Geometric Transformations

$T : \mathbb{R}^3 \to \mathbb{R}^3$

$T(\vec{x}) = \vec{x} + \vec{t}$ translation

Linear geometric transformations

linear combo

$\to x' = f(x, y, z) = ax + by + cz$

$y' = f(x, y, z) = dx + ey + fz$

$z' = f(x, y, z) = gx + hy + iz$

$T_{3\times3}$ $\vec{x}_{3\times1}$

Matrices - 2D array of (real) number

for us

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4.5 & 0.5 \end{bmatrix}$$

$m \times n$

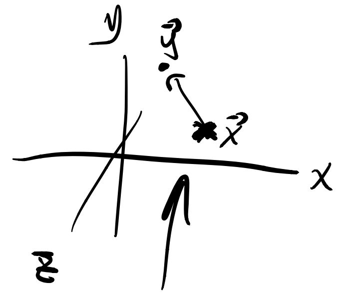
2×3

rows cols

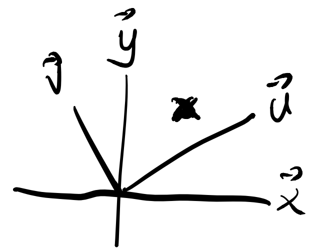
a_{ij} $a_{21} = 3$

Matrix - Vector Mult

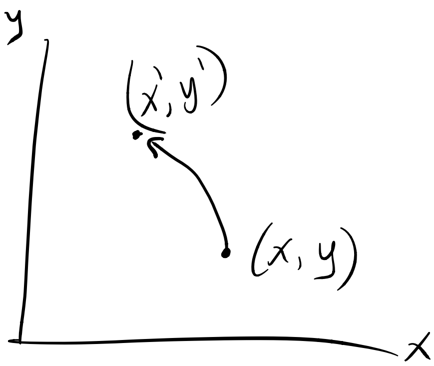
$$\vec{y} = \begin{bmatrix} \vec{r}_1 \cdot \vec{x} \\ r_2 \cdot \vec{x} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \text{—} & r_1 & \text{—} \\ \text{—} & r_2 & \text{—} \\ \text{—} & r_3 & \text{—} \\ & \vdots & \\ \text{—} & r_m & \text{—} \end{bmatrix} \begin{bmatrix} | \\ \vec{x} \\ | \end{bmatrix}$$

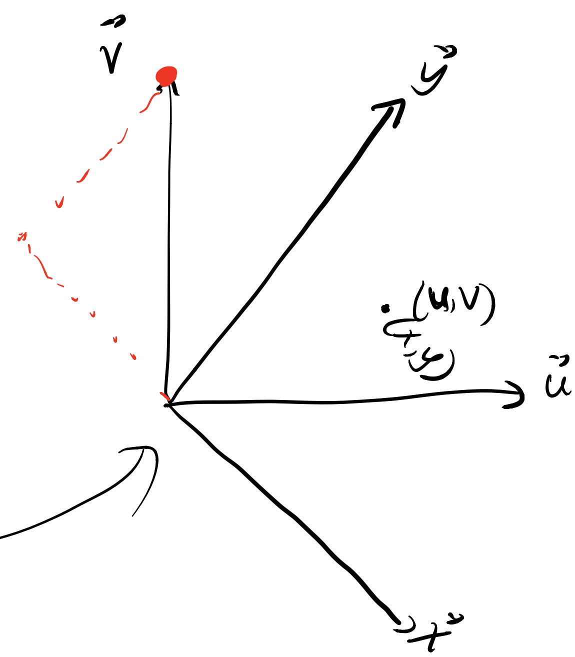


$$\vec{y} = x_1\vec{c}_1 + x_2\vec{c} + \dots + x_n\vec{c}_n = \begin{bmatrix} | & | & & | \\ c_1 & c_2 & \cdots & c_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$



$A_{2\times 2} : \mathbb{R}^2 \to \mathbb{R}^2$

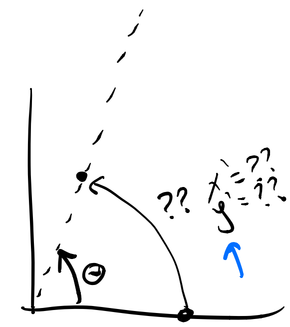




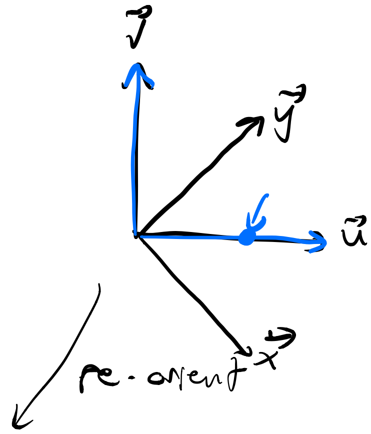
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} | & | \\ \vec{u} & \vec{v} \\ | & | \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

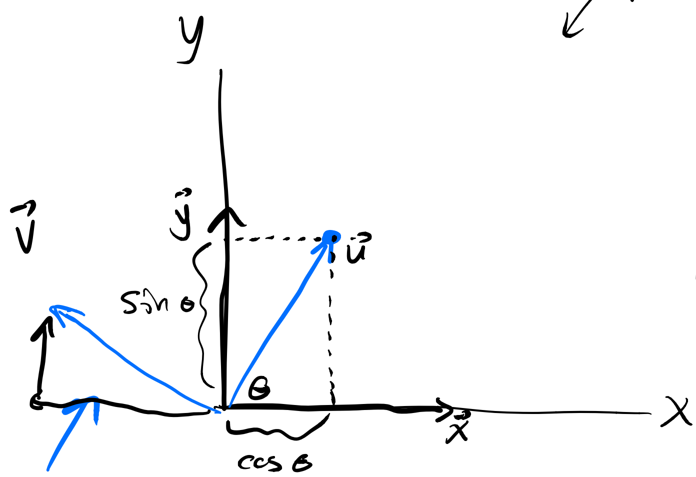
basis to canonical
xform

2x2 Rotation Matrix



Change of basis view:





$$\begin{array}{c} \begin{array}{cc}\vec{u} & \quad \vec{v}\end{array} \\ \begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\end{array}$$