Transforming objects in a scene: need a function (mapping) that describes new position in terms of old position.

\[ T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, \text{ in 2D. } \mathbb{R}^3 \text{ for 3D.} \]

Simple example: \( T(x) = x + \mathbf{e} \) \text{ \underline{translation}}

But first!

Some Review on Matrices.

For our purposes)

Matrix: 2D array of (real) numbers

\[ A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4.5 & 0.5 \end{bmatrix} \] \text{ is a 2-by-3 matrix}

\( A_{ij} = \text{i}^{th} \text{ row, j}^{th} \text{ col. so } A_{21} = 3. \)

Matrix-vector multiplication:

\[ \begin{bmatrix} \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \\ \mathbf{c}_m \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{bmatrix} \]

\[ \mathbf{y}_i = \mathbf{c}_i \cdot \mathbf{x}_i \]

Equivalently:

\[ \begin{bmatrix} \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \ldots & \mathbf{c}_m \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{bmatrix} \]

\[ \mathbf{y} = \mathbf{x}_1 \mathbf{c}_1 + \mathbf{x}_2 \mathbf{c}_2 + \ldots + \mathbf{x}_n \mathbf{c}_n \]
Matrix-Matrix

$$\begin{pmatrix} A & B \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} AB \end{pmatrix}$$

**Example:** multiply $2 \times 2$ matrix $A$ by $x$

$$(AB)_{ij} = (i\text{th row of } A) \cdot (j\text{th col of } B)$$

**Geometric Interpretation:**

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Mapping from $\mathbb{R}^2 \to \mathbb{R}^2$

$$\vec{y} = f(x)$$

Or: a change of Basis!

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

**Properties:**

- Linearity: $T(x+y) = T(x) + T(y)$
- Linear under composition: $S \circ T = ST = M$
- Right to left: $ABx = A(Bx)$
- Associative: $ABC = (AB)C = A(BC)$
- Not commutative: $AB \neq BA$
- $A \cdot 0 = 0$
- $A(Bx) = A(Bx)$
What about Rotation?

Change of basis viewpoint:

Need: $\hat{u}$ vector in $\hat{x}, \hat{y}$ basis
$\hat{v}$ vector in $\hat{x}, \hat{y}$ basis.

From diagram at left by analogous reasoning for $\hat{v}$ vector
Composing Transformations

$T_1 : \mathbb{R}^2 \to \mathbb{R}^2 \quad T_1 \circ T_2 : \mathbb{R}^2 \to \mathbb{R}^2$

$T_2 : \mathbb{R}^2 \to \mathbb{R}^2$

Example: Scale then shear

\[ \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} \]

Backwards: right to left:

\[ \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} \]

Matrix mult:

\[ A(Bx) = (AB)x \]

Associative! (not commutative)