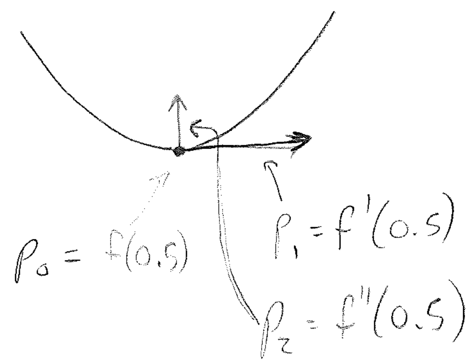
Another type of control point: specify <u>derivative</u> at a pt.

Example: specify midpoint, 1st, and 2nd derivatives at midpt.

$$P_0 = f(0.5) = a_0 + 0.5^1\, a_1 + 0.5^2\, a_2$$

$$P_1 = f'(0.5) = a_1 + 0.5 \cdot 2a_2$$

$$P_2 = f''(0.5) = 2a_2$$



$$C = \begin{pmatrix} 1 & 0.5 & 0.25 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

derivatives?

$f(u) = a_0 + a_1 u + a_2 u^2$

$f'(u) = a_1 + 2a_2 u$

$f''(u) = 2a_2$

Again, $\vec{p} = C\vec{a}$, so $\vec{a} = C^{-1}\vec{p}$

~~84.5~~
26.1

What people actually use - Cubics.

Useful example: a cubic specified by its start, end, and derivatives at start + end.

Hermite Spline

Demo

$$f(u) = a_0 + u a_1 + u^2 a_2 + u^3 a_3$$

$$f'(u) = \qquad a_1 + 2u a_2 + 3u^2 a_3$$

$$\begin{array}{c} f(0) \\ f'(0) \\ f(1) \\ f'(1) \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} = C$$

Bezier Curves: Algebraic intuition

- P_2, P_4 are points specifying derivatives (gradients)
- depends on where curve is wrt origin
- instead, set $P_2 - P_1 = f'(0)$ so the vector between control points gives a tangent vector.
- likewise, $P_4 - P_3 = f'(1)$