Computer Graphics

Lecture 15
2D Linear Transformations

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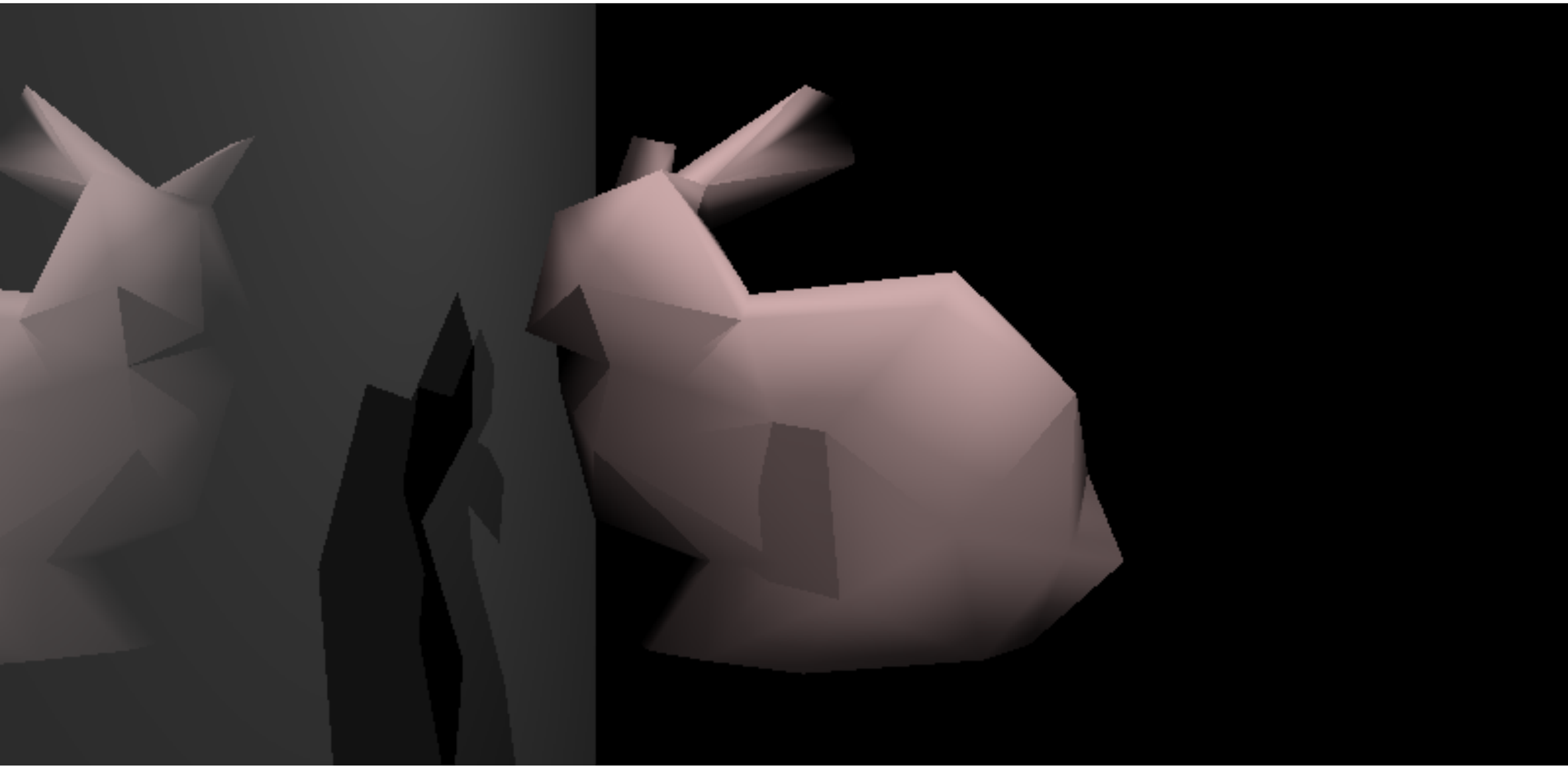
Announcements

- Wednesday office hours moved 1/2 hr later:
  - 11:30 - 12:30 instead of 11-12

Goals

- Have intuition for matrices as
  - Linear functions that map points from one place to another in space.
  - Basis-change machines that convert coordinates expressed in some basis into the canonical basis.
- Know how to construct 2D matrices that perform uniform and nonuniform scaling, reflection, and rotation.
- Know some properties of linear transformations:
  - Linearity, closure under composition, associativity, non-commutativity

Situation: Bunny is sad.



Bunny is sad because it can't move.

Today: Make bunny happy

- How can we manipulate objects in the scene to
  - put them in the right position?
  - scale them to the right size?
  - orient them in the right direction?

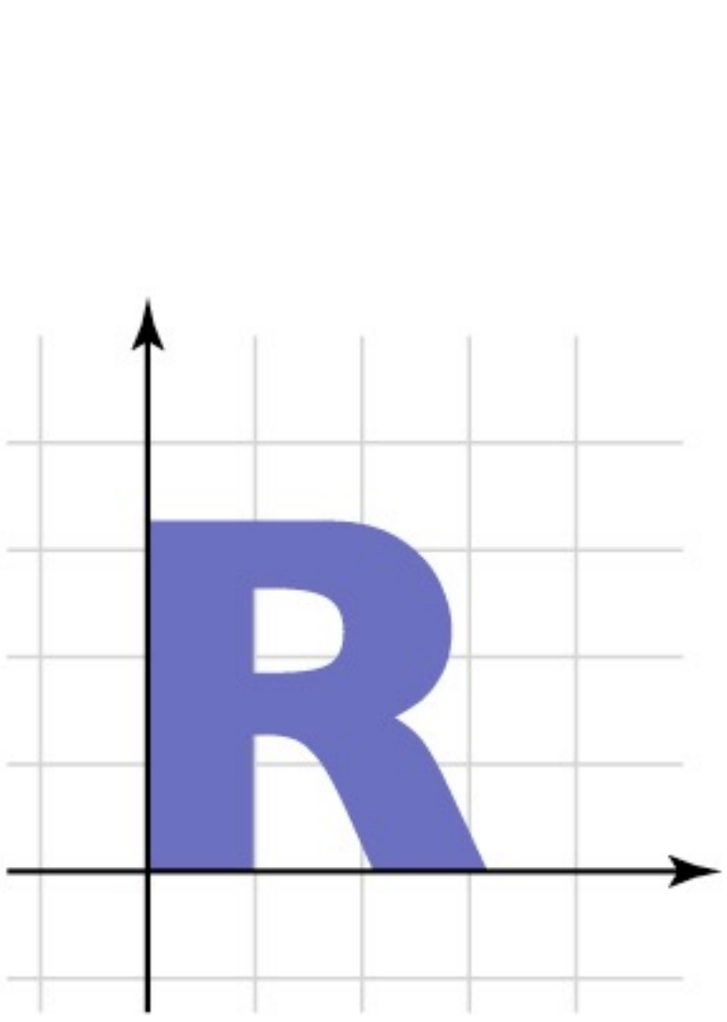
Our answer: matrices.

Geometric Transformations

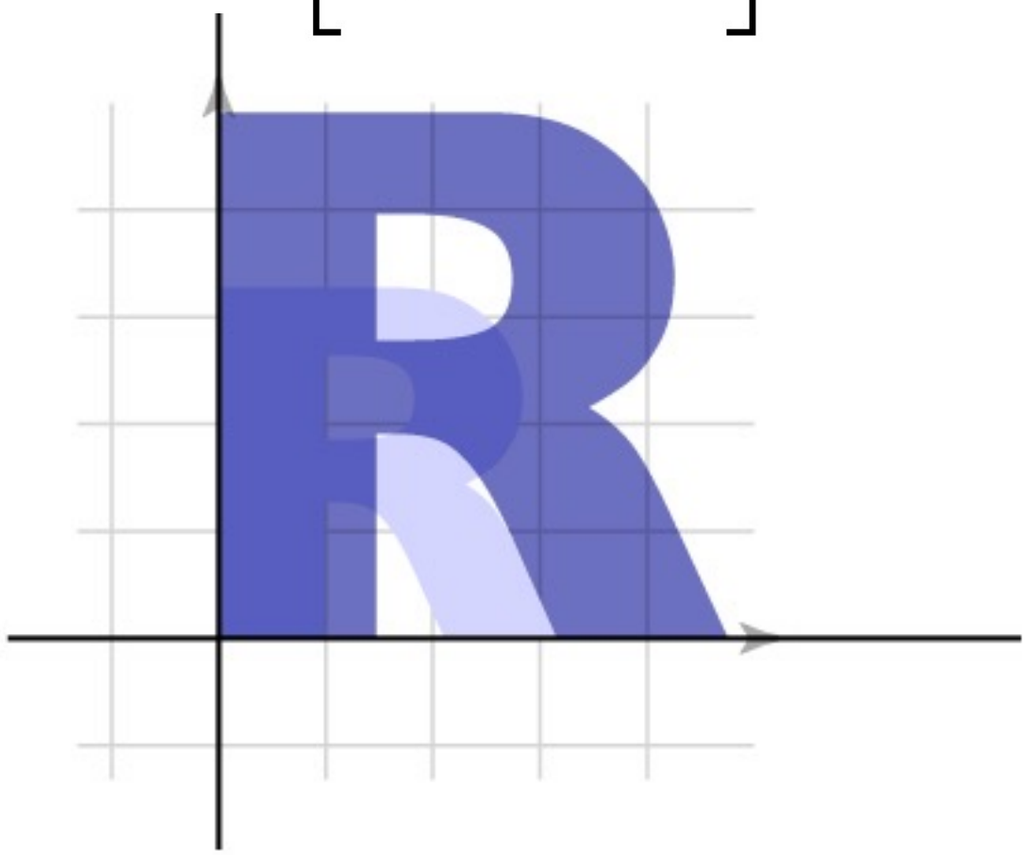
- To the notes!

Linear transformation gallery

- Uniform scale $\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} sx \\ sy \end{bmatrix}$



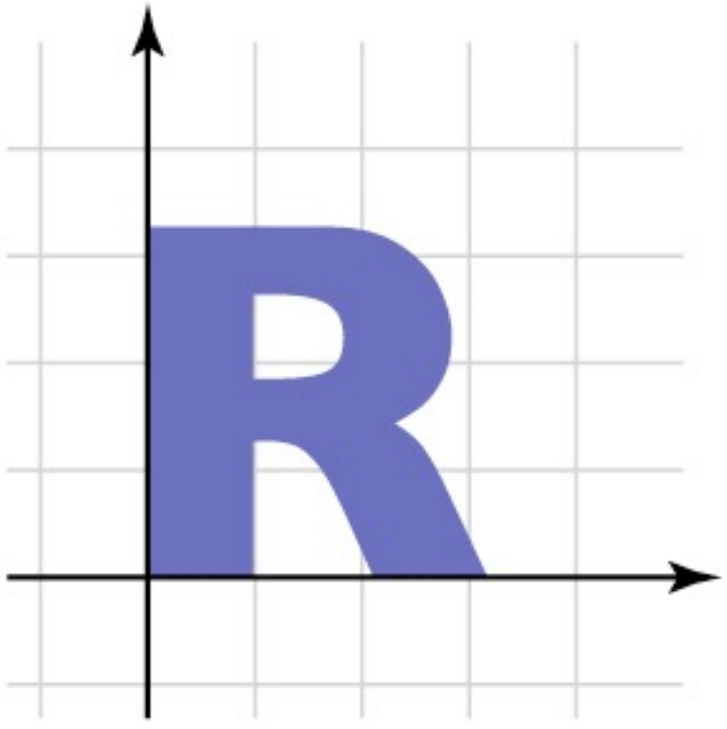
$$\begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$

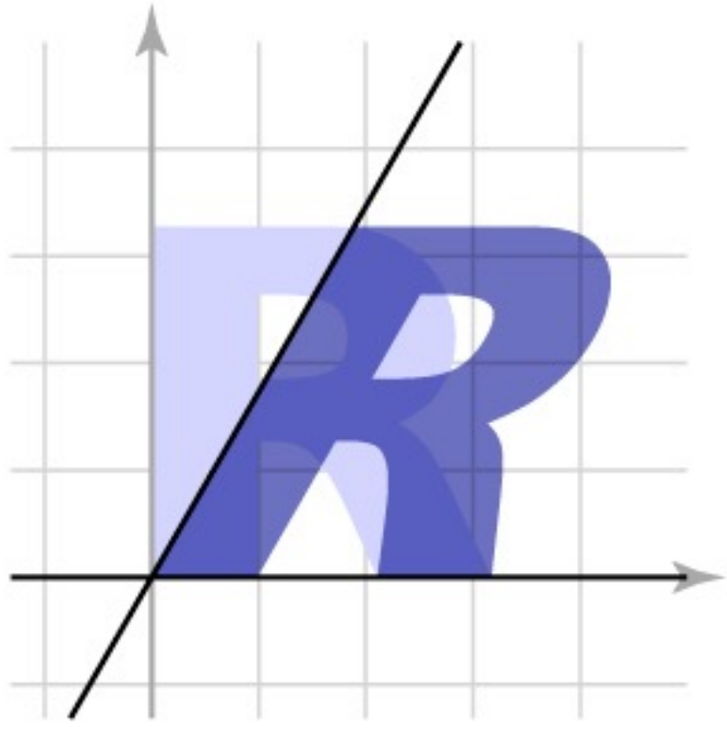


Linear transformation gallery

- Shear $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + ay \\ y \end{bmatrix}$

$$\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$$



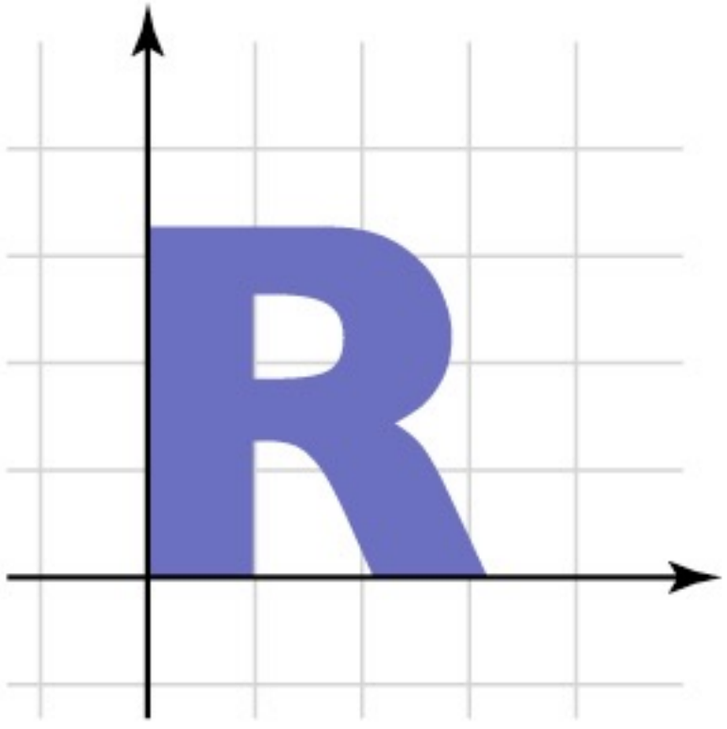


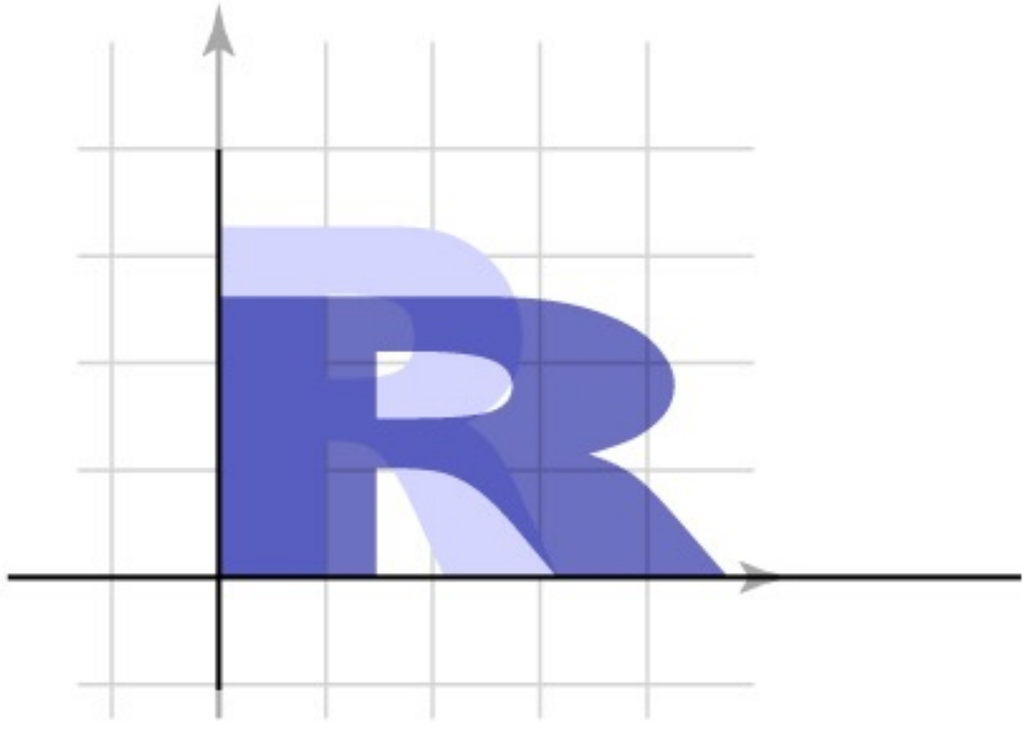
Linear transformation gallery

- Nonuniform scale

$$\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$$

$$\begin{bmatrix} 1.5 & 0 \\ 0 & 0.8 \end{bmatrix}$$

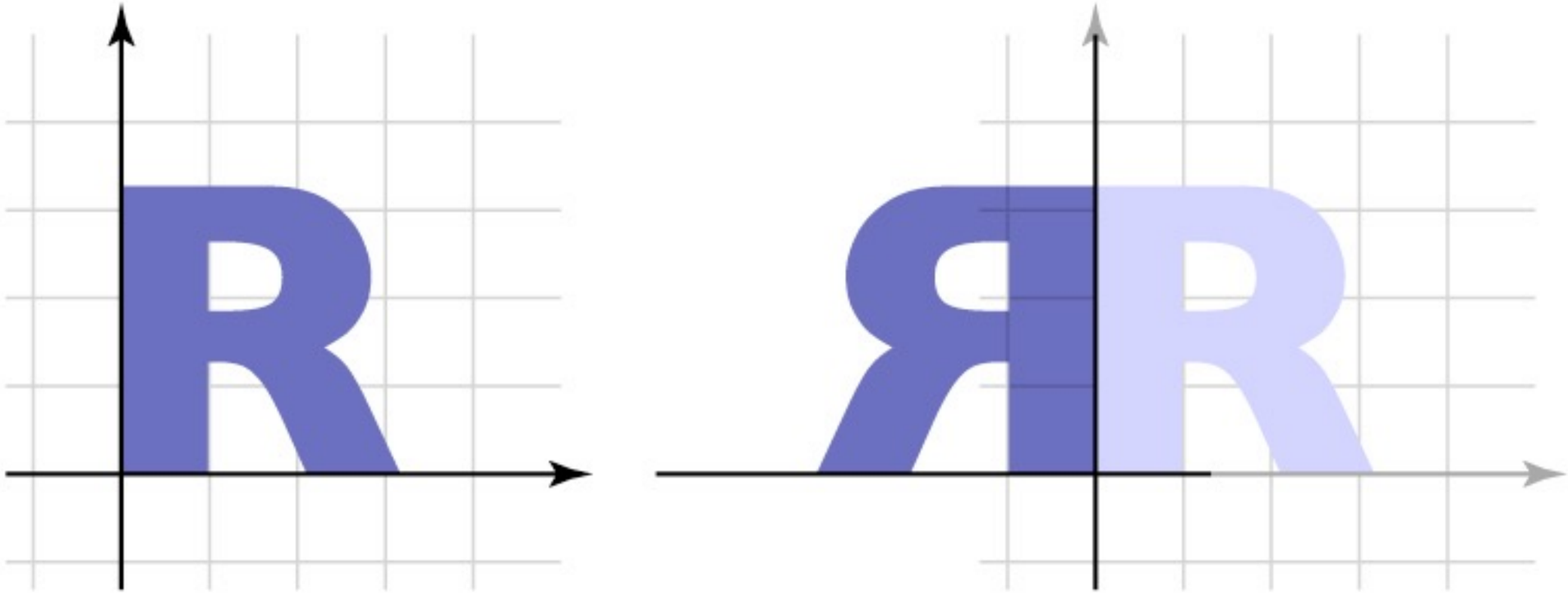




Linear transformation gallery

- Reflection
  - can consider it a special case of nonuniform scale

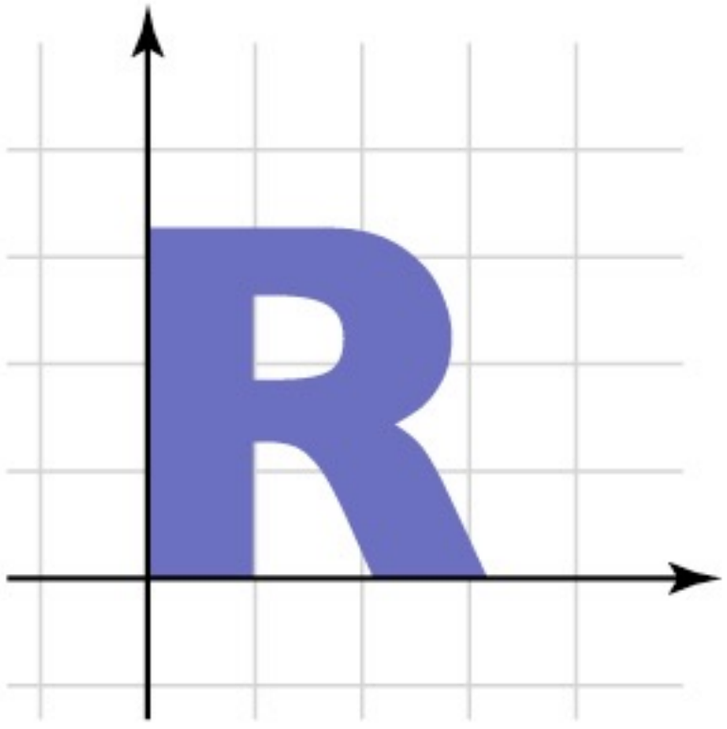
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

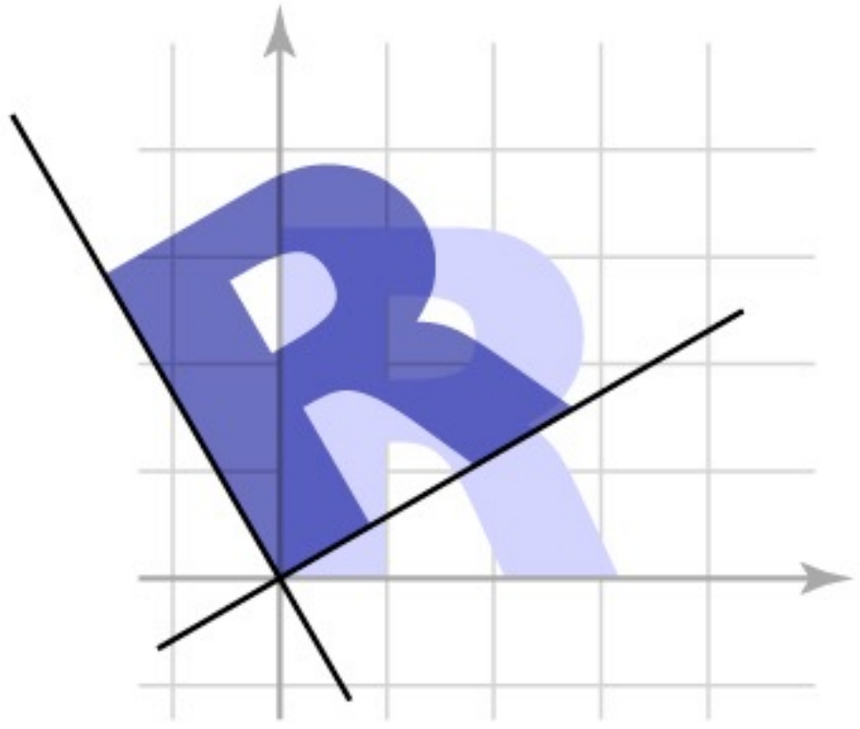


Linear transformation gallery

- Rotation $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$

$$\begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix}$$





2D Matrix Transformations: Properties

- linear
- closed under composition
- associative
- not commutative
- applied right-to-left