Computer Graphics

Lecture 15
2D Linear Transformations
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2D Linear Transformations
Announcements

• Wednesday office hours moved 1/2 hr later:
  • 11:30 - 12:30 instead of 11-12
Goals

• Have intuition for matrices as
  • Linear functions that map points from one place to another in space.
  • Basis-change machines that convert coordinates expressed in some basis into the canonical basis.

• Know how to construct 2D matrices that perform uniform and nonuniform scaling, reflection, and rotation.

• Know some properties of linear transformations:
  • Linearity, closure under composition, associativity, non-commutativity
Situation: Bunny is sad.

Bunny is sad because it can't move.
Today: Make bunny happy

- How can we manipulate objects in the scene to
  - put them in the right position?
  - scale them to the right size?
  - orient them in the right direction?

Our answer: matrices.
Geometric Transformations

• To the notes!
Linear transformation gallery

- Uniform scale

\[
\begin{bmatrix}
  s & 0 \\
  0 & s \\
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
\end{bmatrix}
=
\begin{bmatrix}
  sx \\
  sy \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  1.5 & 0 \\
  0 & 1.5 \\
\end{bmatrix}
\]
Linear transformation gallery

- Shear

\[
\begin{bmatrix}
1 & a \\
0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix}
= 
\begin{bmatrix}
x + ay \\
y \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0.5 \\
0 & 1 \\
\end{bmatrix}
\]

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Linear transformation gallery

- Nonuniform scale

\[
\begin{bmatrix}
s_x & 0 \\ 0 & s_y
\end{bmatrix}
\begin{bmatrix}
x \\ y
\end{bmatrix} =
\begin{bmatrix}
s_xx \\ s_yy
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.5 & 0 \\ 0 & 0.8
\end{bmatrix}
\]
Linear transformation gallery

- Reflection
  - can consider it a special case of nonuniform scale

\[
\begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix}
\]
Linear transformation gallery

• Rotation

\[
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
=
\begin{bmatrix}
x \cos \theta - y \sin \theta \\
x \sin \theta + y \cos \theta
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.866 & -0.5 \\
0.5 & 0.866
\end{bmatrix}
\]
2D Matrix Transformations: Properties

- linear
- closed under composition
- associative
- not commutative
- applied right-to-left