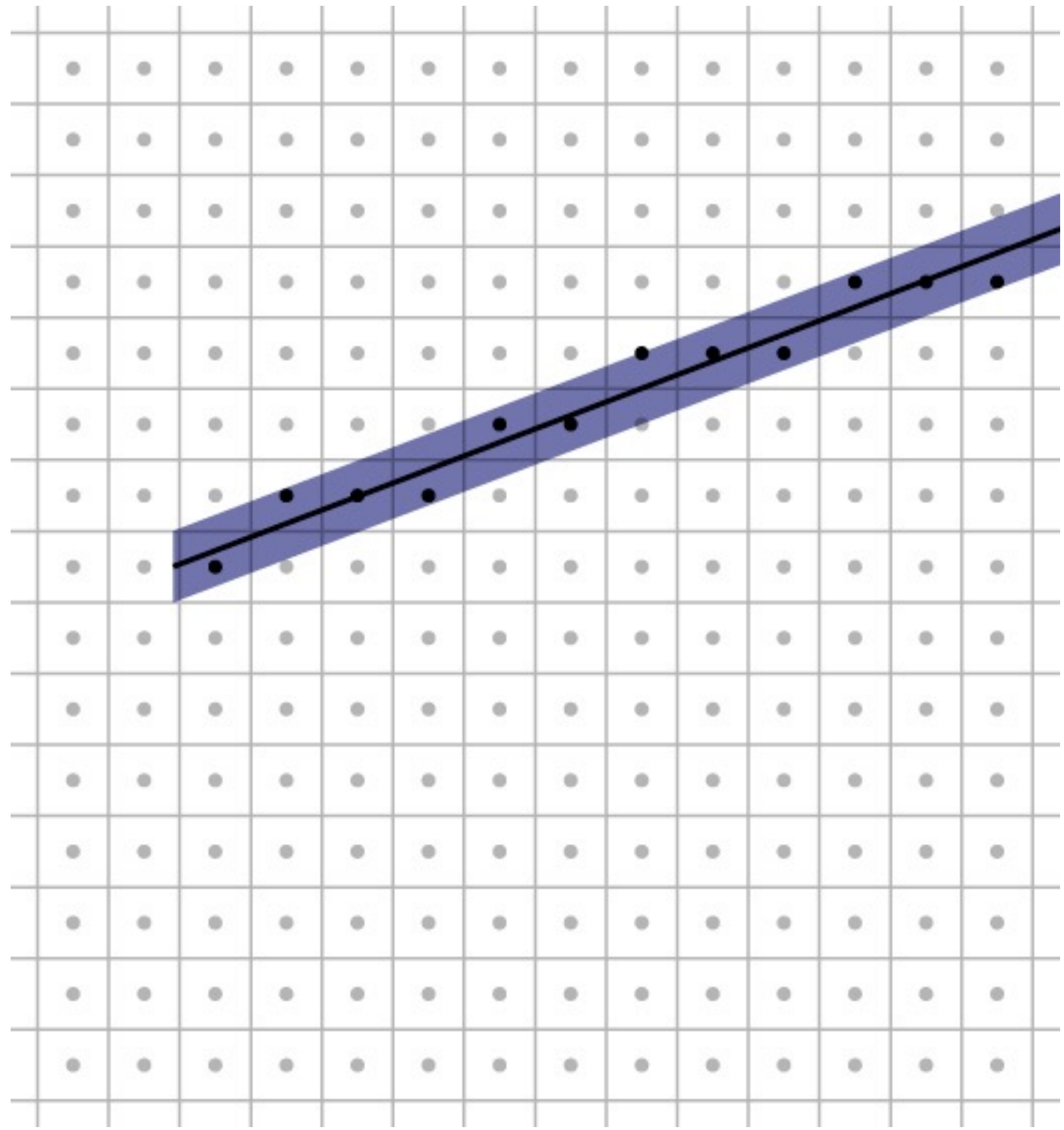

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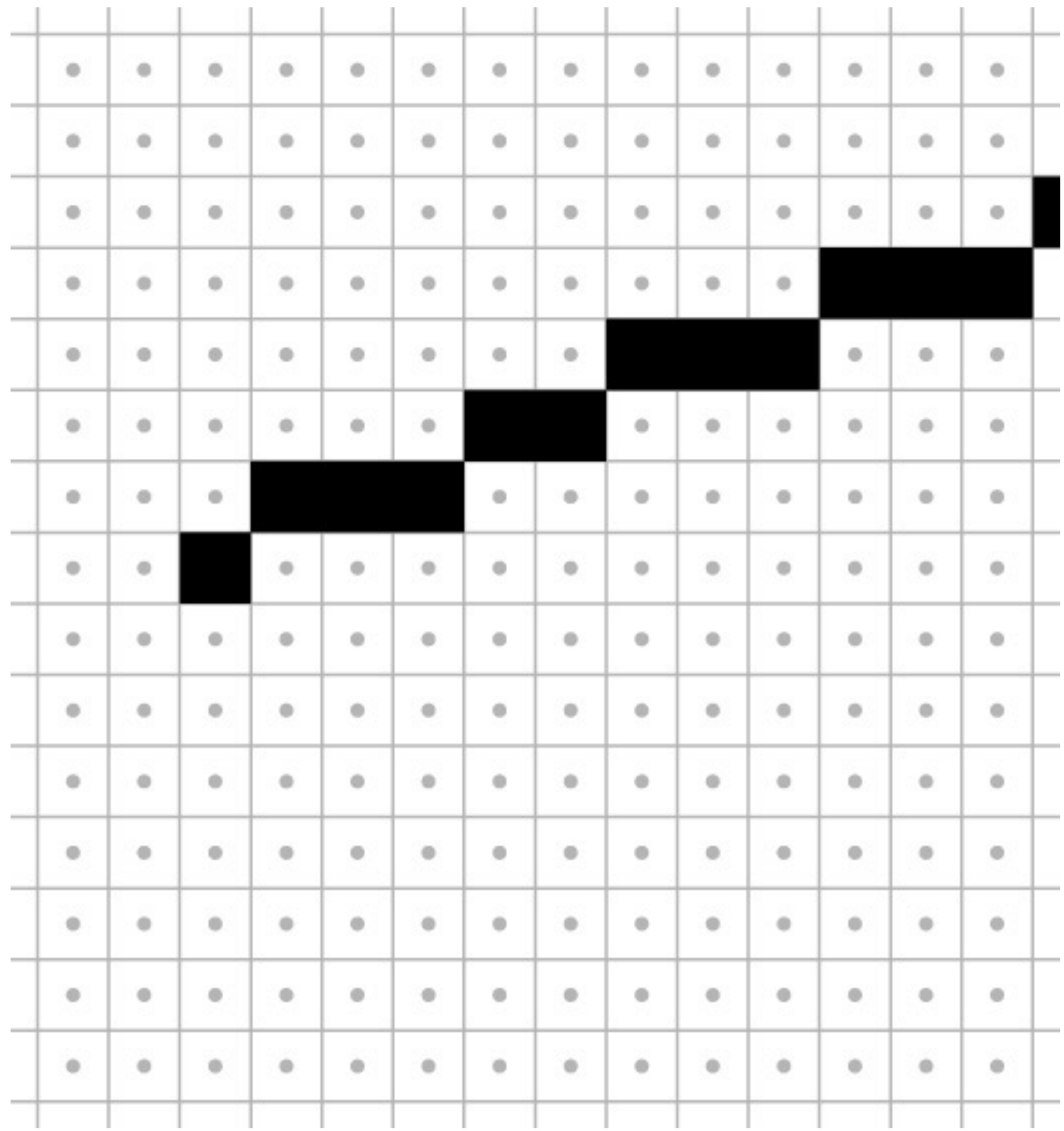
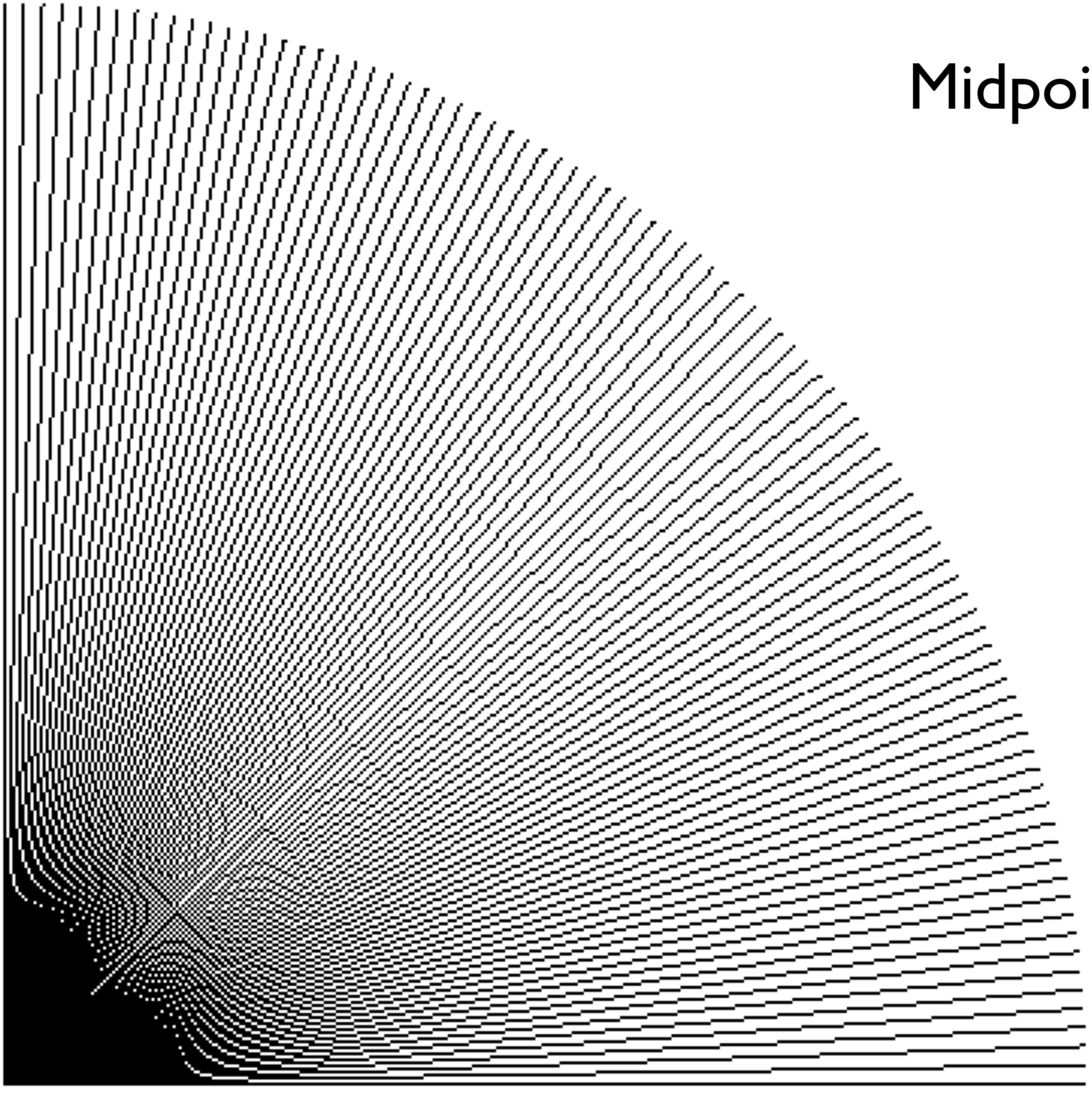
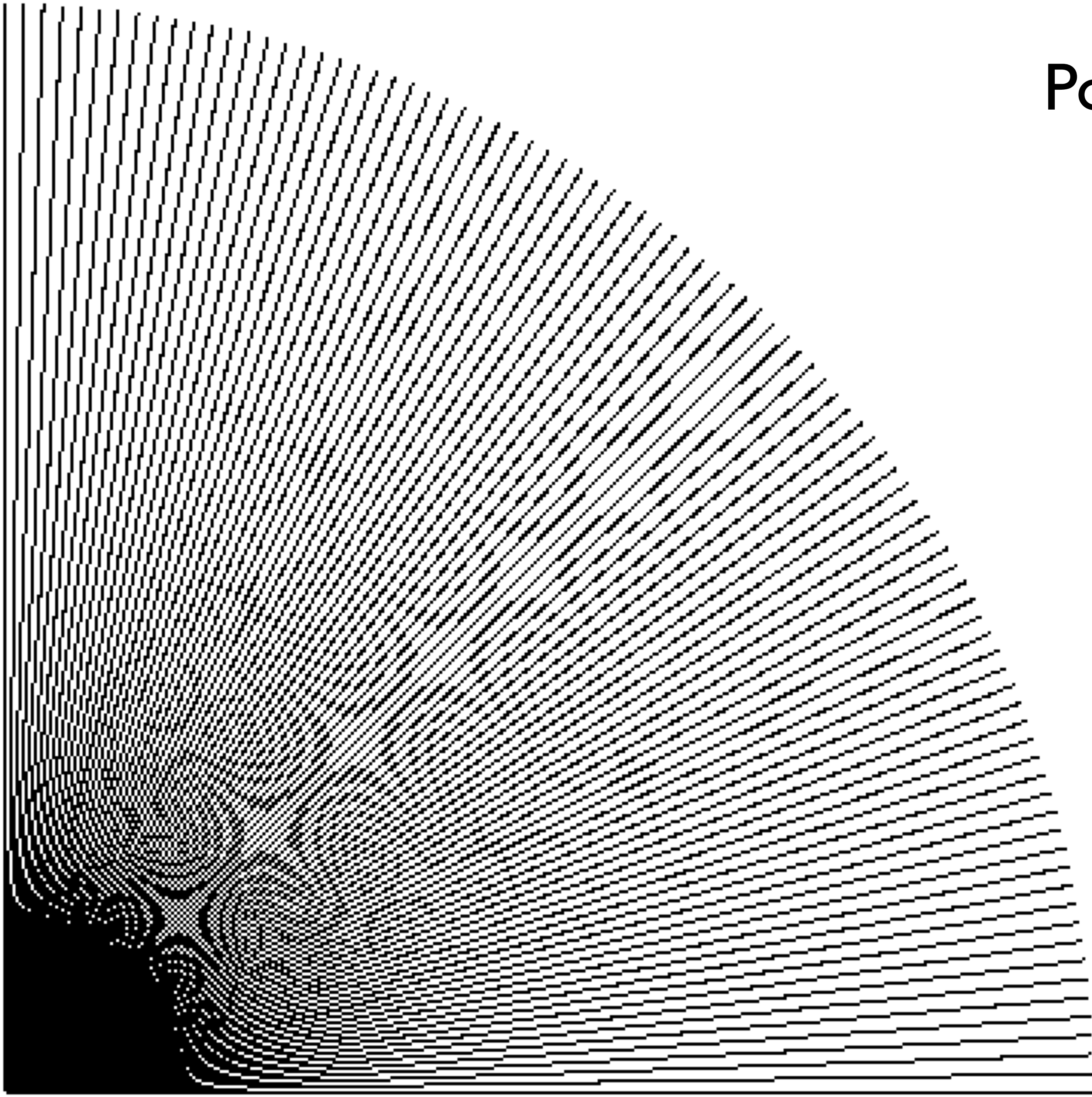




Midpoint algorithm in action

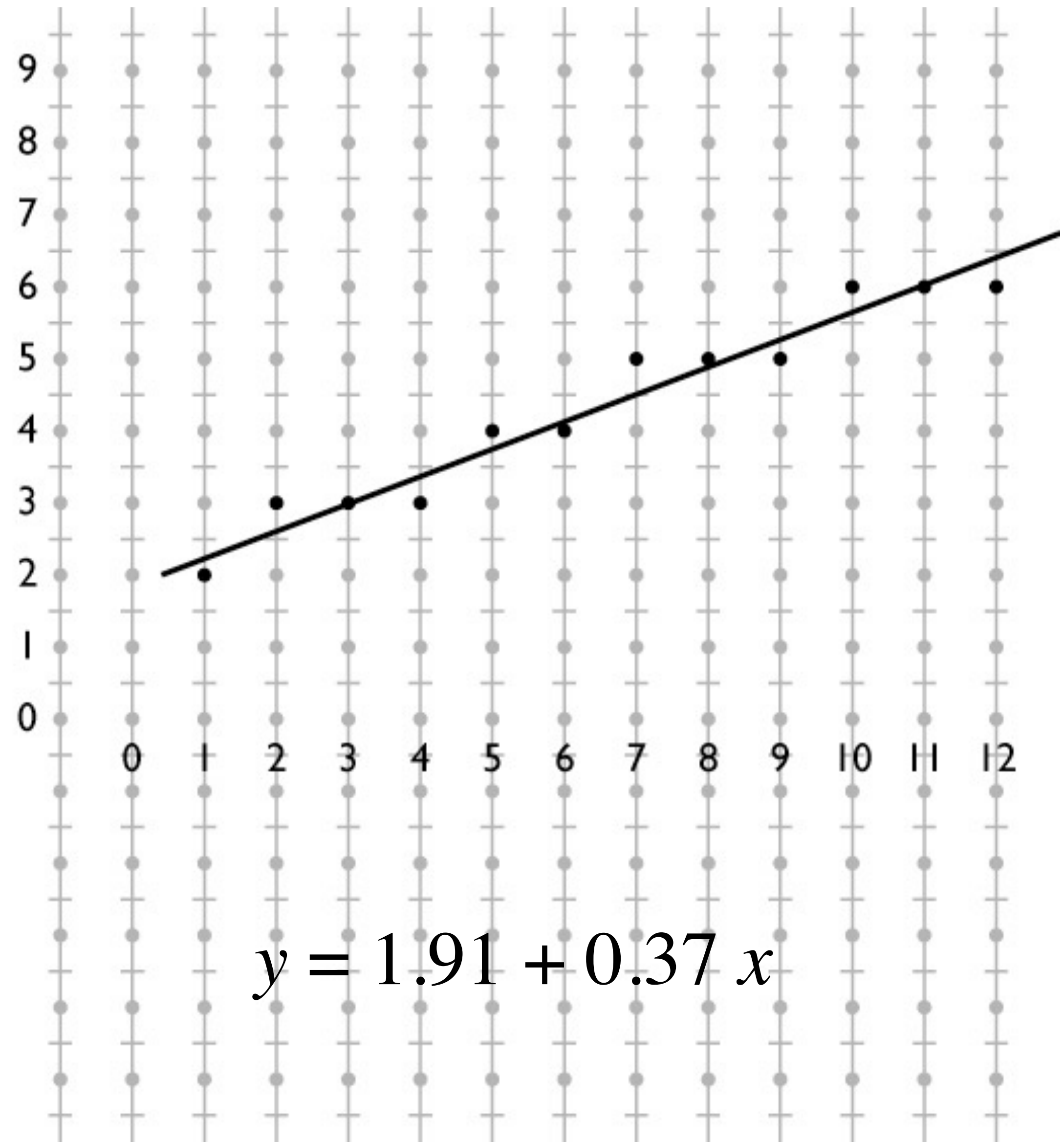


Point sampling in action

Notes:
Midpoint Algorithm

Midpoint Algorithm

- line equation:
  $y = b + m\,x$
- Simple algorithm: evaluate line equation per column
- W.l.o.g. $x_0 < x_1$;
  $0 \leq m \leq 1$



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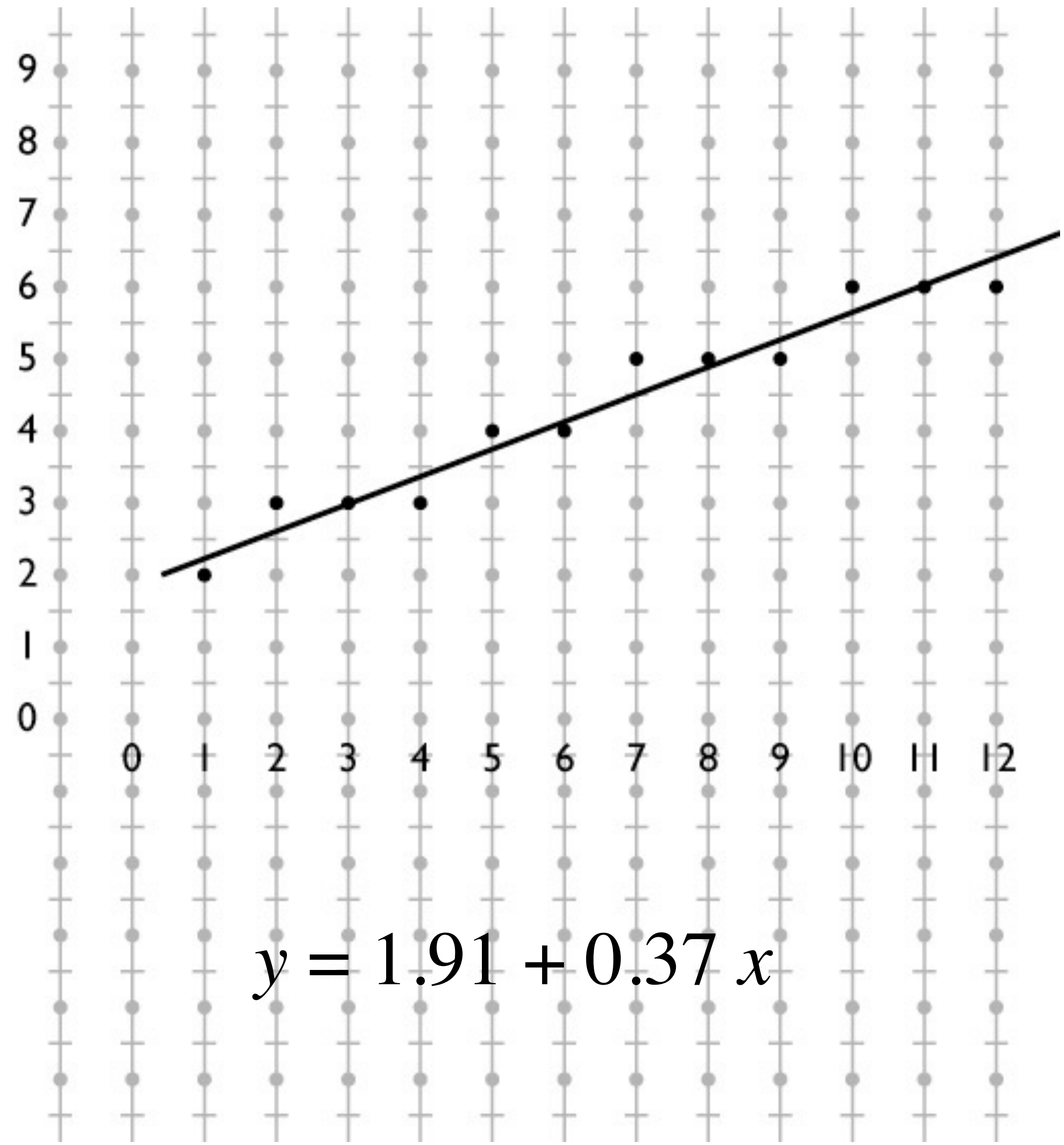

$$y = 1.91 + 0.37\,x$$

Algorithm:

Midpoint Algorithm

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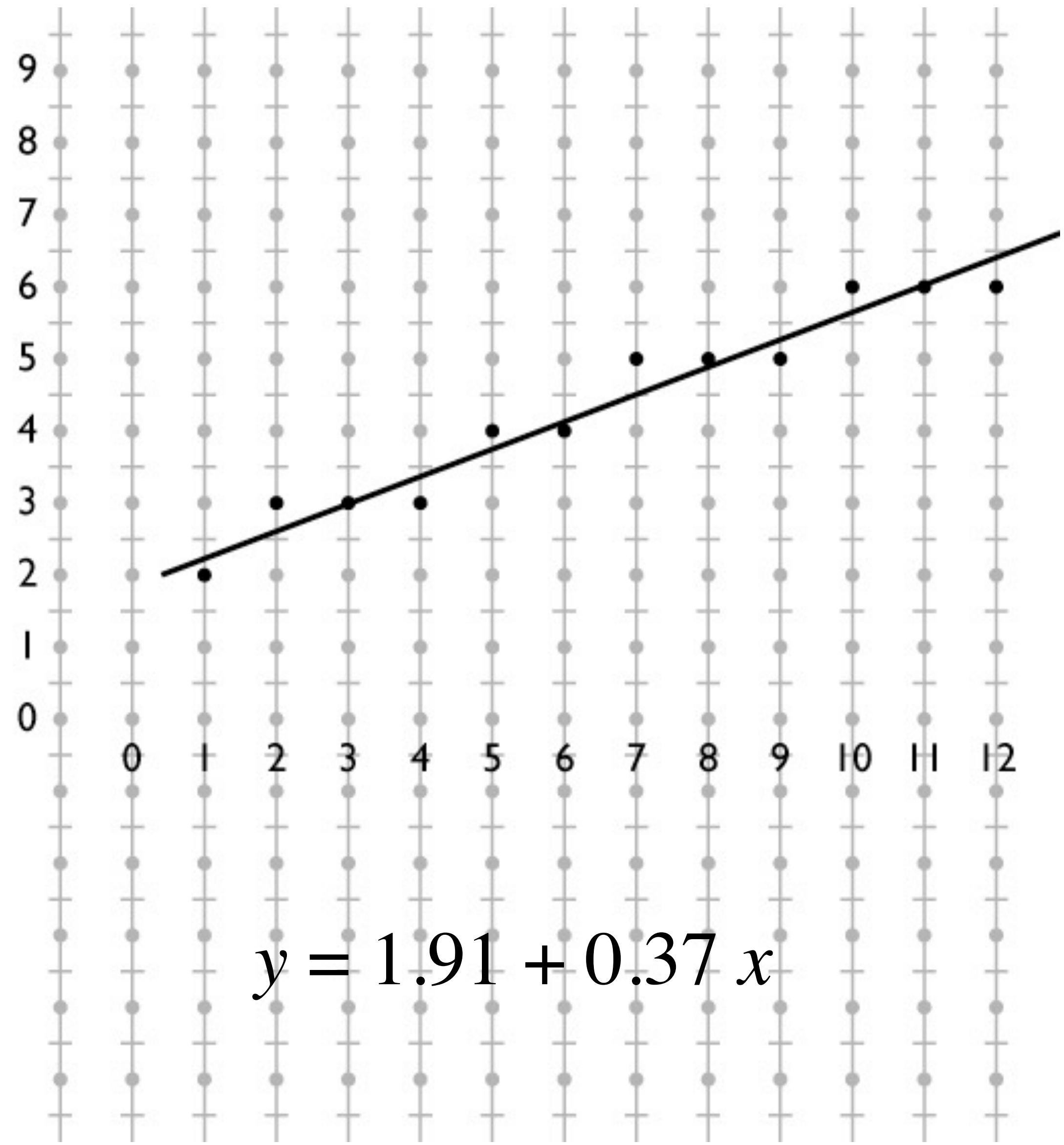
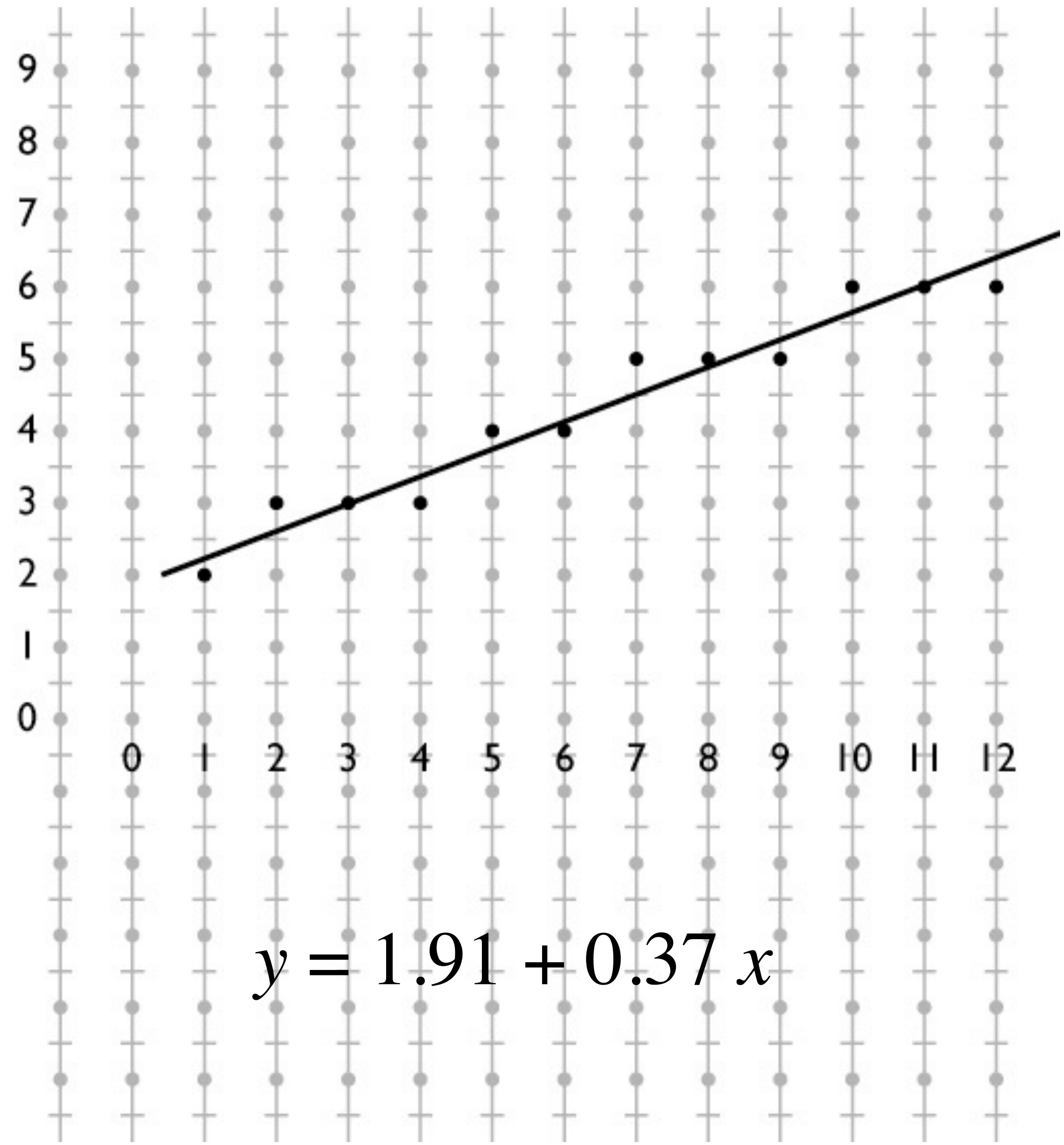


Algorithm:

```
// compute m, b

for x = ceil(x0) to floor(x1)
    y = b + m*x
   // Ex: what goes here?
```


Algorithms for drawing lines

- line equation:
  $y = b + m\,x$
- Simple algorithm: evaluate line equation per column
- W.l.o.g. $x_0 < x_1$;
  $0 \le m \le 1$



Algorithm:

```
// compute m, b

for x = ceil(x0) to floor(x1)
    y = b + m*x
    draw(x, round(y))
```

Optimizing Line Drawing

Can we take stuff out of the inner loop?

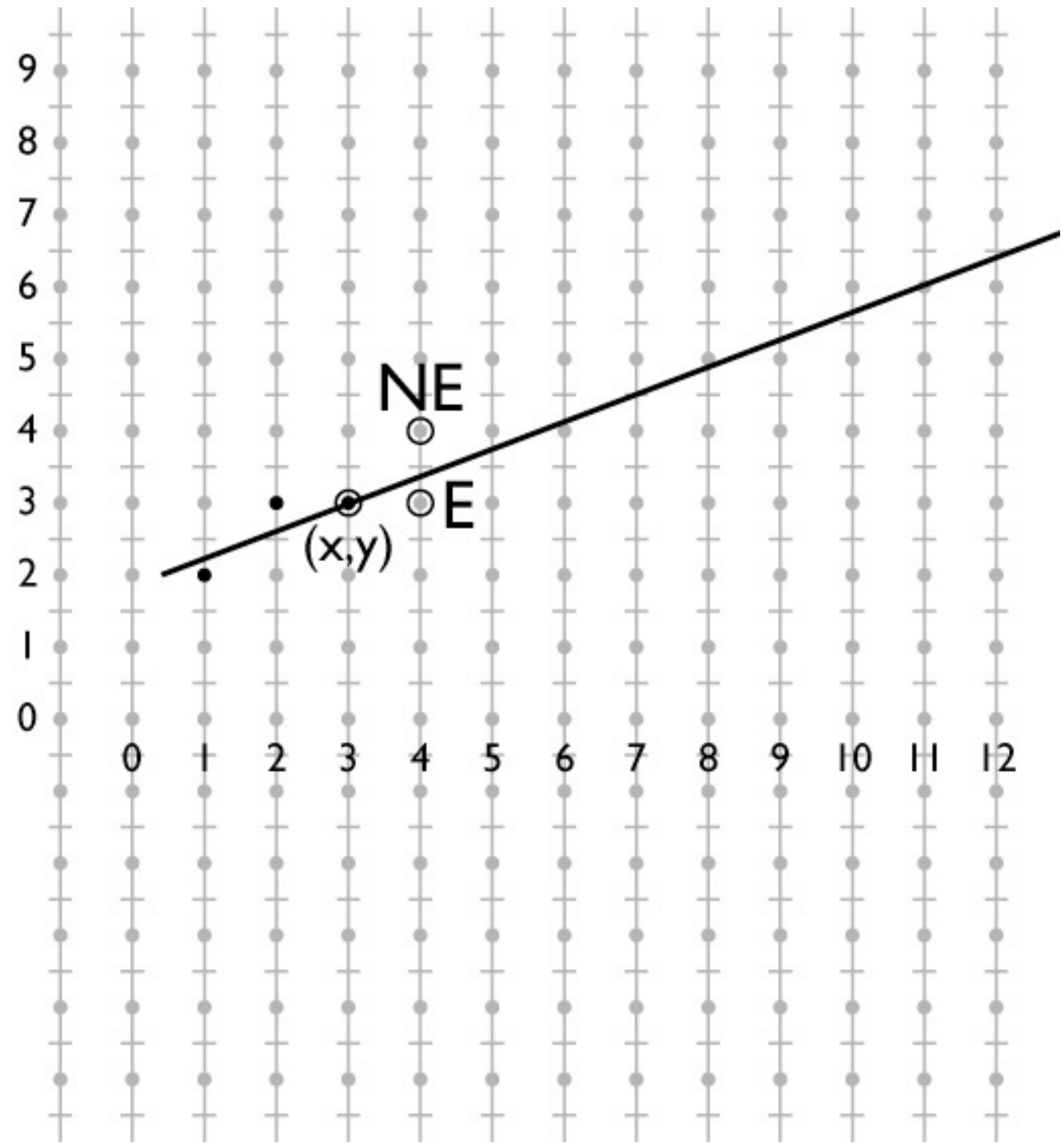
Exercise: optimize this

```
function slow_line(p1, p2):
  // compute m, b
  for x = ceil(x0) to floor(x1)
      y = b + m*x
      draw(x, round(y))
```

```
function fast_line(p1, p2):
  // compute m, b



  for x = ceil(x0) to floor(x1)


      draw(x, round(y))
```

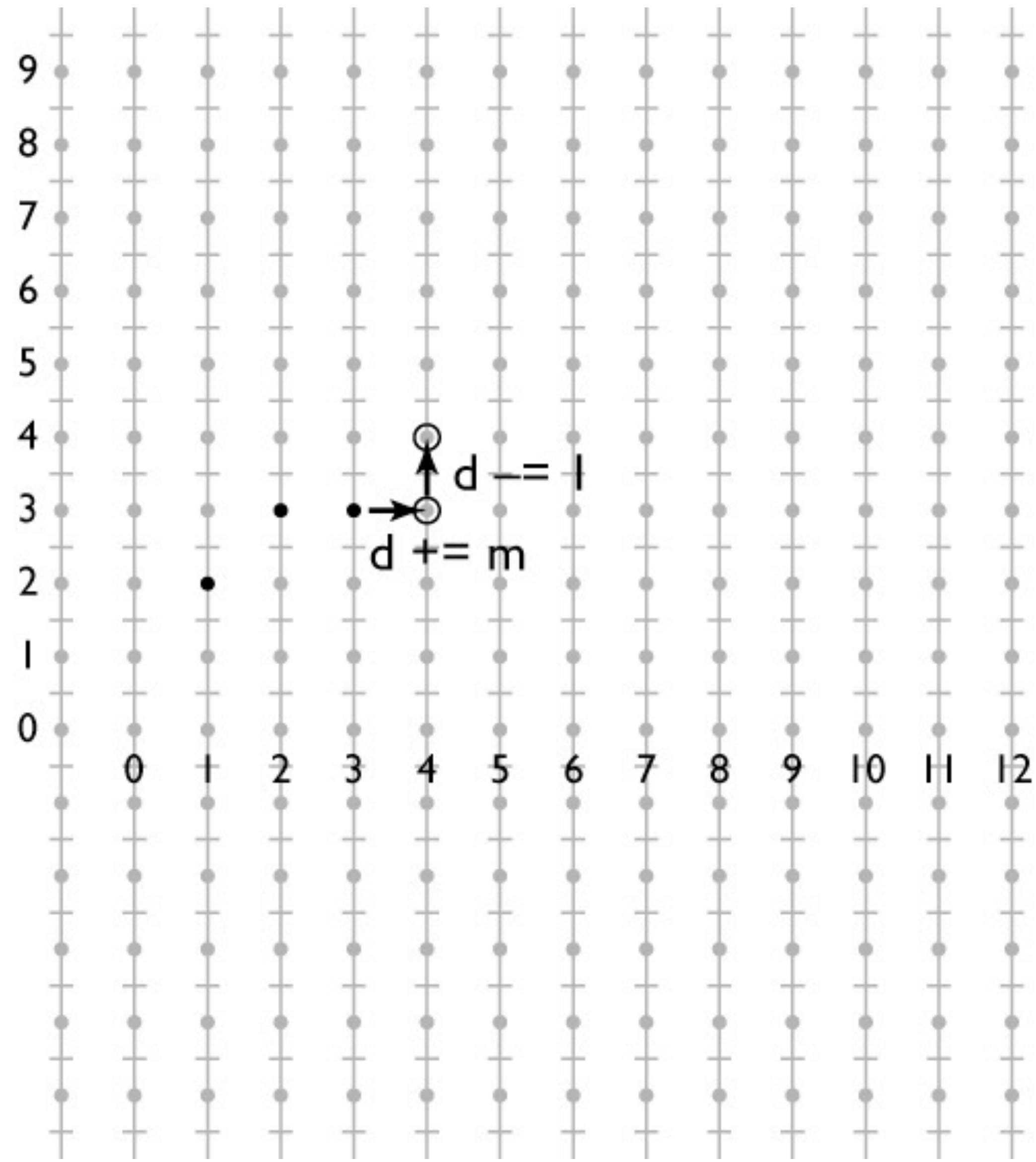
Optimizing Line Drawing Even More

- Rounding is slow too
- At each pixel the only options are E and NE
- Track distance to line:
  - $d = m(x + 1) + b - y$
  - $d > 0.5$ decides between E and NE



Optimizing Line Drawing Even More

- $d = m(x + 1) + b - y$
- Only need to update *d* for integer steps in *x* and *y*
- Do that with addition
- Known as "DDA" (digital differential analyzer)



Linear interpolation

- We often attach attributes to vertices
  - e.g. computed diffuse color of a hair being drawn using lines
  - want color to vary smoothly along a chain of line segments

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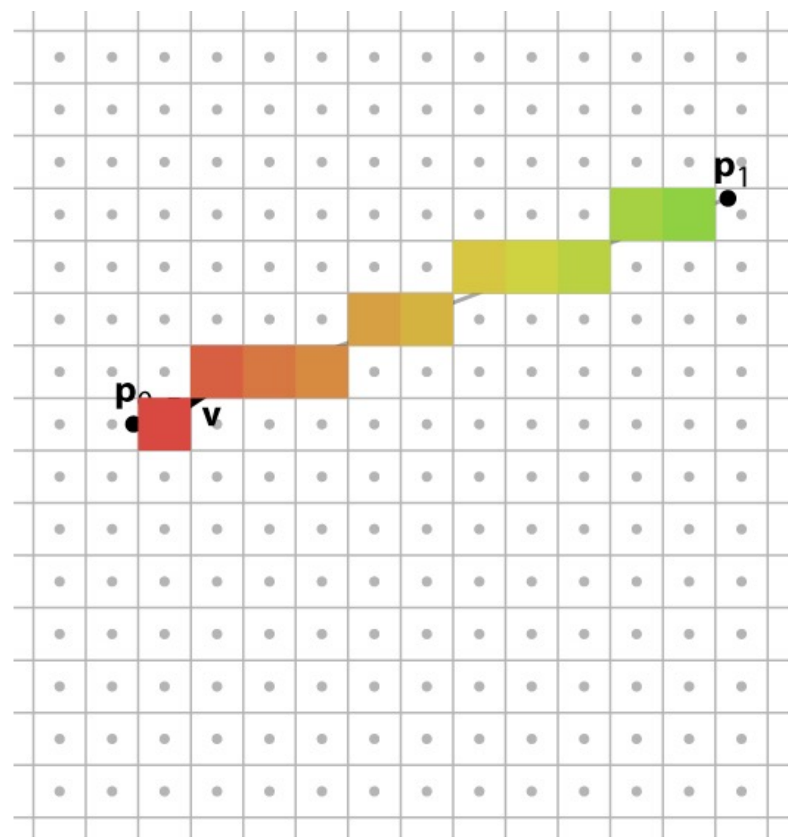
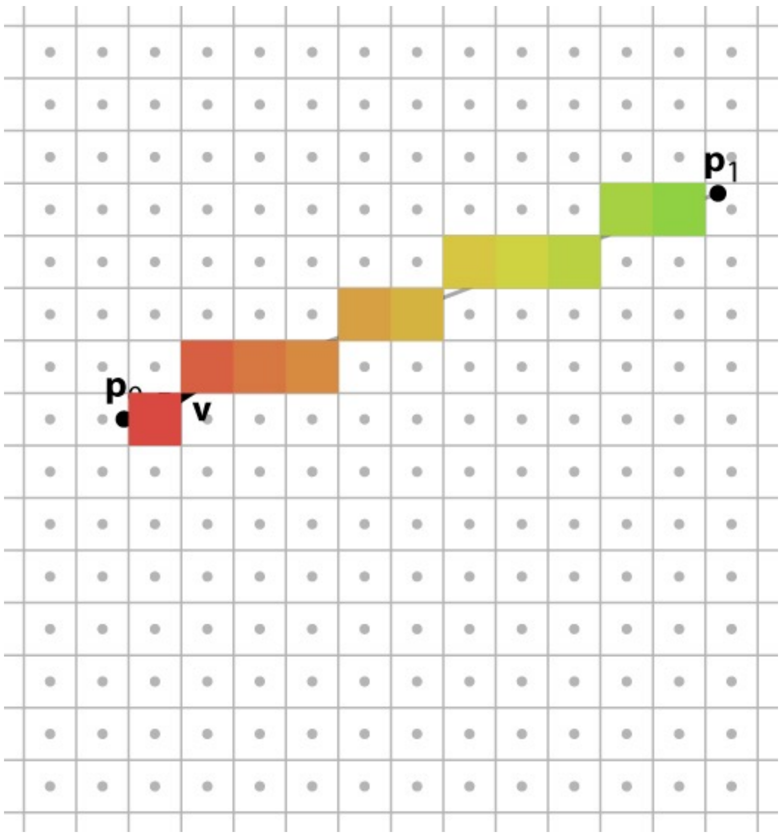


Linear interpolation

- We often attach attributes to vertices
  - e.g. computed diffuse color of a hair being drawn using lines
  - want color to vary smoothly along a chain of line segments

- Same machinery as we used for y works for other values!

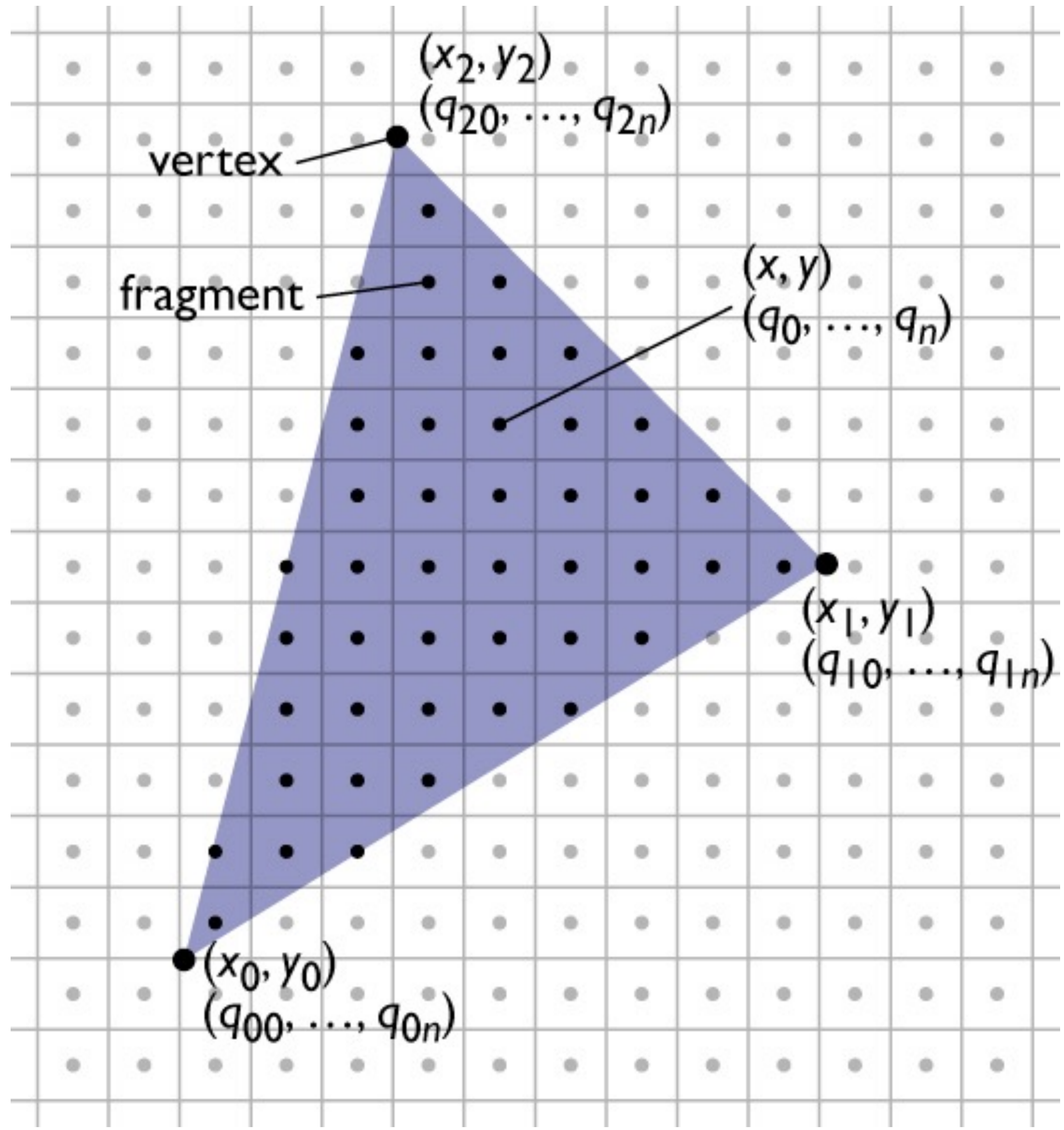


Rasterizing triangles

- Input:
  - three 2D points (the triangle's vertices in pixel space)
    - (x_0, y_0); (x_1, y_1); (x_2, y_2)
  - parameter values at each vertex
    - $q_{00}, \ldots, q_{0n}$; $q_{10}, \ldots, q_{1n}$; $q_{20}, \ldots, q_{2n}$
- Output: a list of fragments, each with
  - the integer pixel coordinates (x, y)
  - interpolated parameter values $q_0, \ldots, q_n$

Rasterizing triangles

- Summary
  1. evaluation of linear functions on pixel grid
  2. functions defined by parameter values at vertices
  3. using extra parameters to determine fragment set



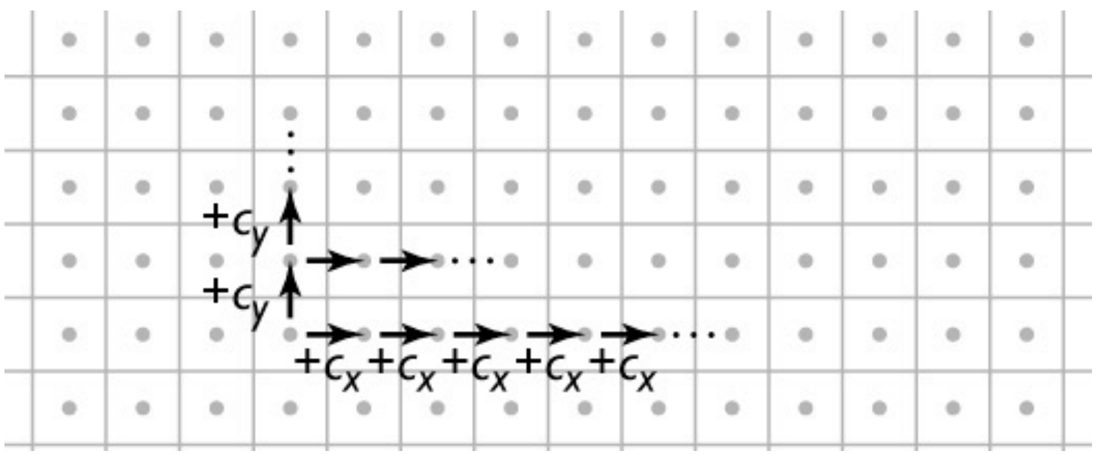
Incremental linear evaluation

- A linear (affine, really) function on the plane is:

$$q(x, y) = c_x x + c_y y + c_k$$

- Linear functions are efficient to evaluate on a grid:

$$q(x+1, y) = c_x(x+1) + c_y y + c_k = q(x, y) + c_x$$
$$q(x, y+1) = c_x x + c_y(y+1) + c_k = q(x, y) + c_y$$



Pixel-walk (Pineda) rasterization

- Conservatively visit a superset of the pixels you want
- Interpolate linear functions
  - barycentric coords (determines when to emit a fragment)
  - colors
  - normals
  - whatever else!