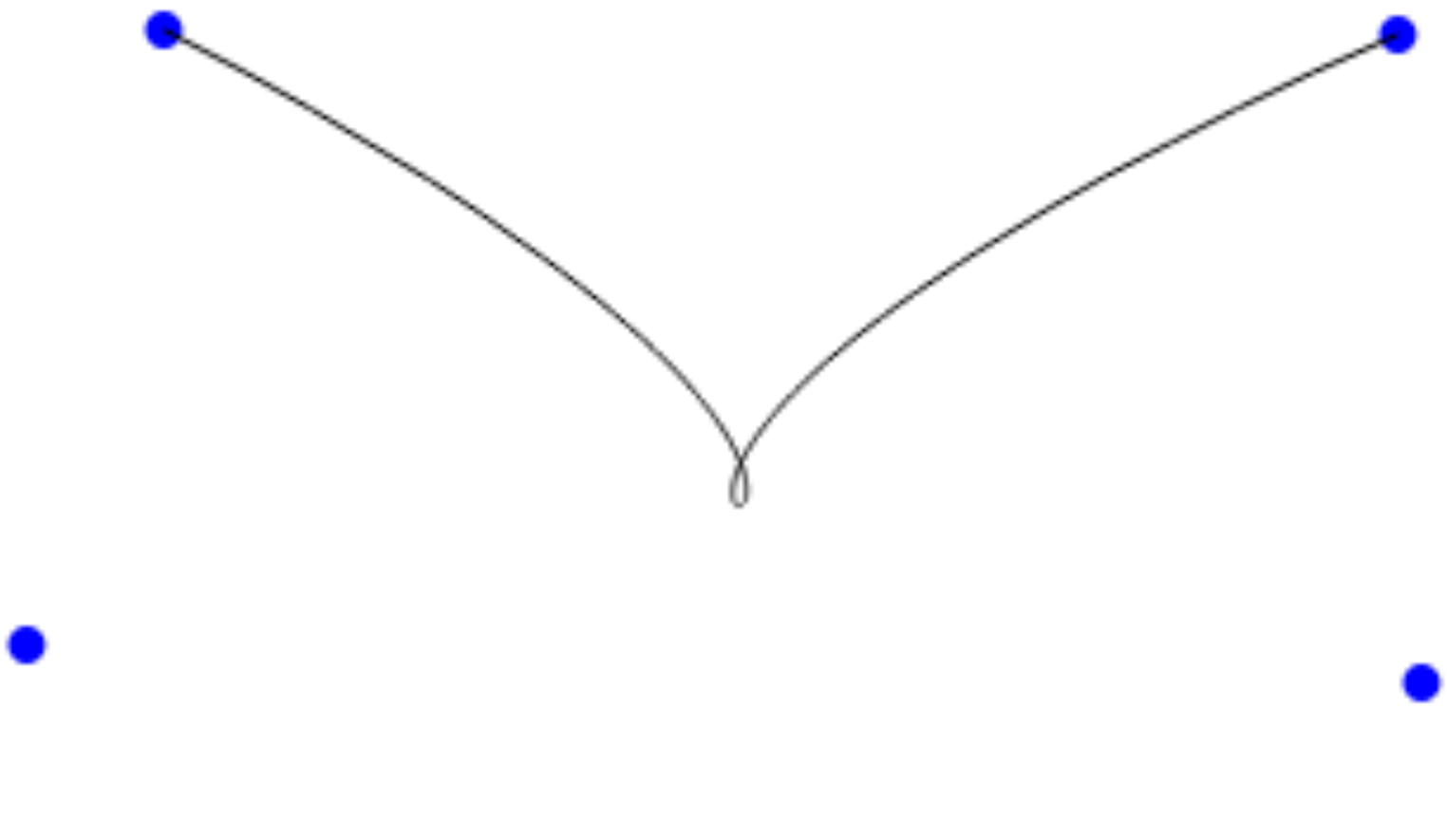


Computer Graphics

Lecture 32

Curve Properties

Spline Lab, continued

Announcements

- Topics for Tuesday?

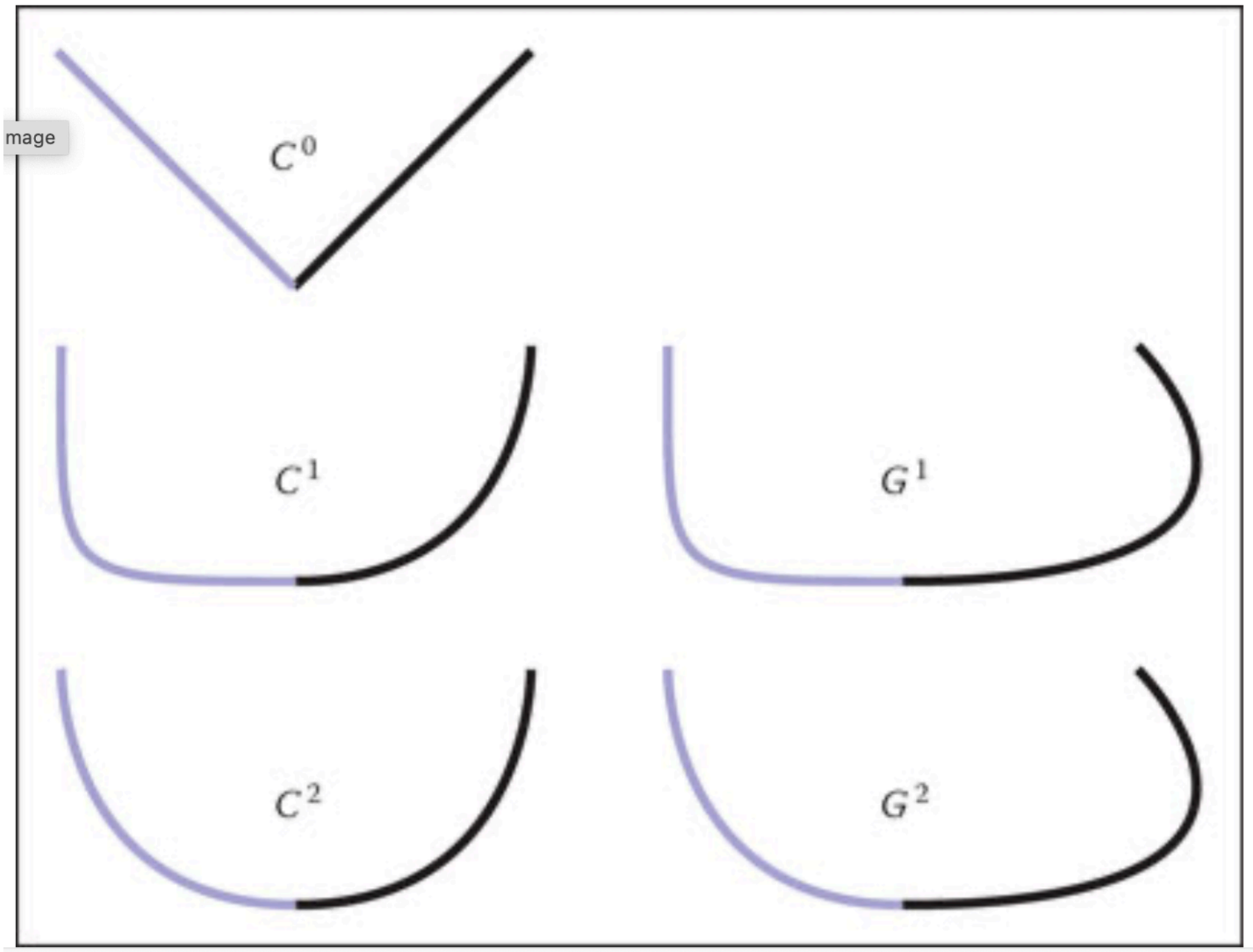
Goals

- Know how to join multiple (e.g., Bezier) spline segments together
- Know the definition of parametric and geometric continuity.

Joining Segments

- http://math.hws.edu/eck/cs424/notes2013/canvas/bezier.html

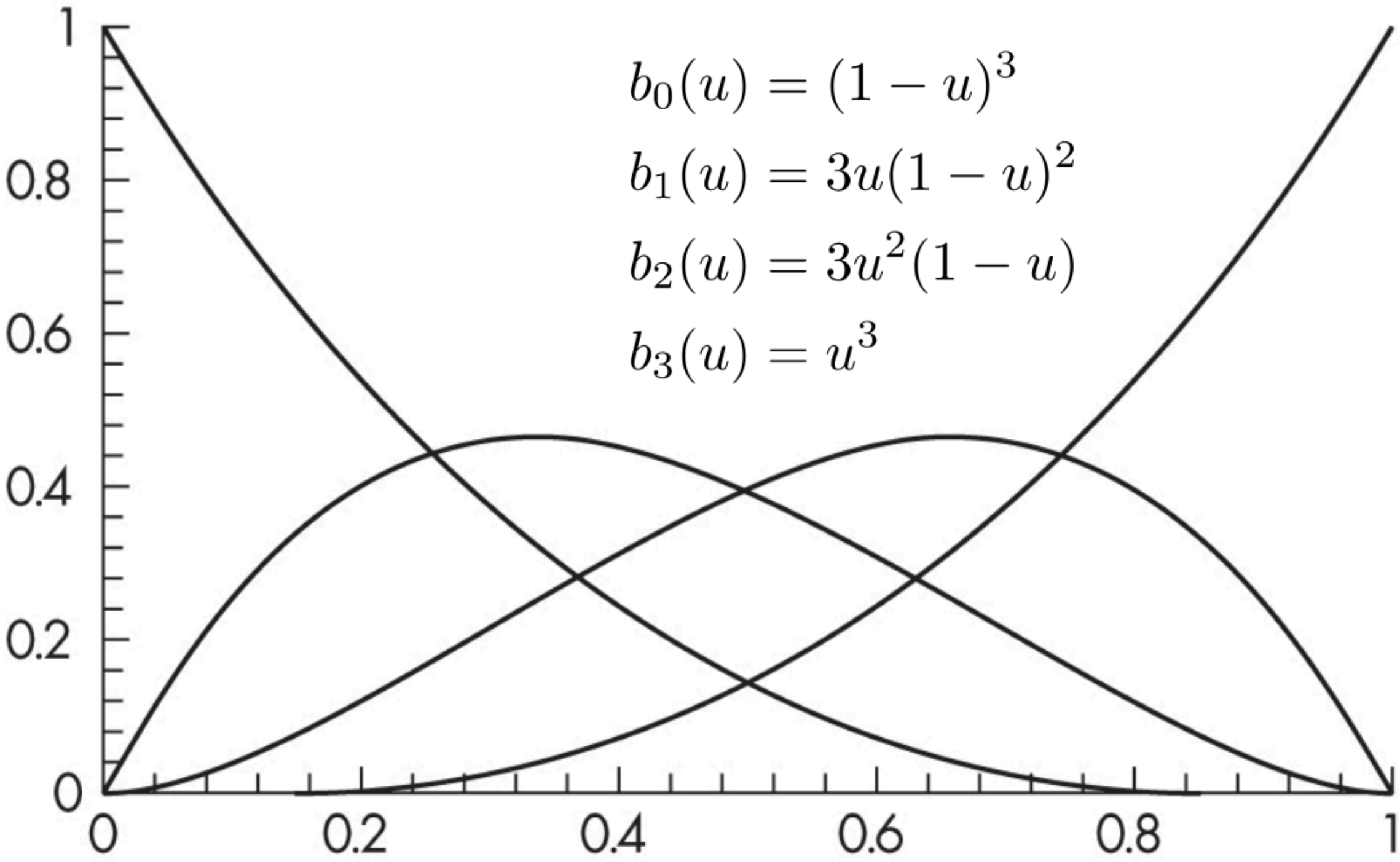
Curve Properties: Continuity



$C^1 : \mathbf{f}_1'(1) = \mathbf{f}_2'(0)$

$G^1 : \mathbf{f}_1'(1) = k\mathbf{f}_2'(0)$

Cubic Bezier blending functions

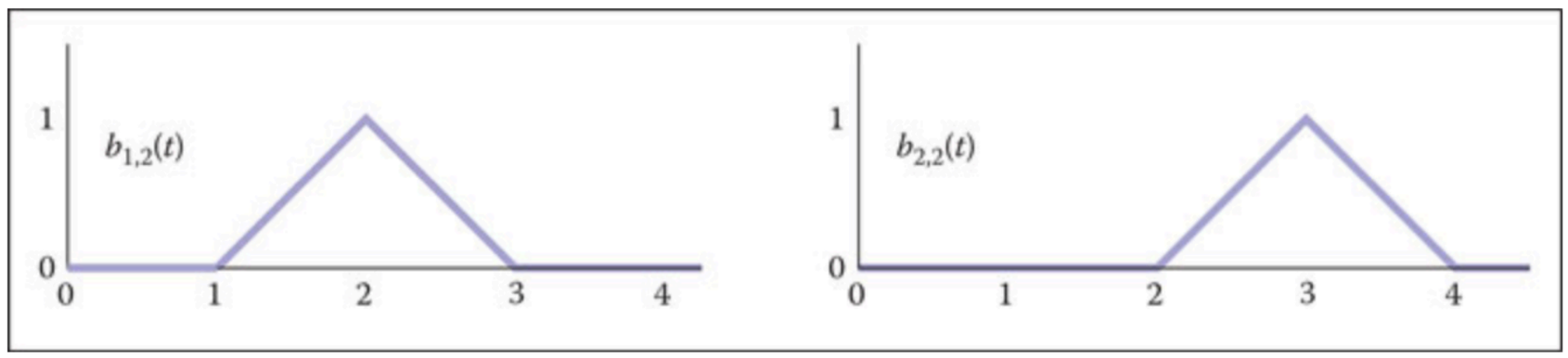


$$b_0(u) = (1-u)^3$$
$$b_1(u) = 3u(1-u)^2$$
$$b_2(u) = 3u^2(1-u)$$
$$b_3(u) = u^3$$

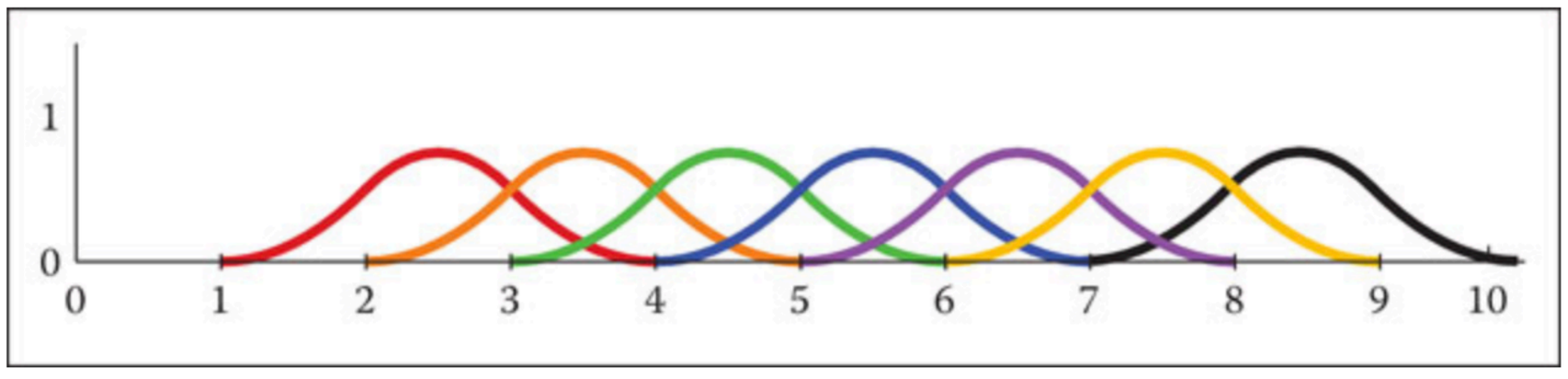
B-Splines

- Offer arbitrary continuity
- The basis polynomials are splines themselves!

k: polynomial order of "bump"

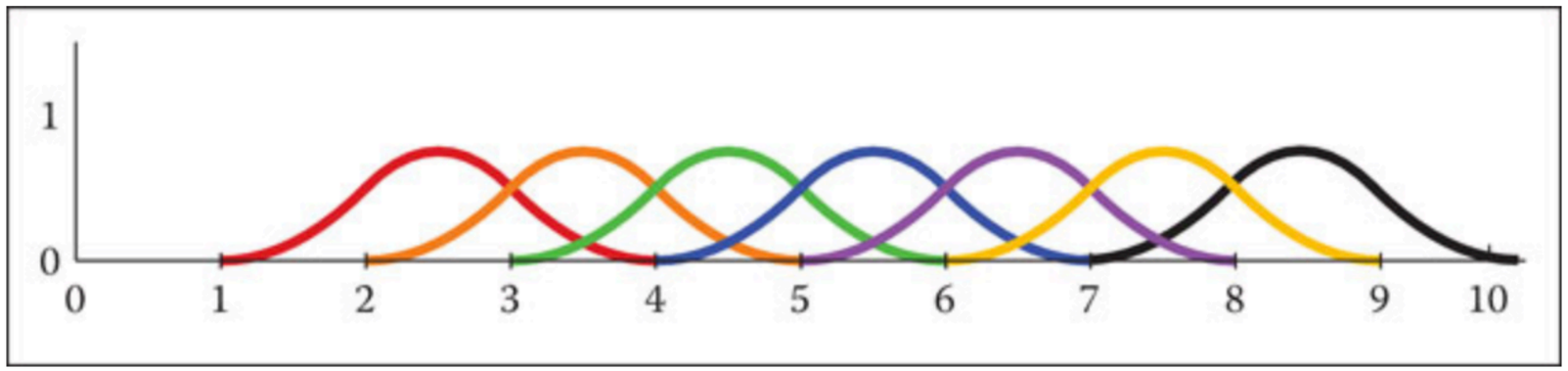


k = 1



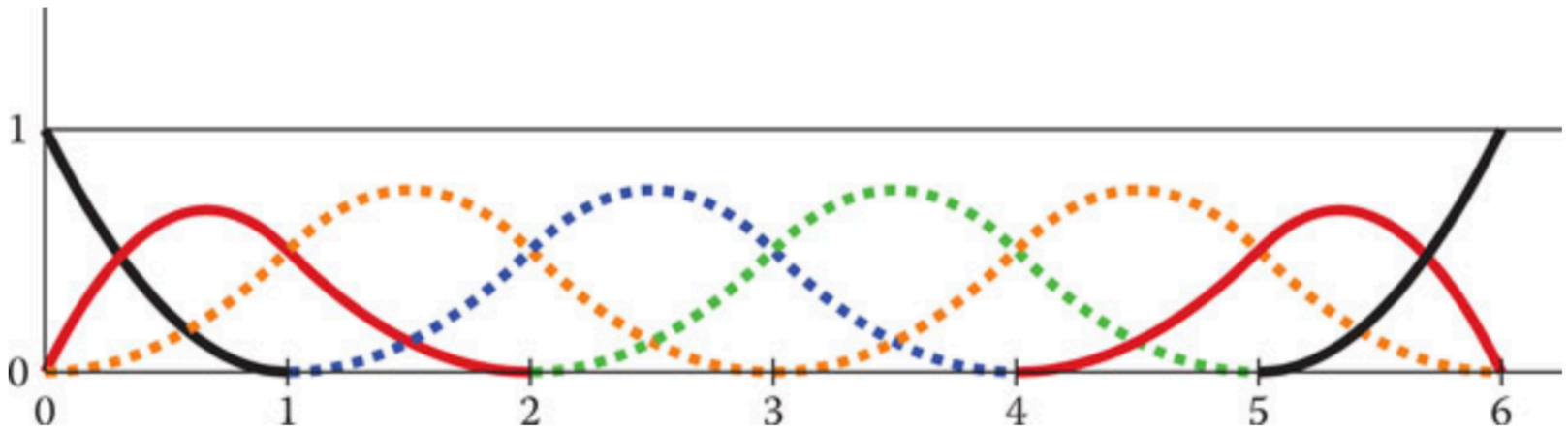
k = 3

Uniform B-Splines

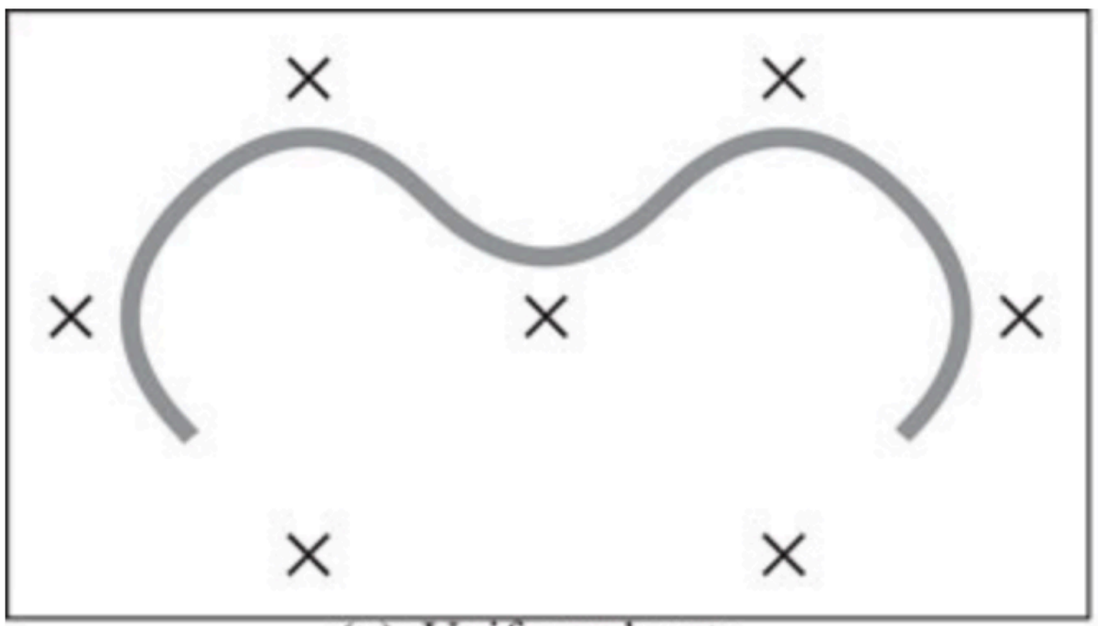


uniform B-spline: "bumps" (knots) evenly spaced

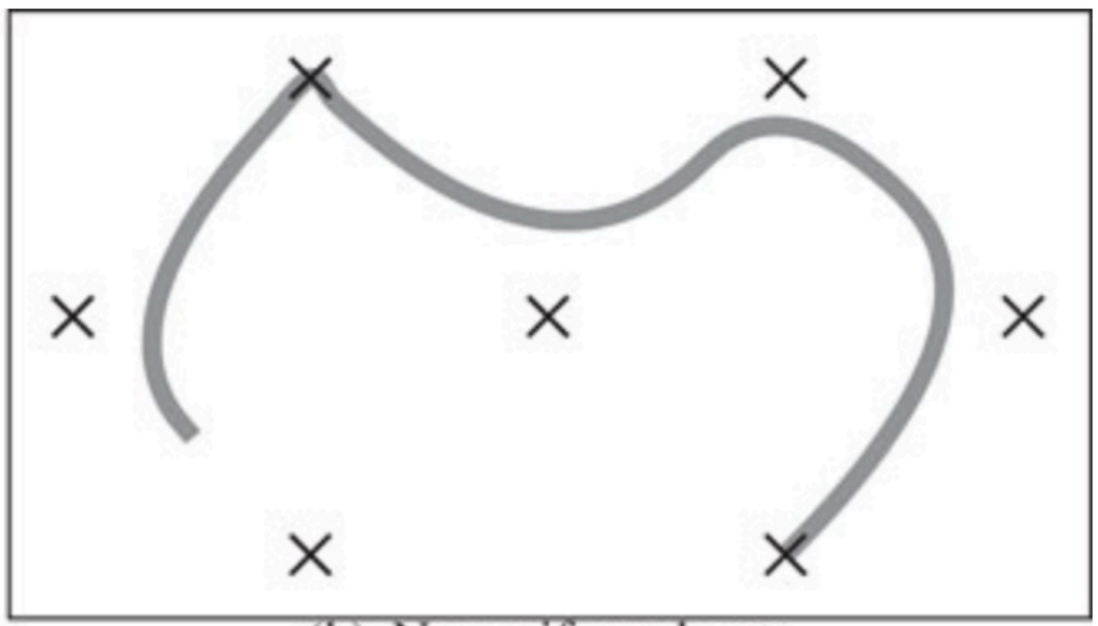
Non-Uniform B-Splines



non-uniform B-spline: "bumps" (knots) are not evenly spaced



(a) Uniform knots



(b) Nonuniform knots

Non-Uniform **Rational** B-Splines: NURBS

- B-spline bases are polynomials - can't represent conic sections e.g., a circle:
  - x^2 + y^2 = 0
- Rational B-splines - use a **ratio** of two polynomials.
  - Numerator and denominator are both B-splines

Curves are great, but.

https://youtu.be/AcFwH161XtM?t=68

https://youtu.be/Zkx1aKv2z8o?t=1080