Transforming objects in a scene: need a function (mapping) that describes new position in terms of old position.

$$T: \mathbb{R}^2 \to \mathbb{R}^2, \text{ in 2D. } \mathbb{R}^3 \text{ for 3D.}$$

Simple example: $T(\vec{x}) = \vec{x} + \vec{t}$ <u>translation</u>

But first!

Some Review on Matrices.

Matrix: 2D array of (for our purposes) (real) numbers

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4.5 & 0.5 \end{bmatrix} \text{ is a } \underset{\text{rows}}{\overset{M}{2}}\text{-}\overset{\text{by}}{\text{by}}\text{-}\underset{\text{cols}}{\overset{N}{3}} \text{ matrix}$$

a_{ij} = ith row, jth col. so $a_{21} = 3$.

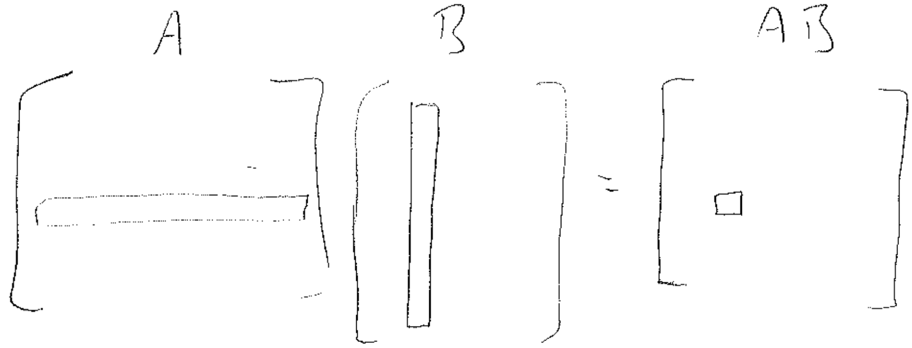
Matrix-vector multiplication:

$$\begin{bmatrix} \\ \vec{y} \\ \\ \end{bmatrix} = \begin{bmatrix} \text{---} \ \vec{r}_1 \ \text{---} \\ \text{---} \ \vec{r}_i \ \text{---} \\ \vdots \\ \text{---} \ \vec{r}_m \ \text{---} \end{bmatrix} \begin{bmatrix} \\ \vec{x} \\ \\ \end{bmatrix} \qquad y_i = \vec{r}_i \cdot \vec{x}$$

Equivalently:

$$\begin{bmatrix} | \\ \vec{y} \\ | \end{bmatrix} = \begin{bmatrix} | & | & & | \\ \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_m \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \vec{y} = x_1 \vec{c}_1 + x_2 \vec{c}_2 + \ldots + x_n \vec{c}_n$$

Matrix-Matrix



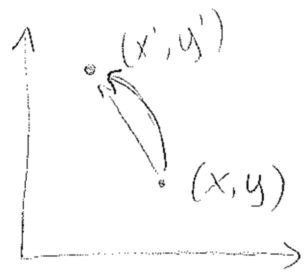
Ex 1 = multiply 2×2 Ax

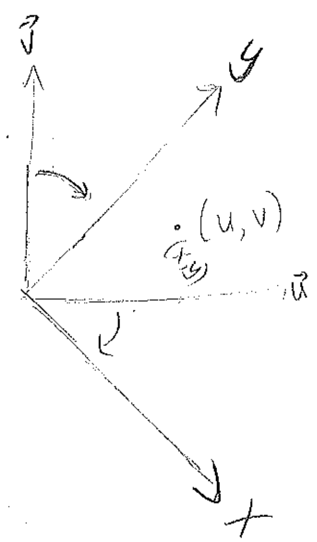
$$(AB)_{ij} = (i\text{th row of } A) \cdot (j\text{th col of } B)$$

Geometric interpretation:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$ Mapping from $\mathbb{R}^2 \to \mathbb{R}^2$

$$\vec{y} = f(\vec{x})$$





Or: a change of Basis!

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} | & | \\ \vec{u} & \vec{v} \\ | & | \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

uv-to-canonical

Associative $ABC = (AB)C = A(BC)$

Not commutative $AB \neq BA$

Right-to-left $ABx = \overset{2}{A} \cdot \overset{1}{(Bx)}$

Properties:

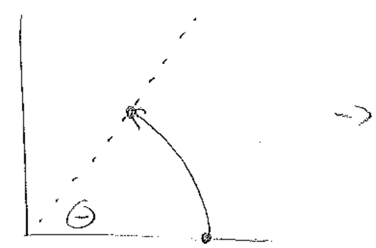
Linear
- $Tx + Ty = T(x+y)$
- $aTx = T(ax)$

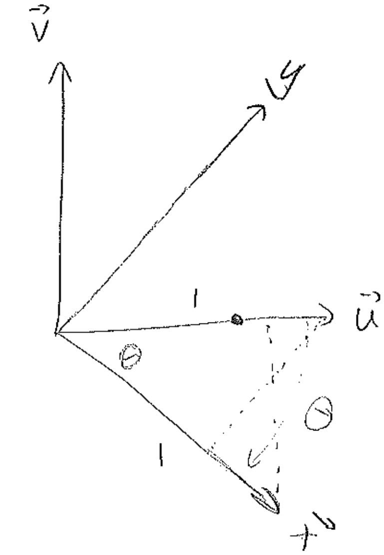
Closed under Composition

$S \circ T = ST = M$ ← still a linear transformation

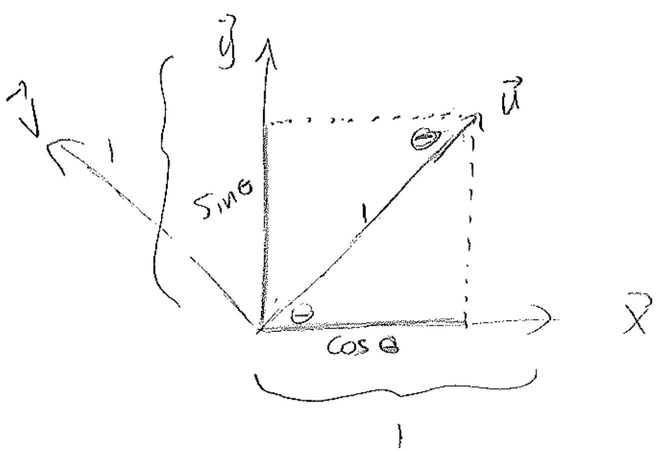
What about Rotation?

Change-of-basis viewpoint:





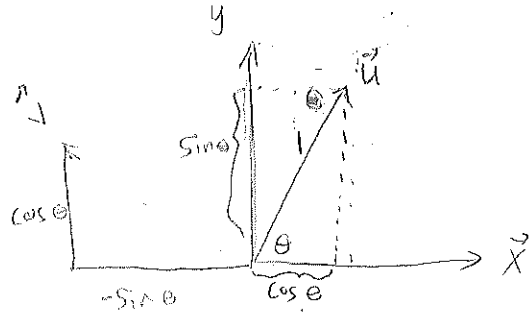
Need: $\vec{u}$ vector in $\vec{x}, \vec{y}$ basis
$\vec{v}$ vector in $\vec{x}, \vec{y}$ basis.



$$\begin{bmatrix} | & | \\ \vec{u} & \vec{v} \\ | & | \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

from diagram at left

by analogous reasoning for $\vec{v}$ vector



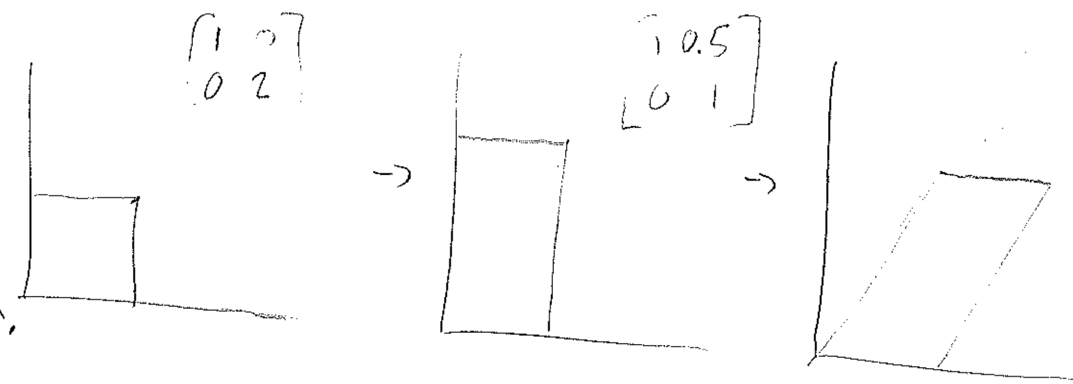
Composing Transformations

$T_1 : \mathbb{R}^2 \to \mathbb{R}^2$

$T_2 : \mathbb{R}^2 \to \mathbb{R}^2$

$T_1 \circ T_2 : \mathbb{R}^2 \to \mathbb{R}^2$

Example: Scale then shear



Backwords - right-to-left!

$$\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Matrix mult:

$A(Bx) = (AB)x$

associative!

(not commutative)