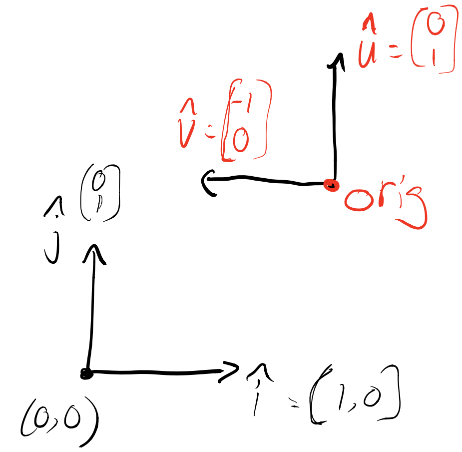
Barycentric Coordinates

An elegant way to parameterize a plane with respect to a triangle that lives in it.

Reminder: Coordinate Frames

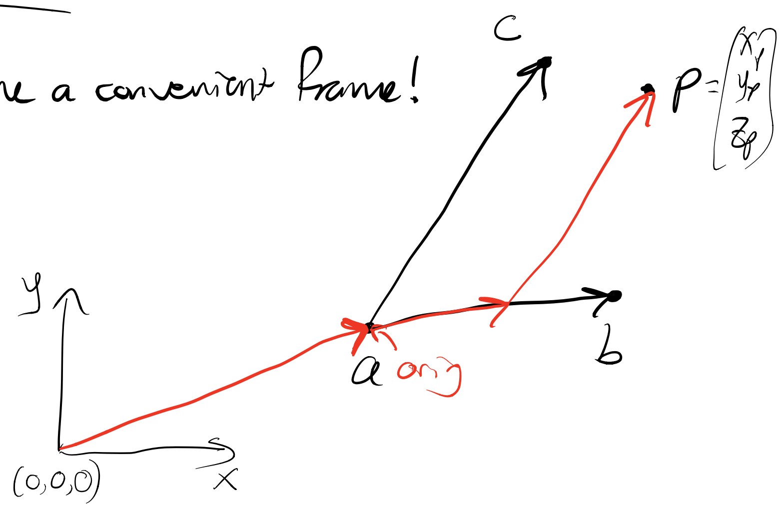
- Basis plus origin



Barycentric Frame

Use the triangle to define a convenient frame!

$$\boxed{a + \beta(b-a) + \gamma(c-a)}$$



Rewrite:

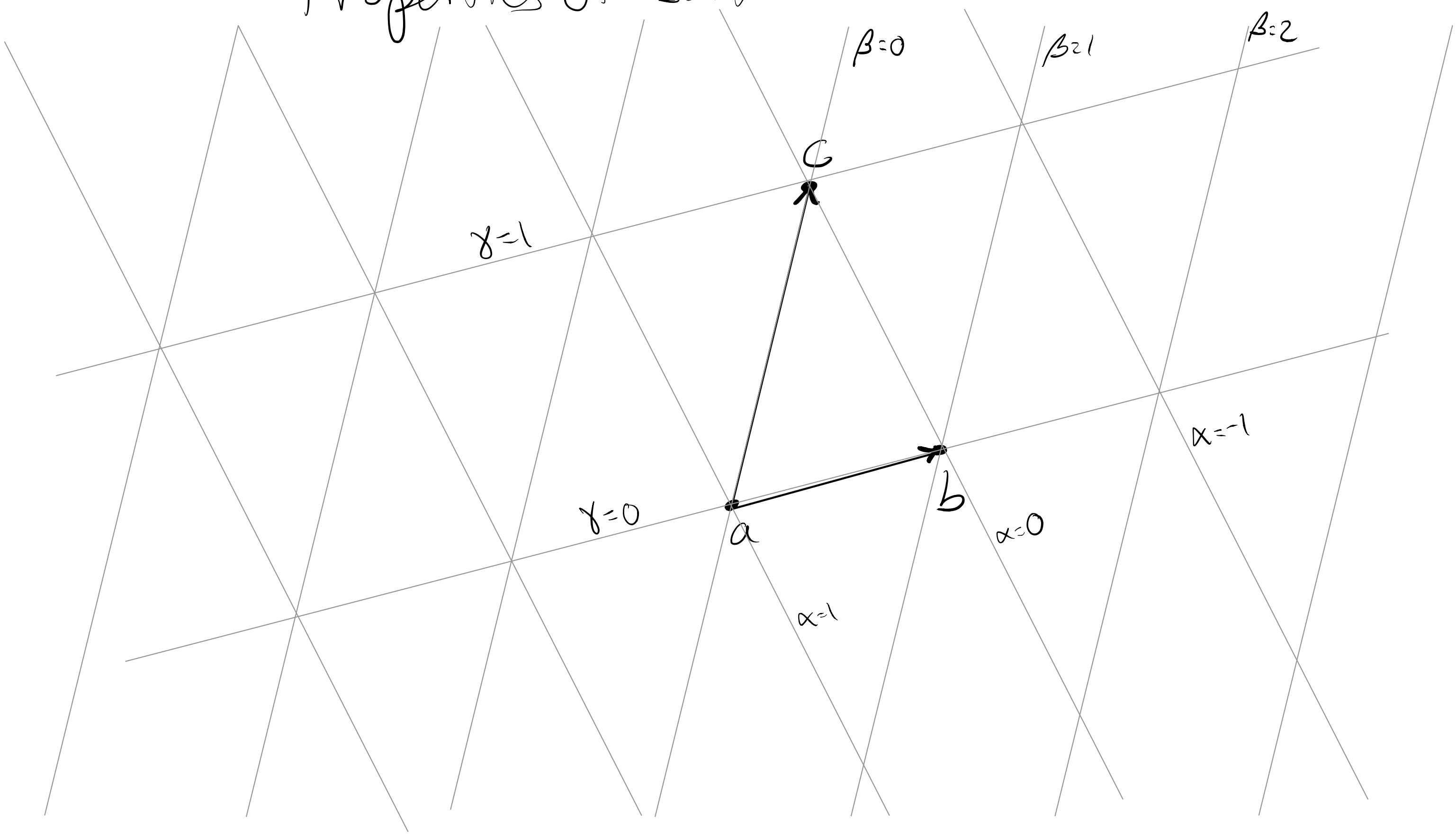
$$p = a + \beta(b-a) + \gamma(c-a)$$

$$= a - \beta a - \gamma a + \beta b + \gamma c$$

$$= \underline{(1-\beta-\gamma)}\, a + \beta b + \gamma c$$

$$\boxed{p = \alpha a + \beta b + \gamma c}$$

Properties of Barycentric Coordinates



- $\alpha + \beta + \gamma = 1$ $\qquad 1 - \beta - \gamma = \alpha$

- α, β, γ give scaled signed distances from edges

  $\alpha = 0 \rightarrow$ on edge cb
  $\alpha = 1 \Rightarrow$ distance 1 (towards a) from edge bc

  $\beta = 0 \Rightarrow$ on edge ac | at b

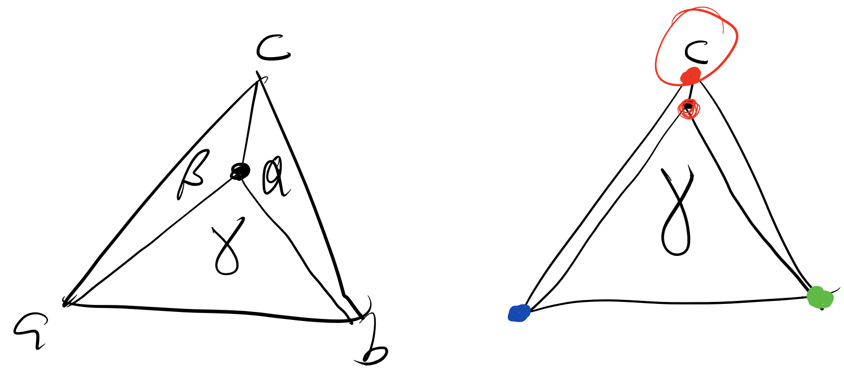
  $\gamma = 0$ on edge ab | at c

- a point is inside abc iff

$$0 < \alpha < 1$$
$$0 < \beta < 1$$
$$0 < \gamma < 1$$

point-in-triangle is solved!

- coords are proportional to areas of subtriangles



interpolation is solved!

V_a, V_b, V_c

$$\boxed{V_p = \alpha V_a + \beta V_b + \gamma V_c}$$

α, β, γ

Barycentric Ray-Tri

Plane: $p(\beta, \gamma) = a + \beta(b-a) + \gamma(c-a)$

Ray: $r(t) = \vec{p} + t\vec{d}$

$$p(\beta, \gamma) = r(t)$$