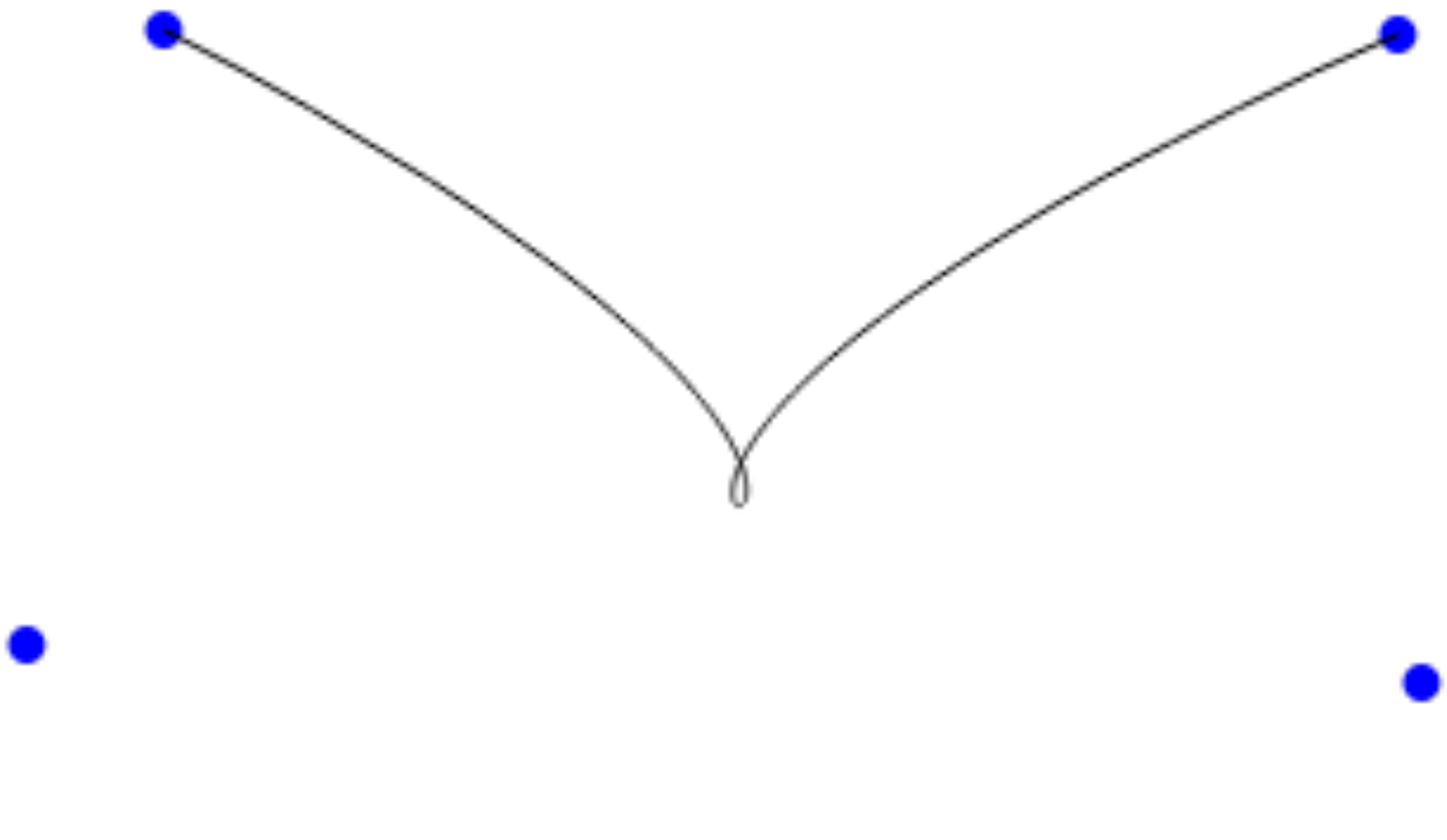


Computer Graphics

Lecture 30

Bézier Curves

de Casteljau's Algorithm

Announcements

- MS1 tonight
- Class tomorrow in CF 420 - spline lab!

Goals

- Be able to derive the basis matrix for cubic Bézier curves.
- Understand why it's called the basis matrix
- Understand some geometric properties of Bézier splines:
  - Evaluation by linear interpolation
  - Subdivision and drawing using de Casteljau's algorithm

Bezier Curves: Demo

- https://celloexpressions.com/geometry/bezier-curves-splines/560.html

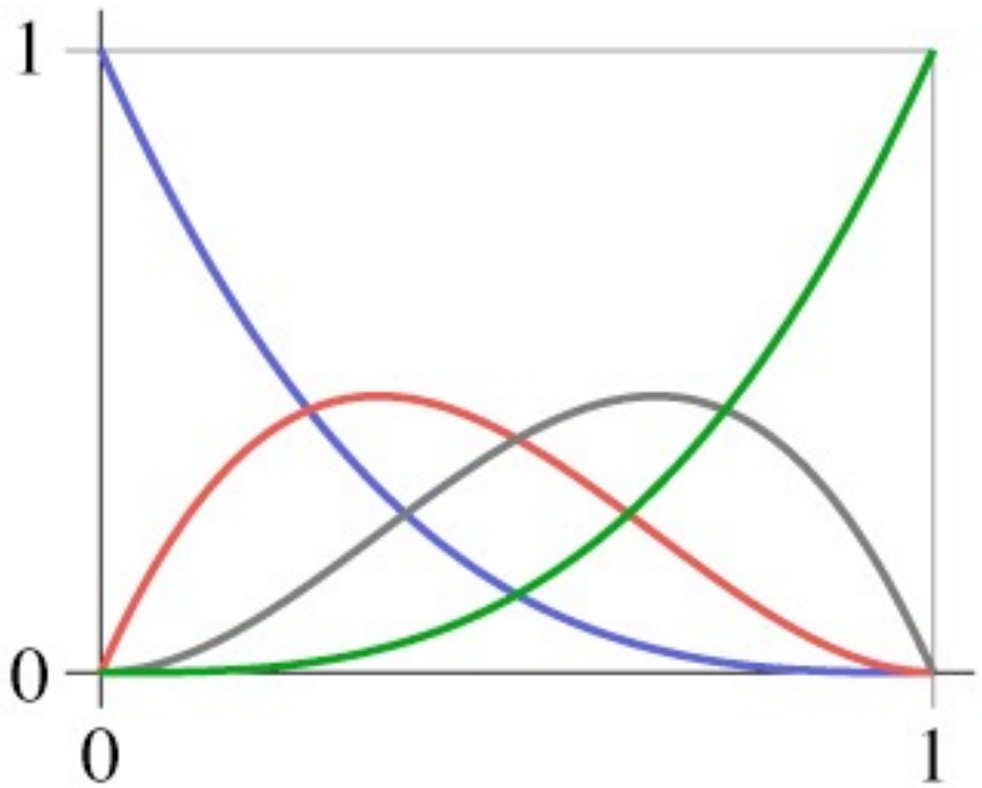
- or

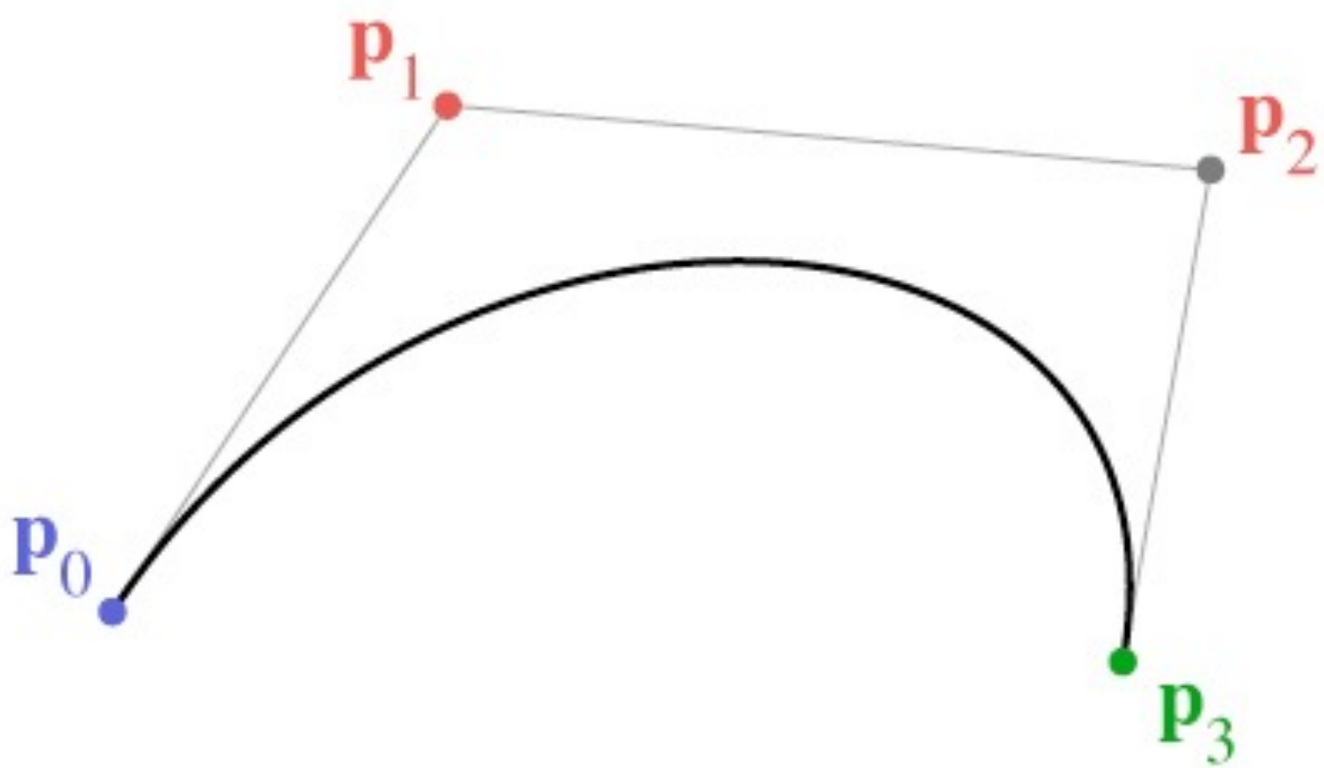
- https://math.hws.edu/eck/cs424/notes2013/canvas/bezier.html

Why is it called a "Basis Matrix"?

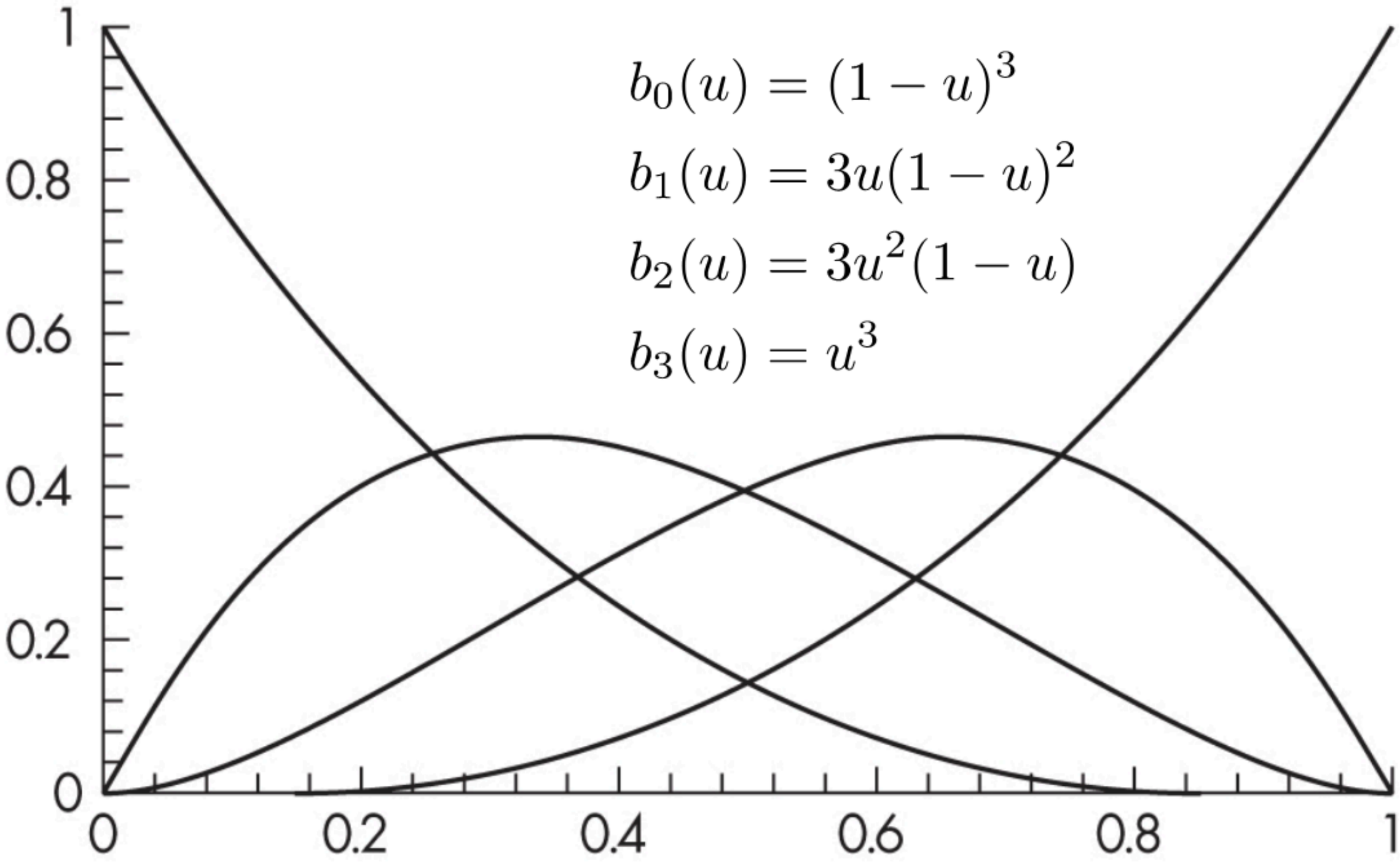
- We have: $f(u) = \mathbf{u}^T B \mathbf{p}$
- For computational purposes, we'll want to precompute Bp.
  - This is the vector of a_i's that weights each power of u
- How would we interpret $\mathbf{u}^\mathbf{T} B$?
  - A polynomial that specifies the weight on each control point.

Blending Functions



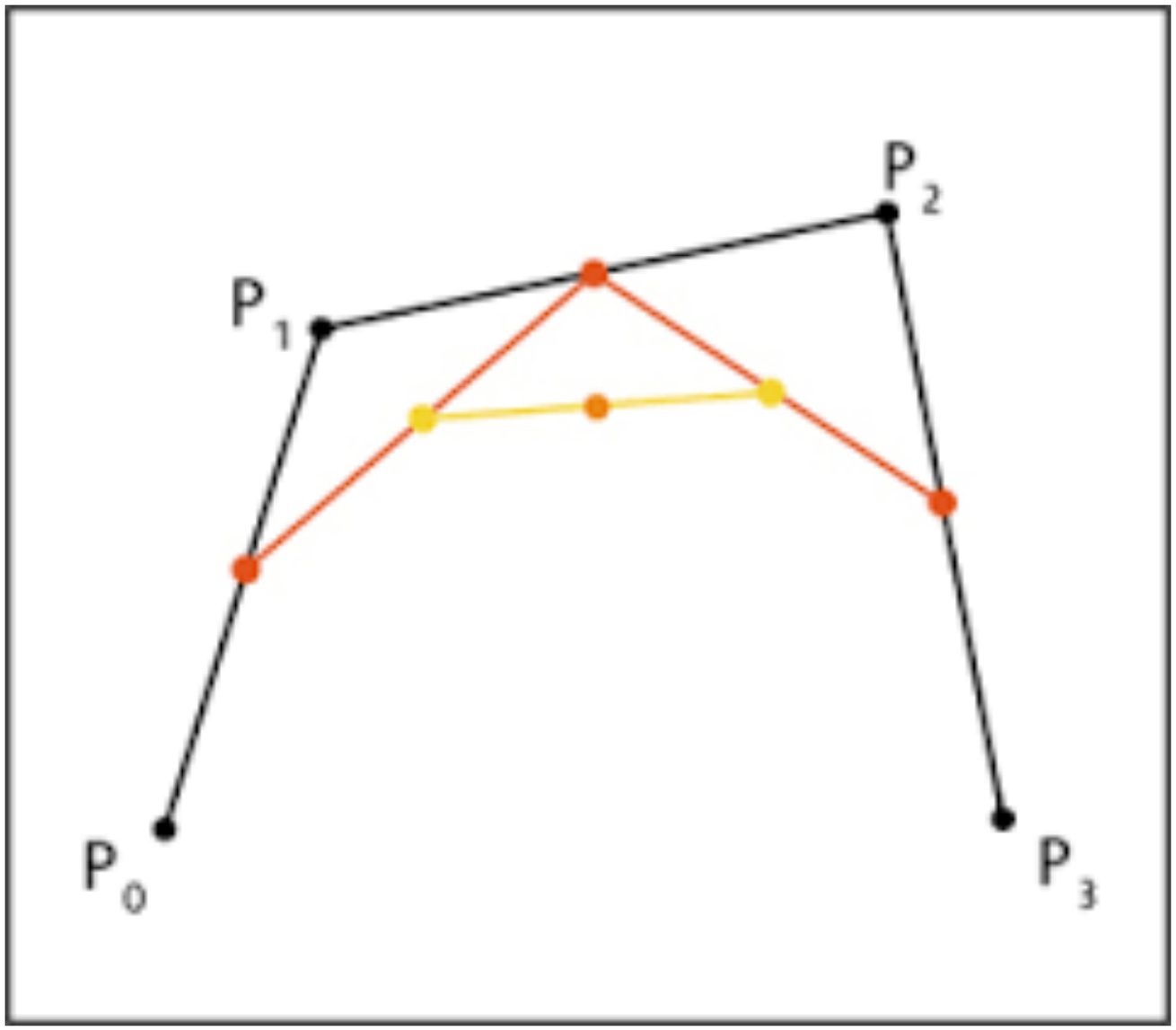


Cubic Bezier blending functions



$$b_0(u) = (1-u)^3$$
$$b_1(u) = 3u(1-u)^2$$
$$b_2(u) = 3u^2(1-u)$$
$$b_3(u) = u^3$$

Bezier Curves: Geometry



Coolest / most satisfying animation of the quarter

https://www.jasondavies.com/animated-bezier/