Bezier Splines solve 2 problems w/Hamle:

1. non-intuitive tangent tails - use $p_2 - p_3$ instead of $p_1$
2. weak tangents - multiply by 3

(Bezier basis matrix) + Demo

Basis Matrix - why?

$$f(u) = \mathbf{U}(\mathbf{B}^T)$$

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Per-point weights - basis functions + Plot (any order exists)

Bezier Geometry:

Evaluation - all you need is loop!

Subdivision
Bézier Basis Matrix and Blending Functions

\[ p_0 = f(0) = a_0 \]

\[ p_3 = f(1) = a_0 + a_1 + a_2 + a_3 \]

\[ 3(p_1 - p_0) = f'(0) = a_1 \]

\[ 3(p_3 - p_2) = f'(1) = a_1 + 2a_2 + 3a_3 \]

Do some algebra...

\[ B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \]

\[ f(u) = \tilde{a} \cdot \tilde{u} \]

\[ \tilde{a} = B \tilde{p} \]

\[ f(u) = \tilde{u}^T B \tilde{p} \]

Thought: What does \( \tilde{u}^T B \) represent?

Exercise: Compute.

Slide: Plot the blending functions.

Better curves exist for any order!
Bézier - Geometric View

Cubic

Animated Demo

One more crazy cool thing:

Midpoints become new control points for half the curve!

Recursively subdivide, and you have an arbitrarily close piecewise linear approximation to the curve!

Drawing algorithm - subdivide until some error metric is small, draw line segments.
Bernstein: Algebraic Details

\[ f(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3 \]

\[ P_0 = f(0) = a_0 \]

\[ P_3 = f(1) = a_0 + a_1 + a_2 + a_3 \]

\[ 3(P_1 - P_0) = f'(0) = a_1 \quad \rightarrow \quad P_1 = \frac{1}{3} a_1 + \frac{2}{3} P_0 \]

\[ 3(P_3 - P_2) = f'(1) = a_1 + 2a_2 + 3a_3 \quad \rightarrow \quad 3\rho_2 = 3\rho_3 - a_1 - 2a_2 - 3a_3 \]

\[ \frac{1}{3}\rho_2 = \frac{1}{3}(a_0 + a_1 + a_2 + a_3) - \frac{1}{3} a_1 - \frac{2}{3} a_2 - \frac{3}{3} a_3 \]

\[ \rho_2 = a_0 + a_1 + a_2 + a_3 - \frac{1}{3} a_1 - \frac{2}{3} a_2 - a_3 \]