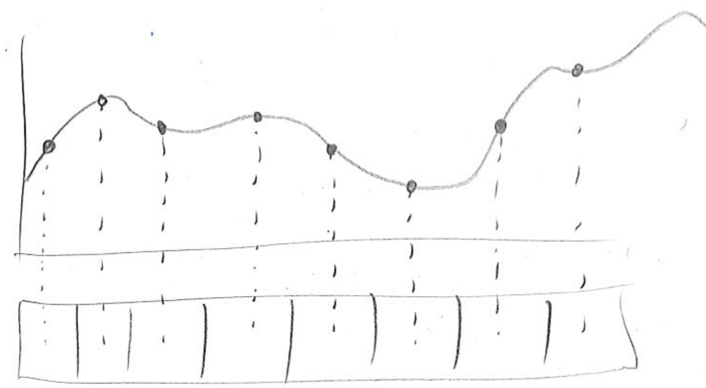
480 - Computer Graphics

Continuous vs Discrete Functions

$I : \mathbb{R}^2 \to \mathbb{R}^3$ - continuous function, "ideal image"

$f : \mathbb{R} \to \mathbb{R}$



Array :

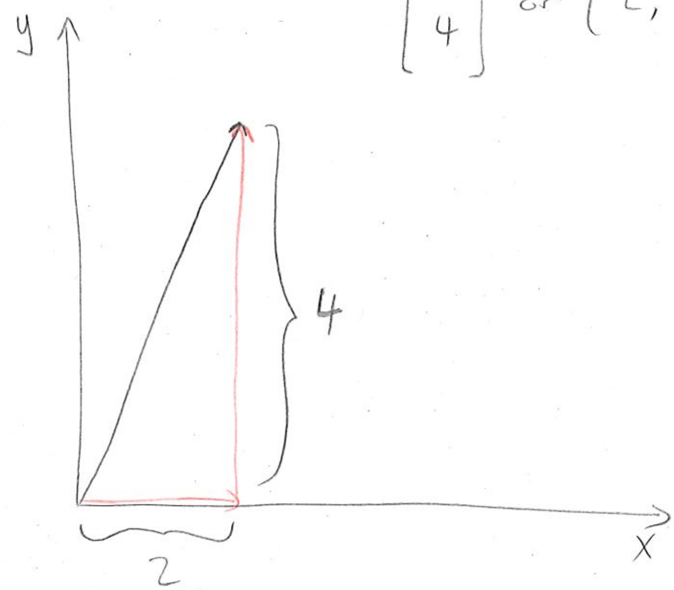
sampled representation (discrete)

480 - Computer Graphics

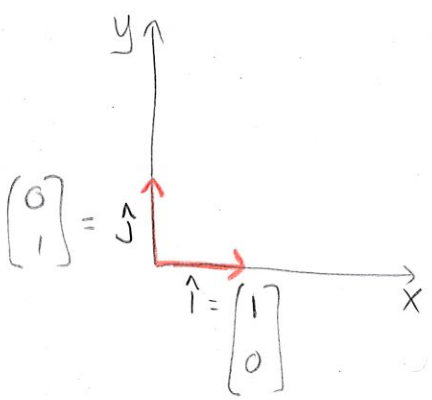
L02.1

Review: Vectors

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} \text{ or } (2, 4)$$



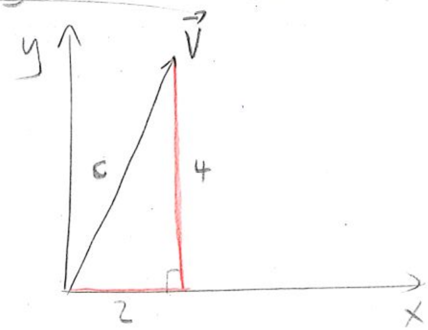
The Canonical Basis



$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2\hat{i} + 4\hat{j}$$

- addition is elementwise
- scalar mult applies to all dims

Vector Magnitude: length of the vector



$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

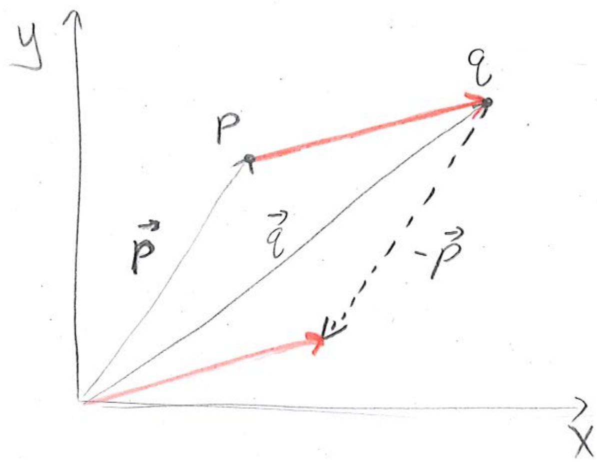
Applies in any dimensions:

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} : \|\vec{u}\| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

L02.2

Vectors are <u>displacements</u>

Absent context, we draw them with the <u>tail</u> at the <u>origin</u>.



The vector from point p to point q is $q-p$

The Dot product

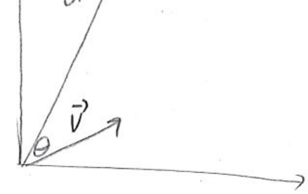
One version of "$\vec{u}$ times $\vec{v}$"

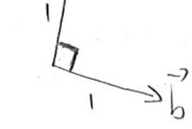
Let $\vec{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$, $\vec{v} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$. Then:

$$\vec{u} \cdot \vec{v} = \underbrace{u_x v_x + u_y v_y}_{\text{This is a scalar!}} = \vec{u}^T \vec{v} = \|\vec{u}\|\,\|\vec{v}\| \cos \theta$$

angle between $\vec{u}, \vec{v}$

"how much do these vectors agree?"





What is $\vec{a} \cdot \vec{b}$?

A. 1

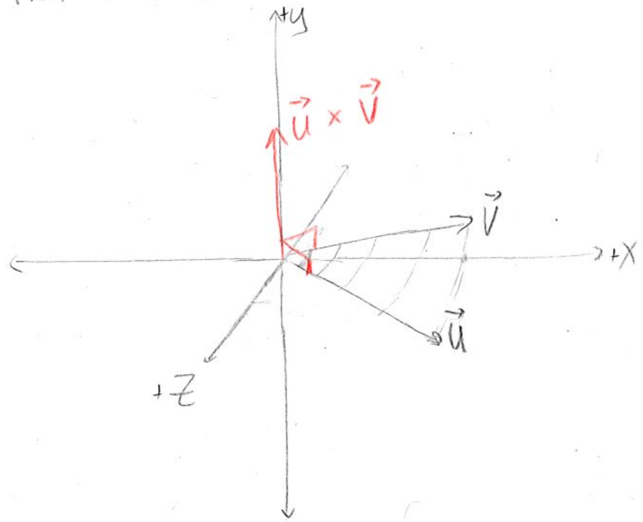
B. 0

C. -1

D. Can't know

The Cross-Product

Another version of "$\vec{u}$ times $\vec{v}$"; only works in 3D

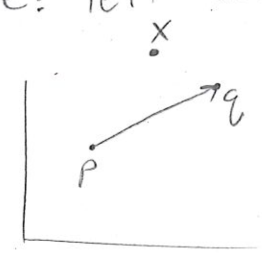


$\vec{u} \times \vec{v}$ is orthogonal to both $\vec{u}$ and $\vec{v}$ (direction given by right hand rule)

its magnitude is $\|\vec{u}\|\|\vec{v}\| \sin\theta$

Point-in-triangle (2D)

Primitive: left-on-right test



is x to the left, right, or on the line between p and q?

Point in triangle: