

[Richard Zakia]

# Computer Graphics

Lecture 6

**Projections**

**More General Cameras**

# Announcements

- CS Department TGIF - today at 4pm!  
Details: <https://cs.wwu.edu/tgif-department-social>
- HW1 is out - due Monday 1/25
  - #3 will help with A1
- A1 - if you don't have a partner yet, meet me in the In-Class Voice channel on Discord directly after class.

# Announcements

- No class Monday - MLK Day
- I'll hold office hours Tuesday 10-11am to replace Monday's.
- Today's office hours will be cut short: 2:00-2:30.
- Tuesday's lecture - live, no videos
  - You may want to review Chapter 2.2 on quadratic equations.



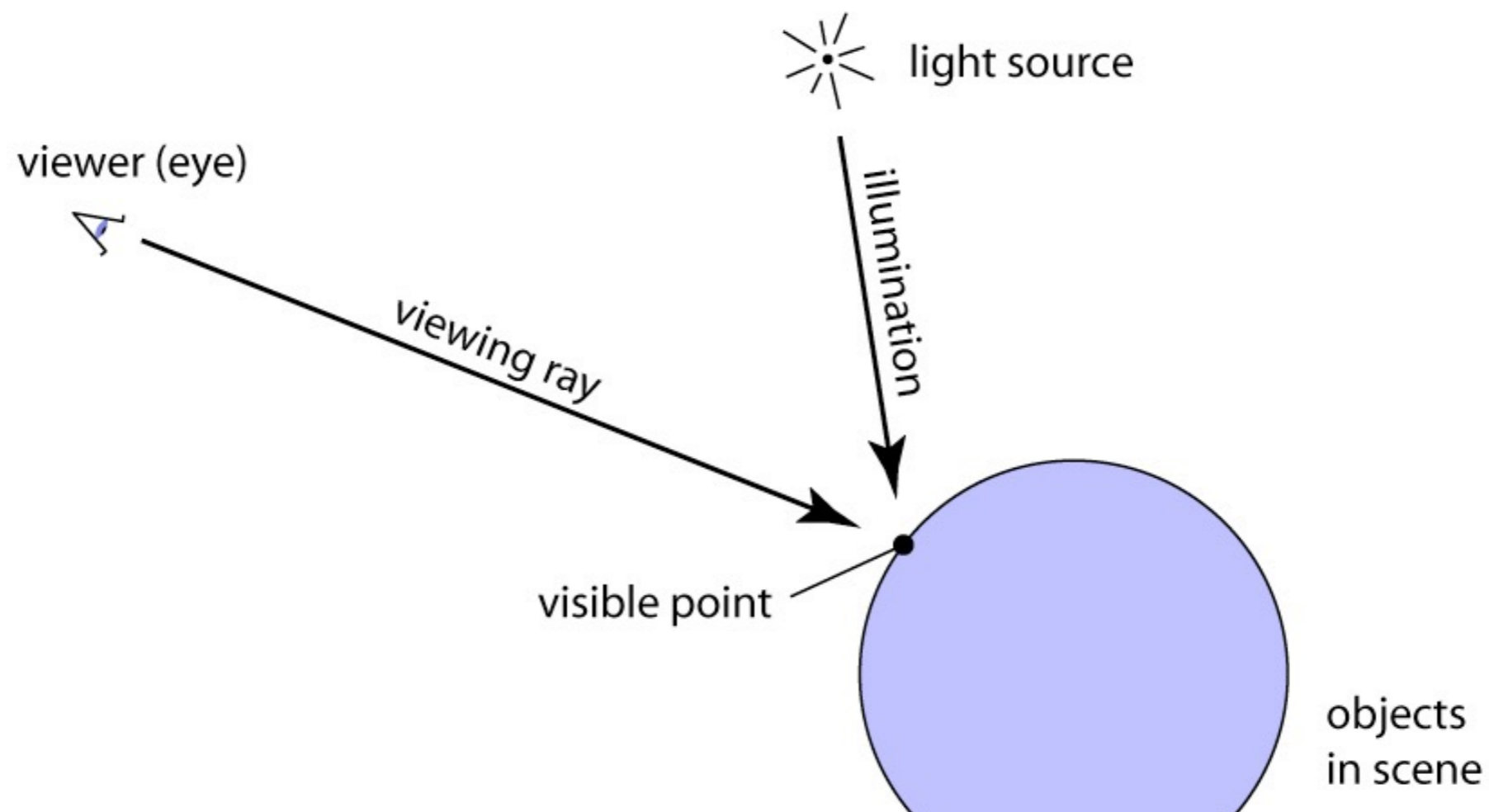
# Ray Tracing: Pseudocode

for each pixel:

generate a viewing ray for the pixel

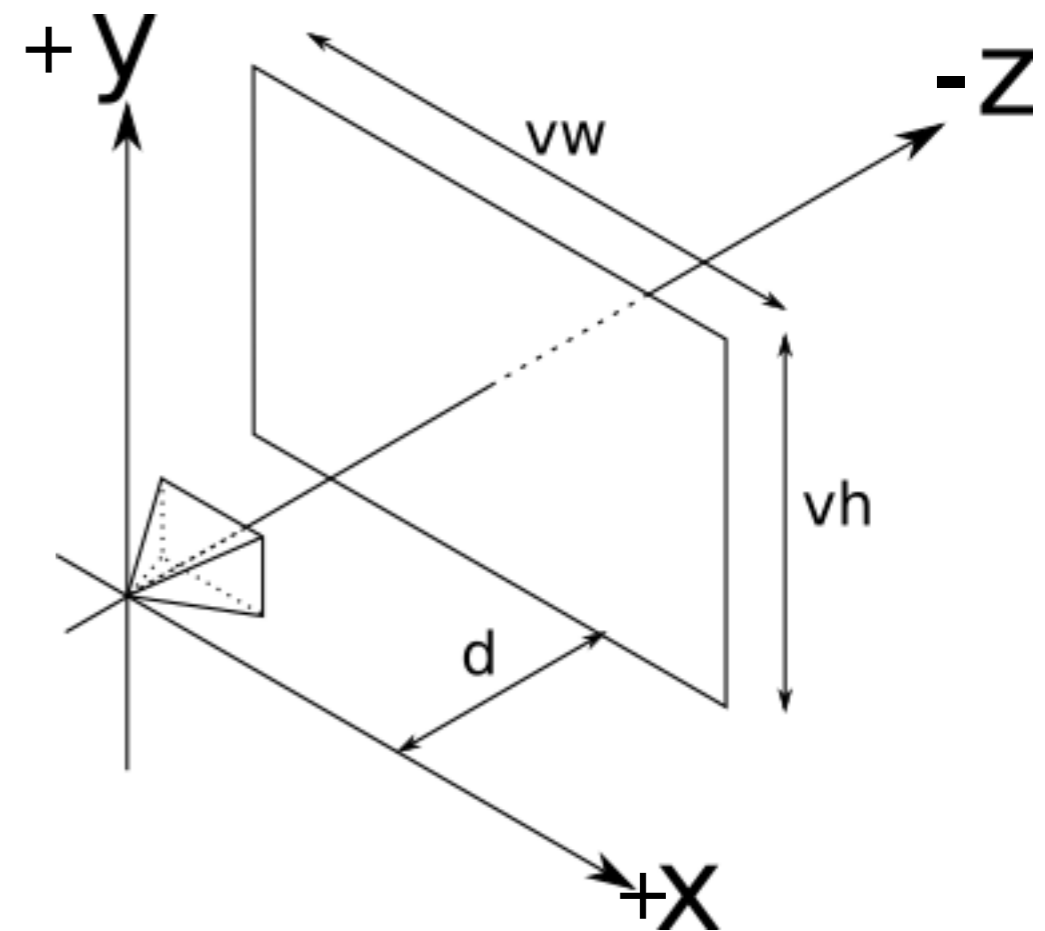
find the closest object it intersects

determine the color of the object



# A "canonical" perspective camera

- Eye is at the origin  $(0, 0, 0)$
- Looking down the **negative** z axis
- Viewport is aligned with the xy plane
- $vh = vw = 1$
- $d = 1$



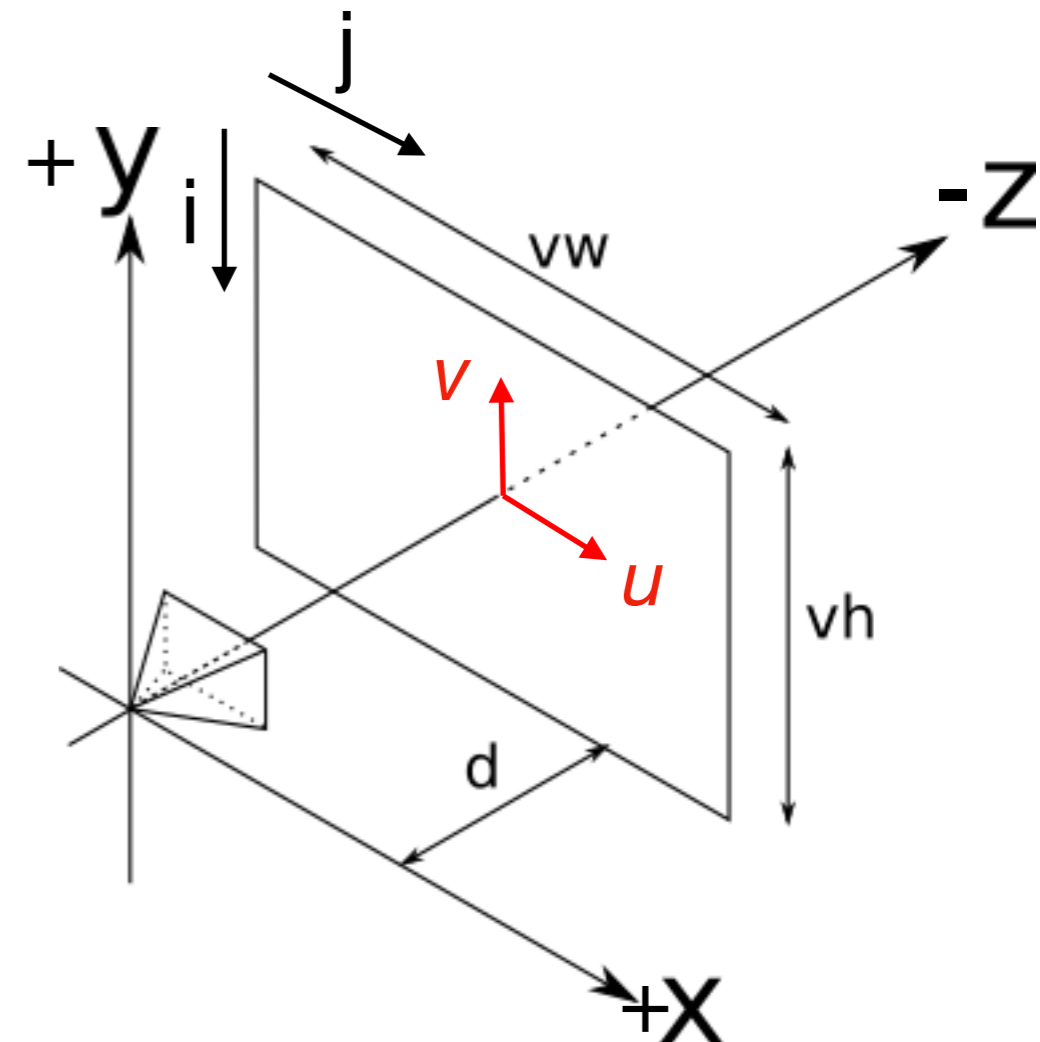
# More General Cameras

$$u = \frac{j - \frac{1}{2}}{W} - \frac{1}{2}$$
$$v = - \left( \frac{i - \frac{1}{2}}{H} - \frac{1}{2} \right)$$

Origin ( $\mathbf{p}$ ): (0, 0, 0)  
Direction ( $\mathbf{d}$ ): ( $u, v, -1$ )

**Let's break some assumptions!**

- $\mathbf{d} = \mathbf{1}$
- $vh = vw = 1$
- Eye is at the origin (0, 0, 0)
- Looking down the **negative** z axis



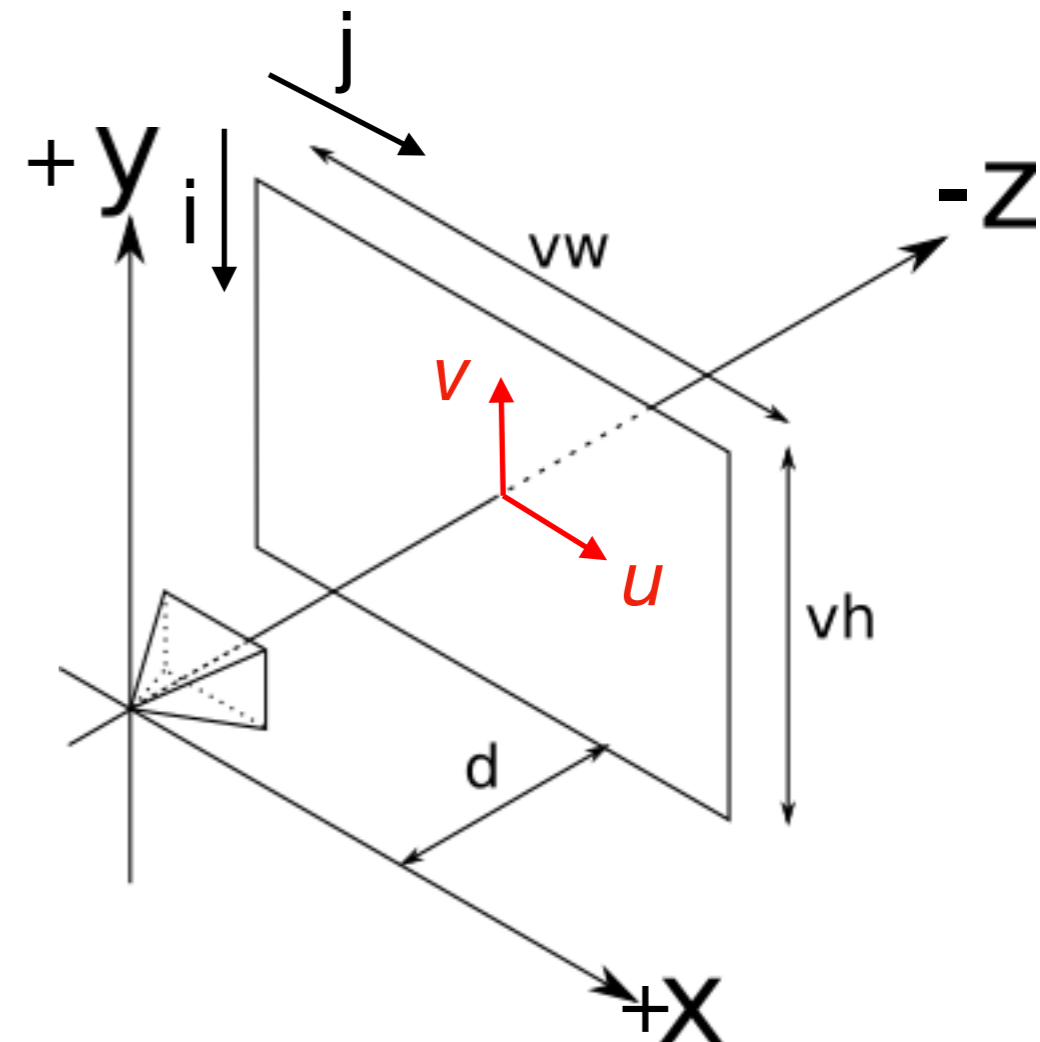
# More General Cameras

$$u = \frac{j - \frac{1}{2}}{W} - \frac{1}{2}$$
$$v = - \left( \frac{i - \frac{1}{2}}{H} - \frac{1}{2} \right)$$

Origin (**p**): (0, 0, 0)  
Direction (**d**): (u, v, -d)

**Let's break some assumptions!**

- **d = 1**
- $vh = vw = 1$
- Eye is at the origin (0, 0, 0)
- Looking down the **negative z axis**





# More General Cameras

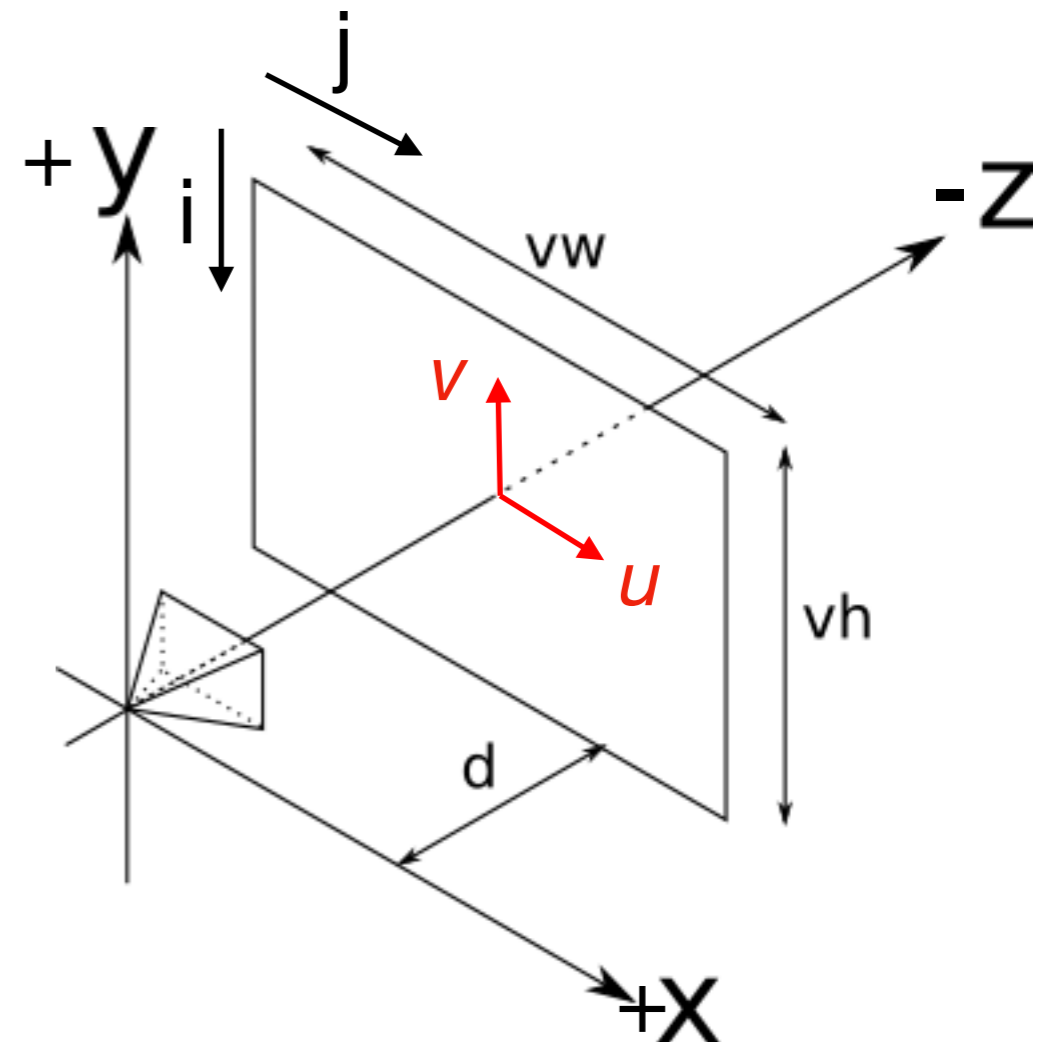
$$u = \frac{j - \frac{1}{2}}{W} - \frac{1}{2}$$

$$v = - \left( \frac{i - \frac{1}{2}}{H} - \frac{1}{2} \right)$$

Origin ( $\mathbf{p}$ ): (0, 0, 0)  
 Direction ( $\mathbf{d}$ ): ( $u, v, -1$ )

**Let's break some assumptions!**

- $d = 1$
- $vh = vw = 1$
- Eye is at the origin (0, 0, 0)
- Looking down the **negative** z axis



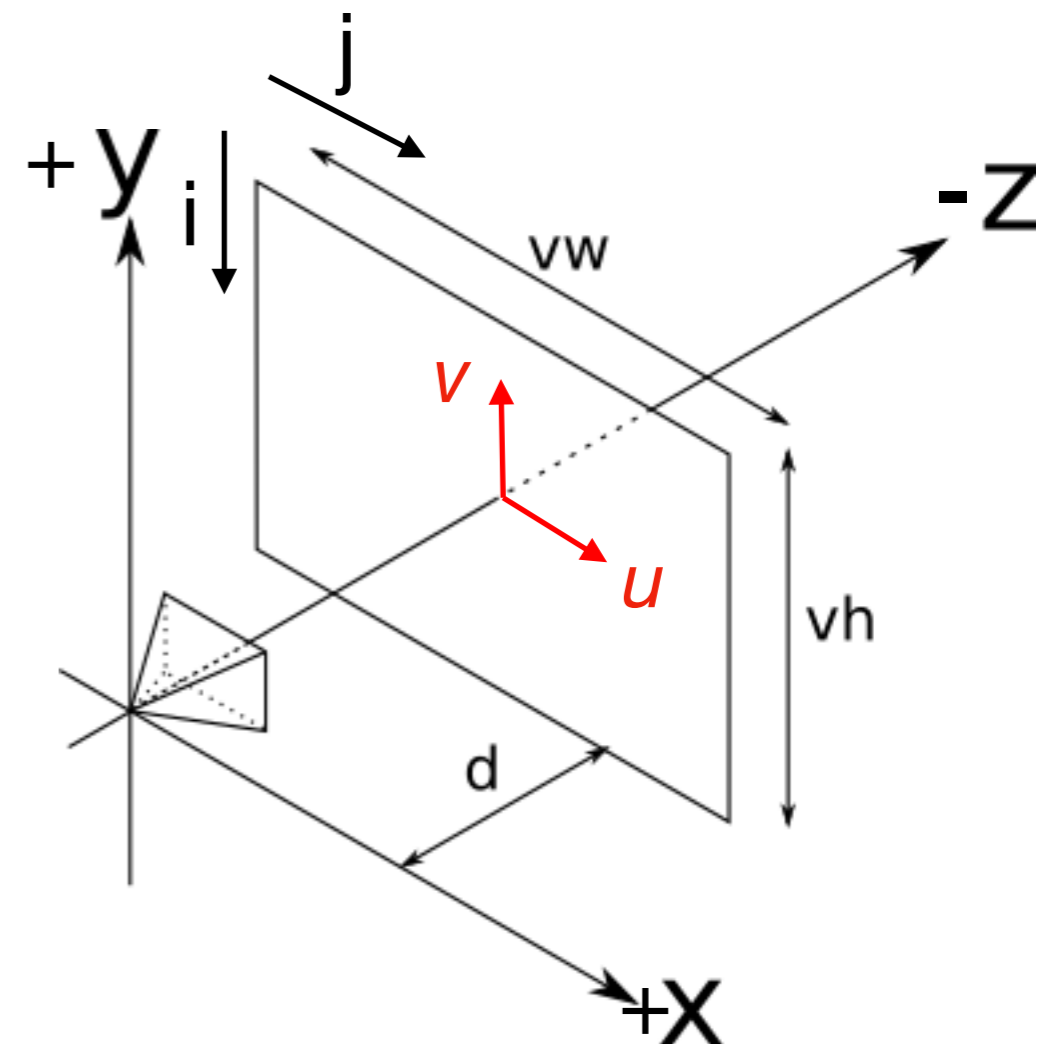
# More General Cameras

$$u = \left( \frac{j - \frac{1}{2}}{W} - \frac{1}{2} \right) * vw$$
$$v = - \left( \frac{i - \frac{1}{2}}{H} - \frac{1}{2} \right) * vh$$

Origin (**p**): (0, 0, 0)  
Direction (**d**): (u, v, -1)

**Let's break some assumptions!**

- $d = 1$
- $vh = vw = 1$
- Eye is at the origin (0, 0, 0)
- Looking down the **negative** z axis



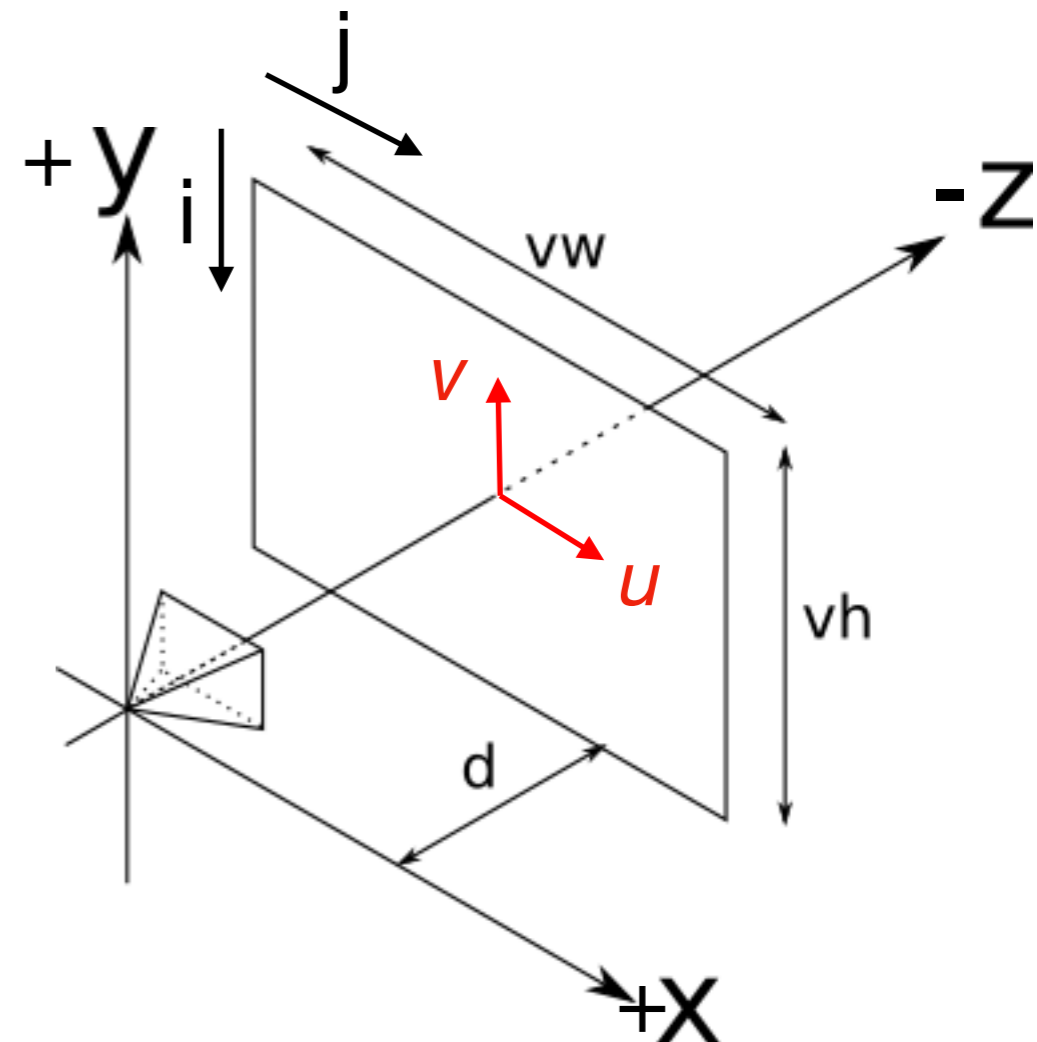
# More General Cameras

$$u = \frac{j - \frac{1}{2}}{W} - \frac{1}{2}$$
$$v = - \left( \frac{i - \frac{1}{2}}{H} - \frac{1}{2} \right)$$

Origin (**p**): (0, 0, 0)  
Direction (**d**): (u, v, -1)

**Let's break some assumptions!**

- $d = 1$
- $vh = vw = 1$
- **Eye is at the origin (0, 0, 0)**
- Looking down the **negative z axis**



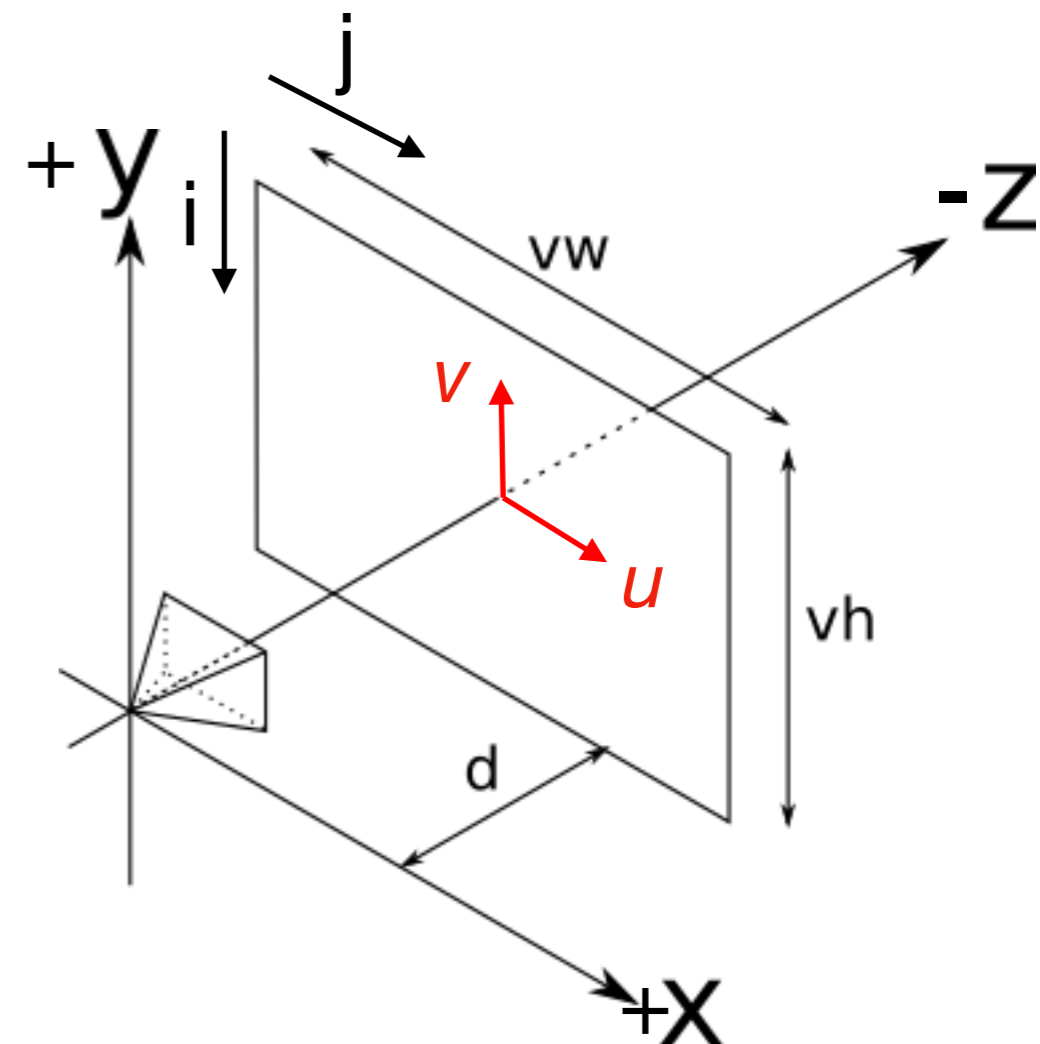
# More General Cameras

$$u = \frac{j - \frac{1}{2}}{W} - \frac{1}{2}$$
$$v = - \left( \frac{i - \frac{1}{2}}{H} - \frac{1}{2} \right)$$

Origin (**p**):  $(e_x, e_y, e_z)$   
Direction (**d**):  $(u, v, -1)$

**Let's break some assumptions!**

- $d = 1$
- $vh = vw = 1$
- **Eye is at the origin  $(0, 0, 0)$**
- Looking down the **negative z axis**



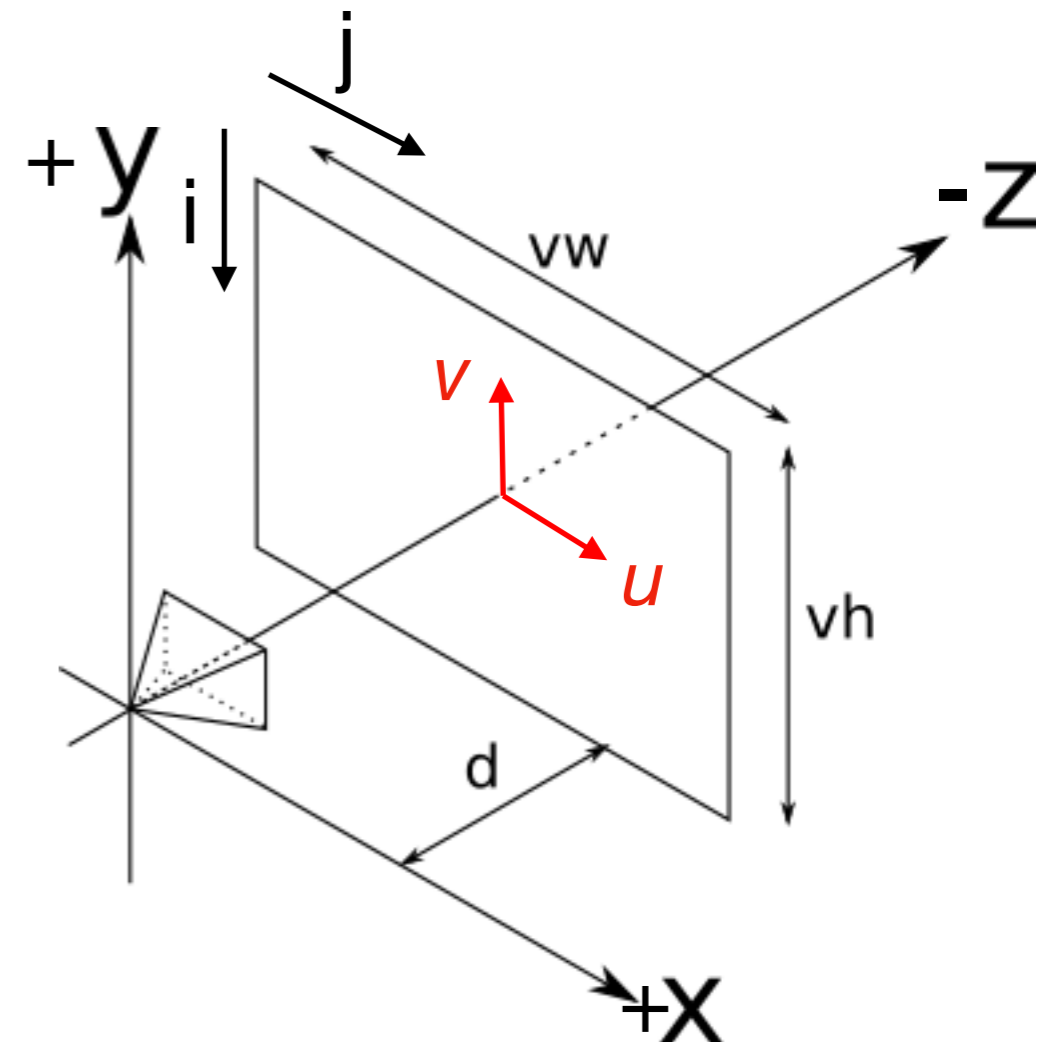
# More General Cameras

$$u = \frac{j - \frac{1}{2}}{W} - \frac{1}{2}$$
$$v = - \left( \frac{i - \frac{1}{2}}{H} - \frac{1}{2} \right)$$

Origin (**p**): (0, 0, 0)  
Direction (**d**): (u, v, -1)

**Let's break some assumptions!**

- $d = 1$
- $vh = vw = 1$
- Eye is at the origin (0, 0, 0)
- **Looking down the negative z axis**



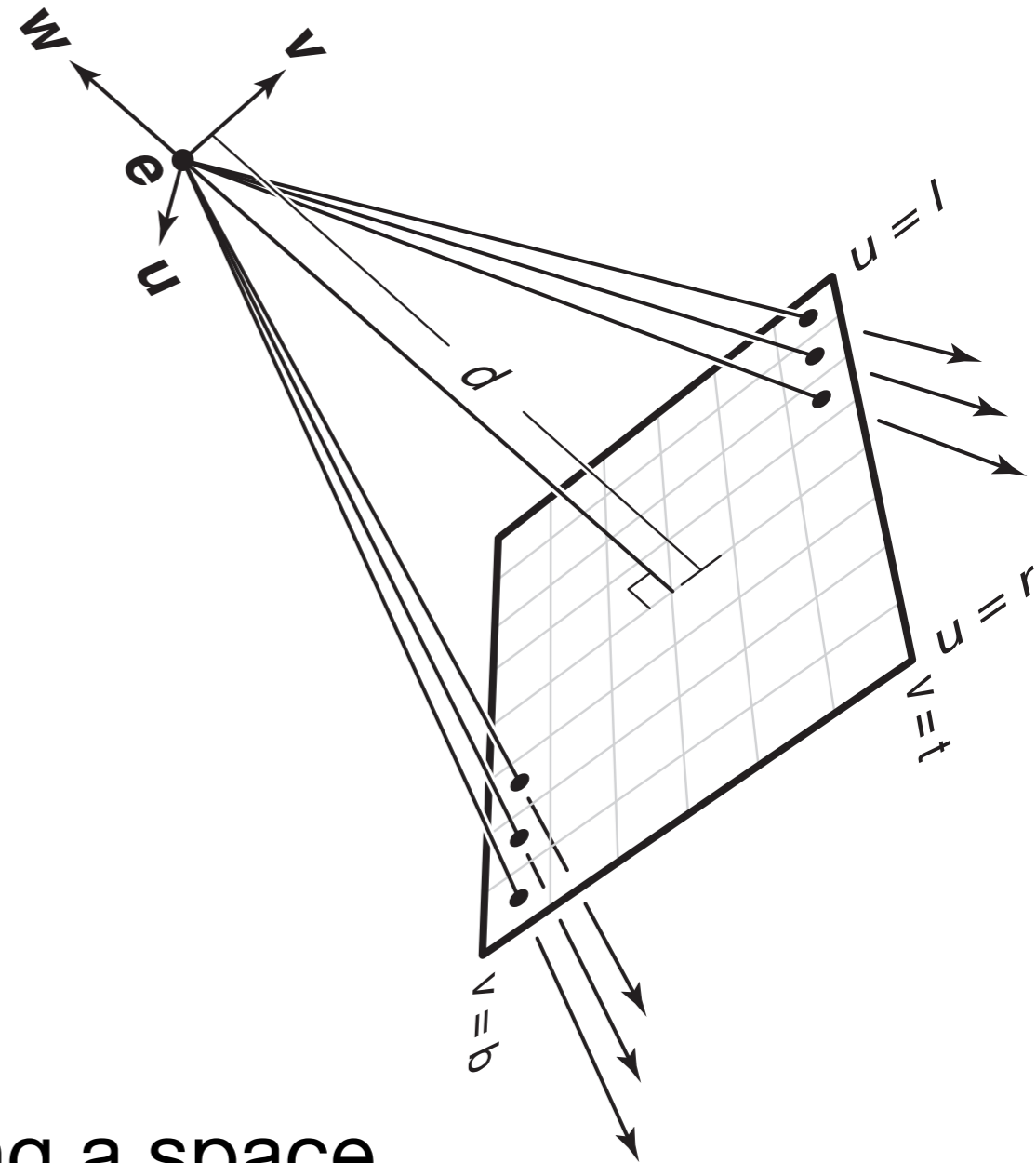
# What if I want to point the camera somewhere else?

The key idea:

1. Find a **basis** where the camera *is* in canonical pose.
2. Change basis back to familiar x-y-z coordinates.

Pedantic side note:

- a **basis** is a set of vectors spanning a space.
- a **frame** is a basis plus an **origin** point.



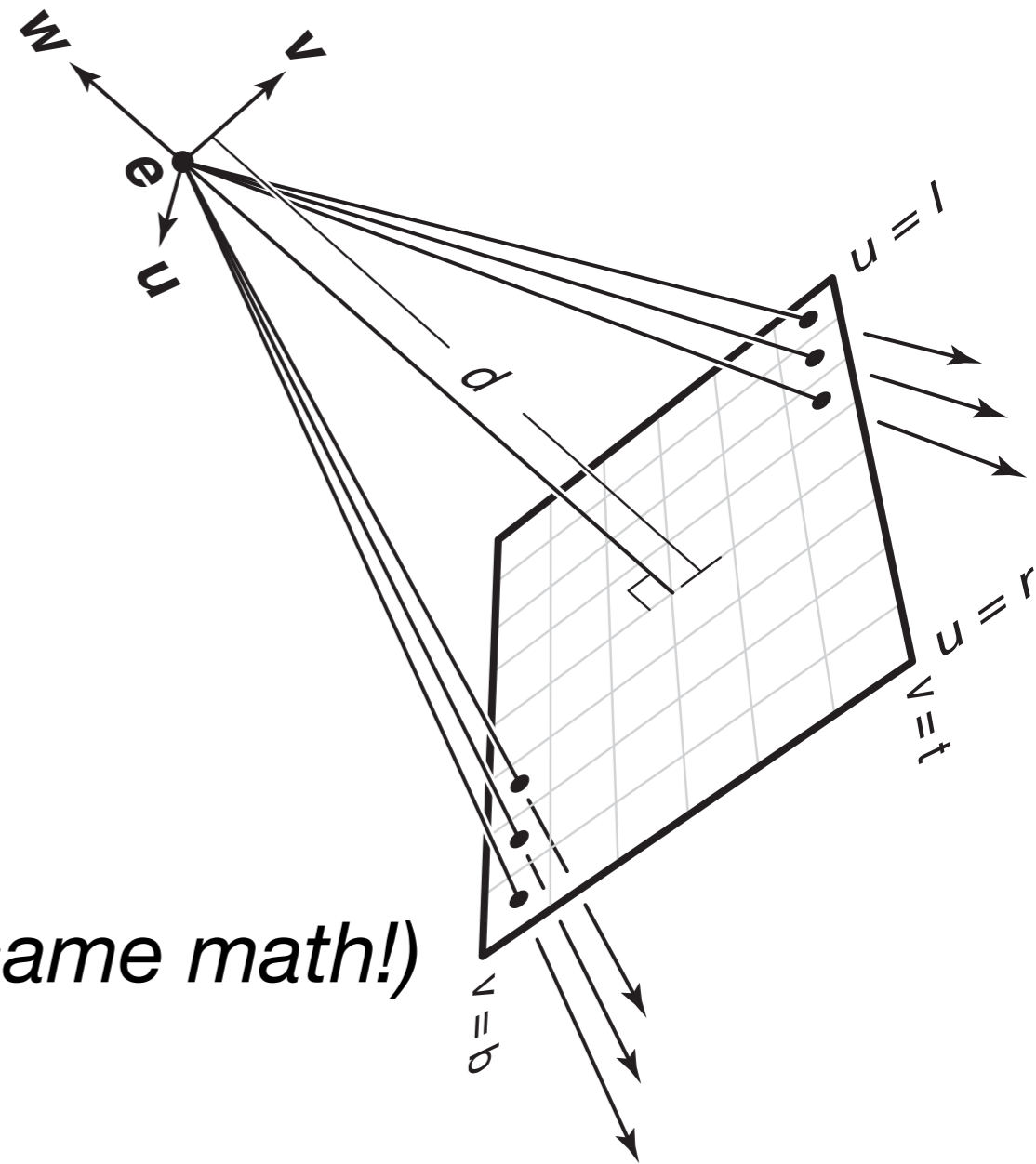
# What if I want to point the camera somewhere else?

The camera's pose is defined by a **coordinate frame**:

- **e** is the position of the eye
- **u** points right from **e**
- **v** points up from **e**
- **w** points back from **e**

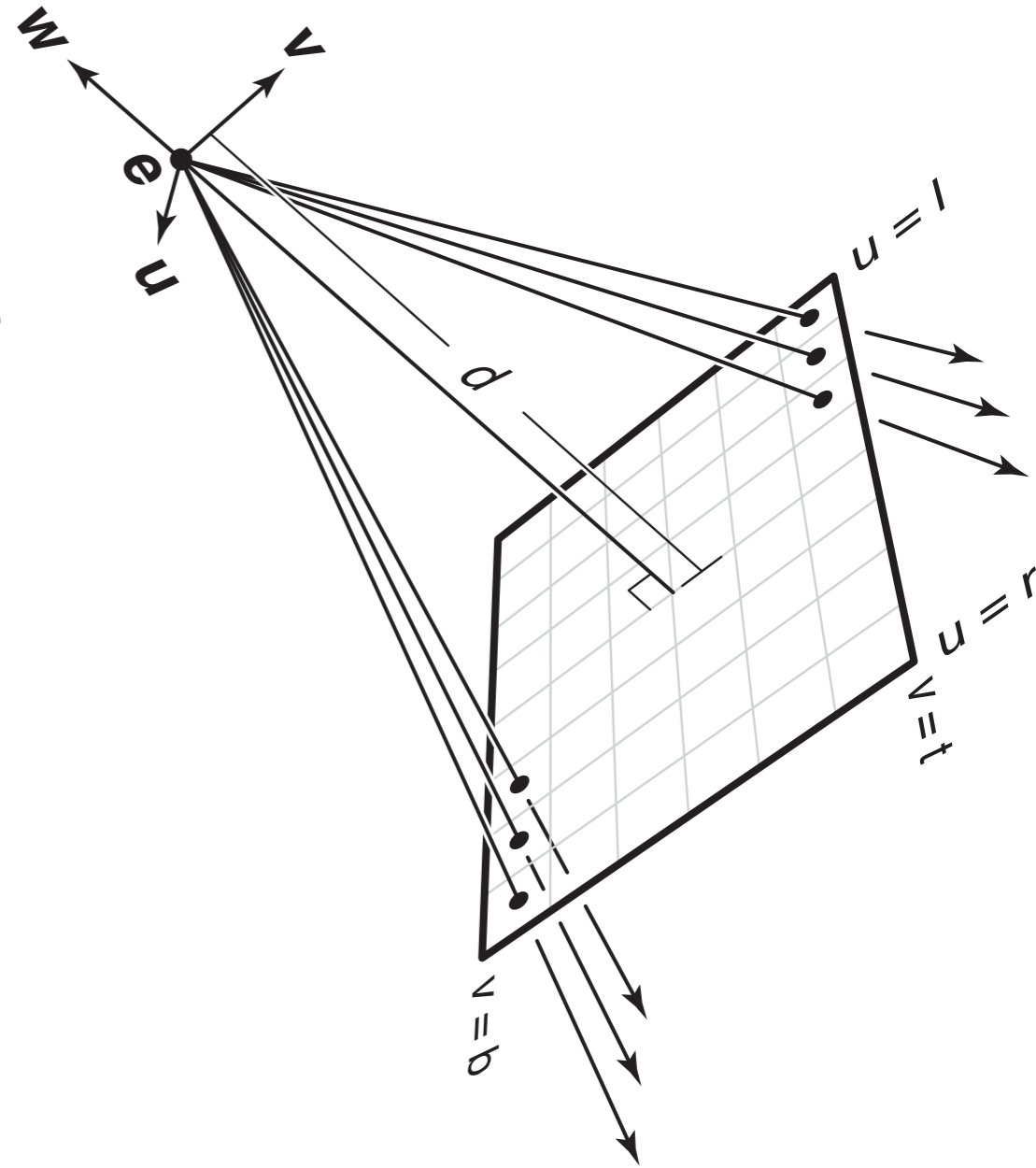
Given this, we can generate a viewing ray as follows:

1. Turn  $(i,j)$  into  $u, v$  instead of  $x, y$  (*same math!*)
2. Viewing ray in  $(x, y, z)$  world is:  
origin = eye  
direction =  $u * \mathbf{u} + v * \mathbf{v} + -d * \mathbf{w}$



# Creating A Camera Basis

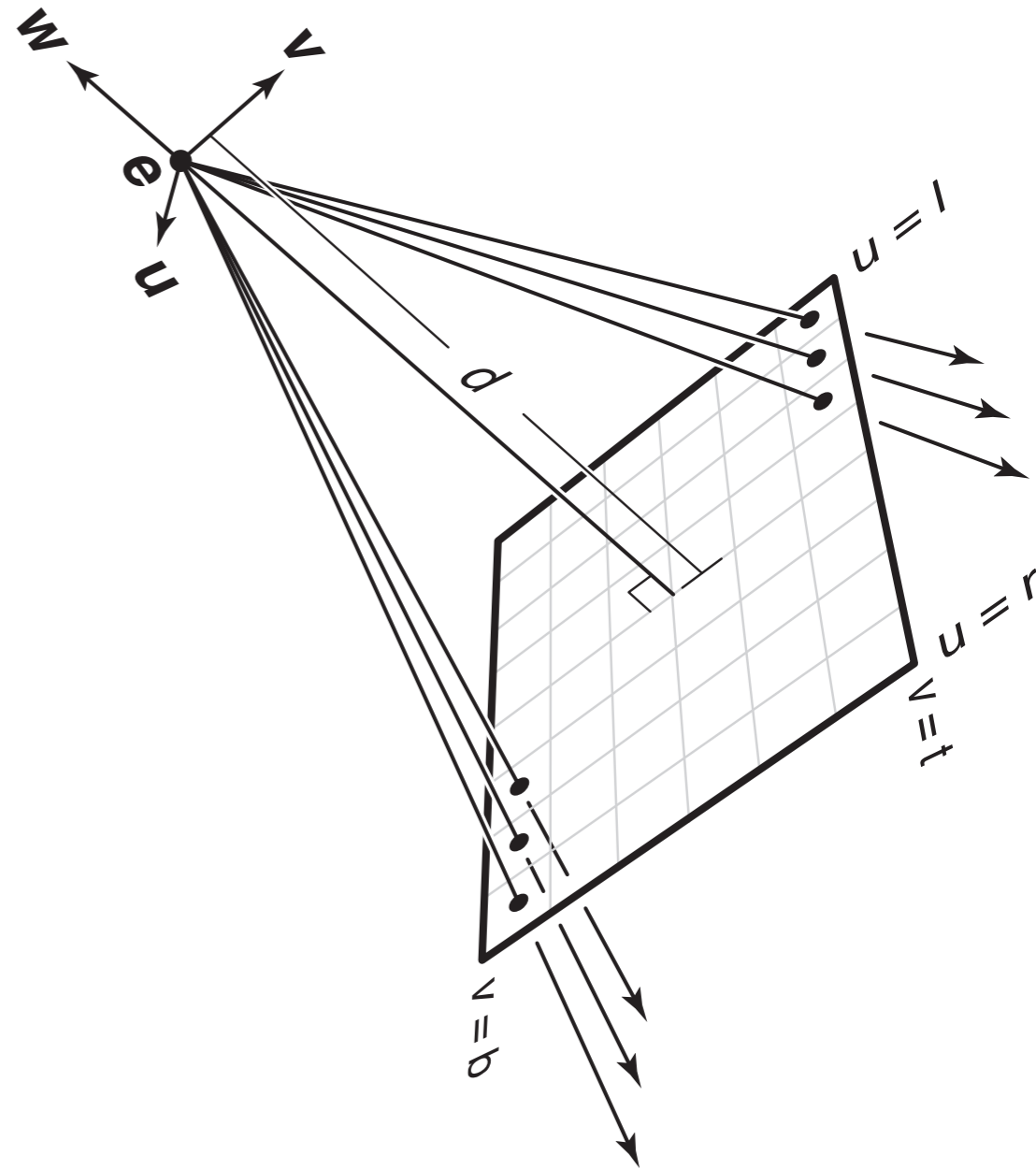
- Ask the modeler to specify  $\mathbf{e}$ ,  $\mathbf{u}$ ,  $\mathbf{v}$ ,  $\mathbf{w}$  : makes the math simple, but not very intuitive modeling.
- Better: position a camera based on:
  - eye
  - view direction or point?





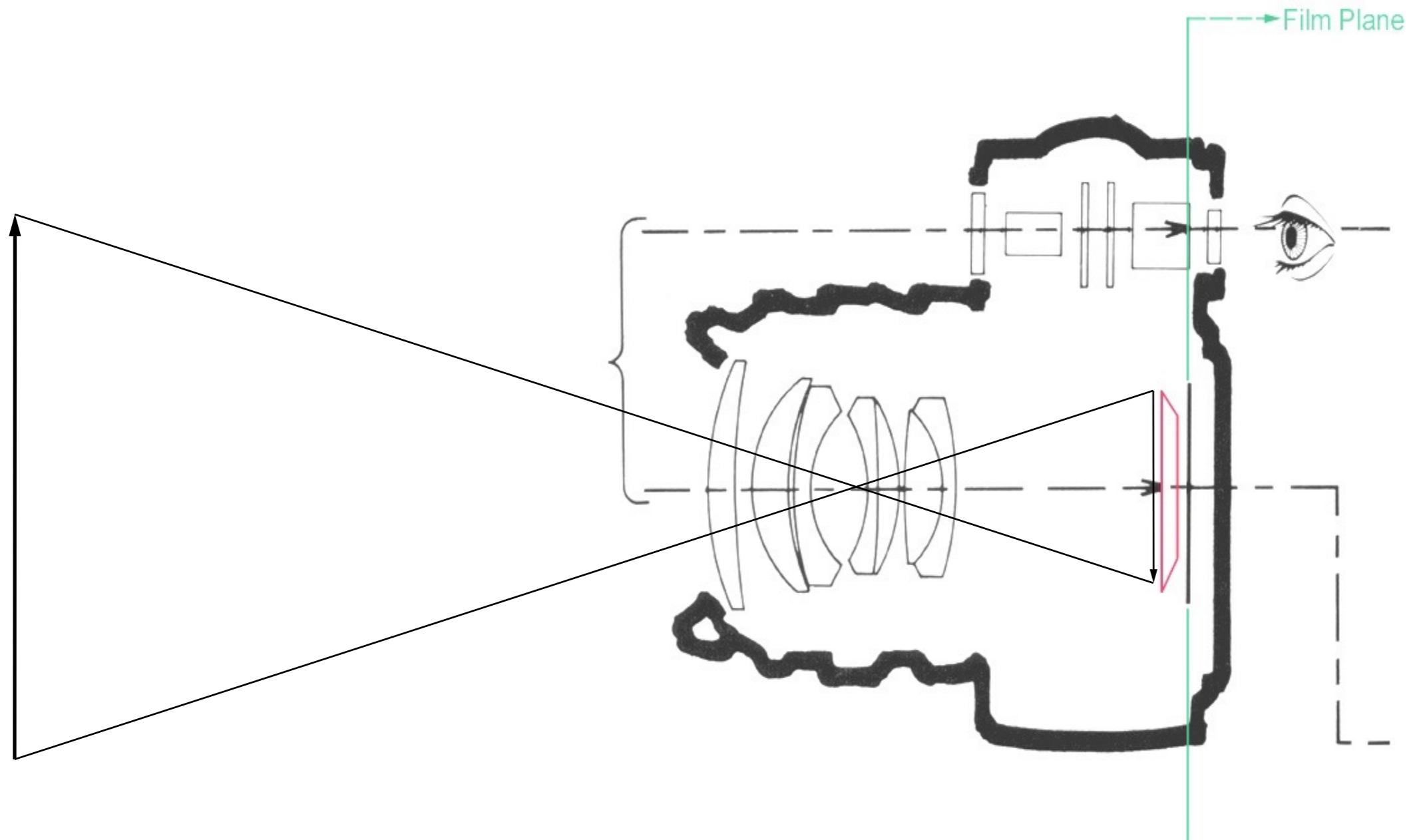
# Creating A Camera Basis

- **eye** - position of eye
- **view** direction - direction camera is looking
- **"up"** vector - points "up" in the scene, but not necessarily in image space.



# Creating a Basis

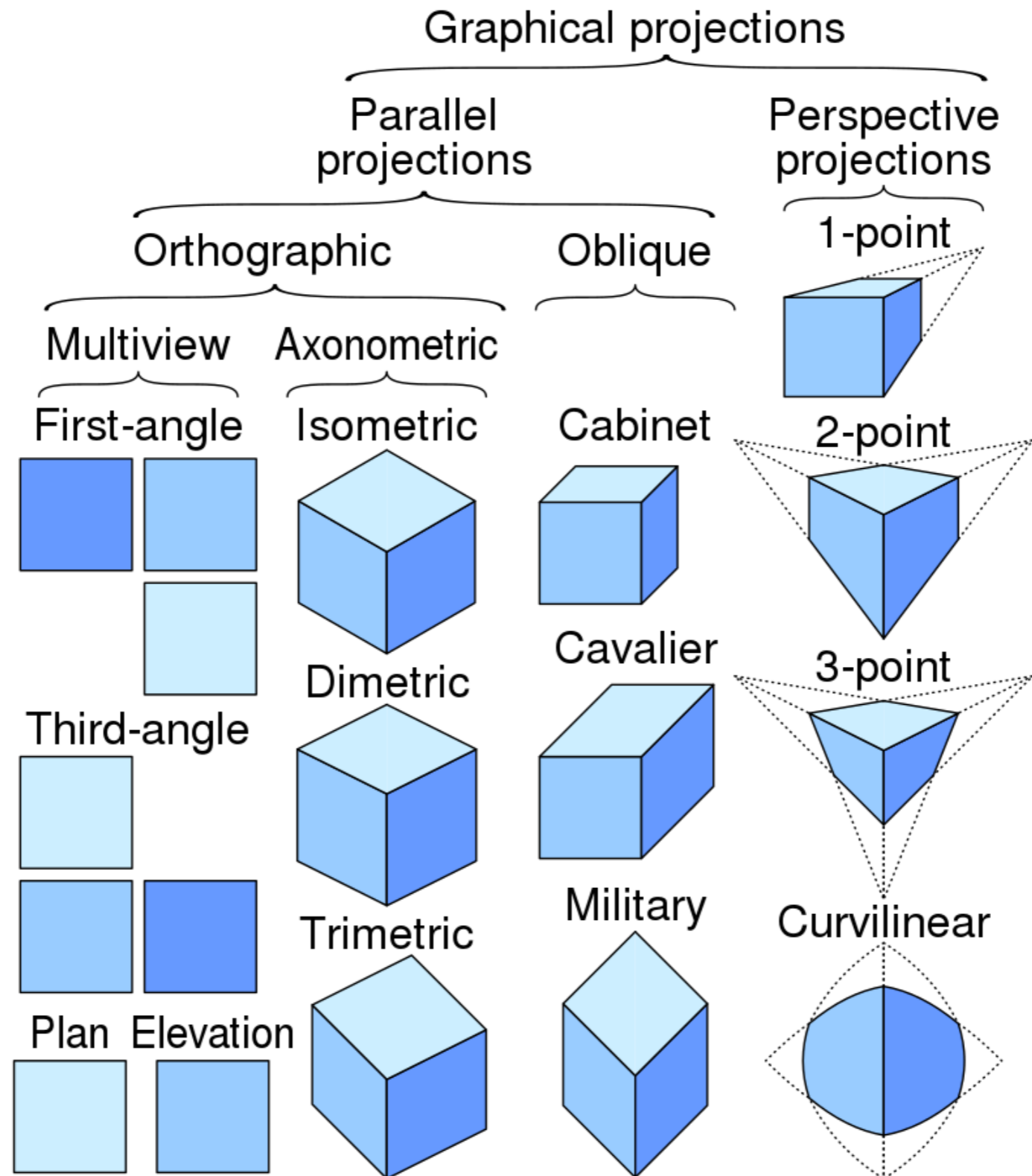
# Perspective Cameras: IRL



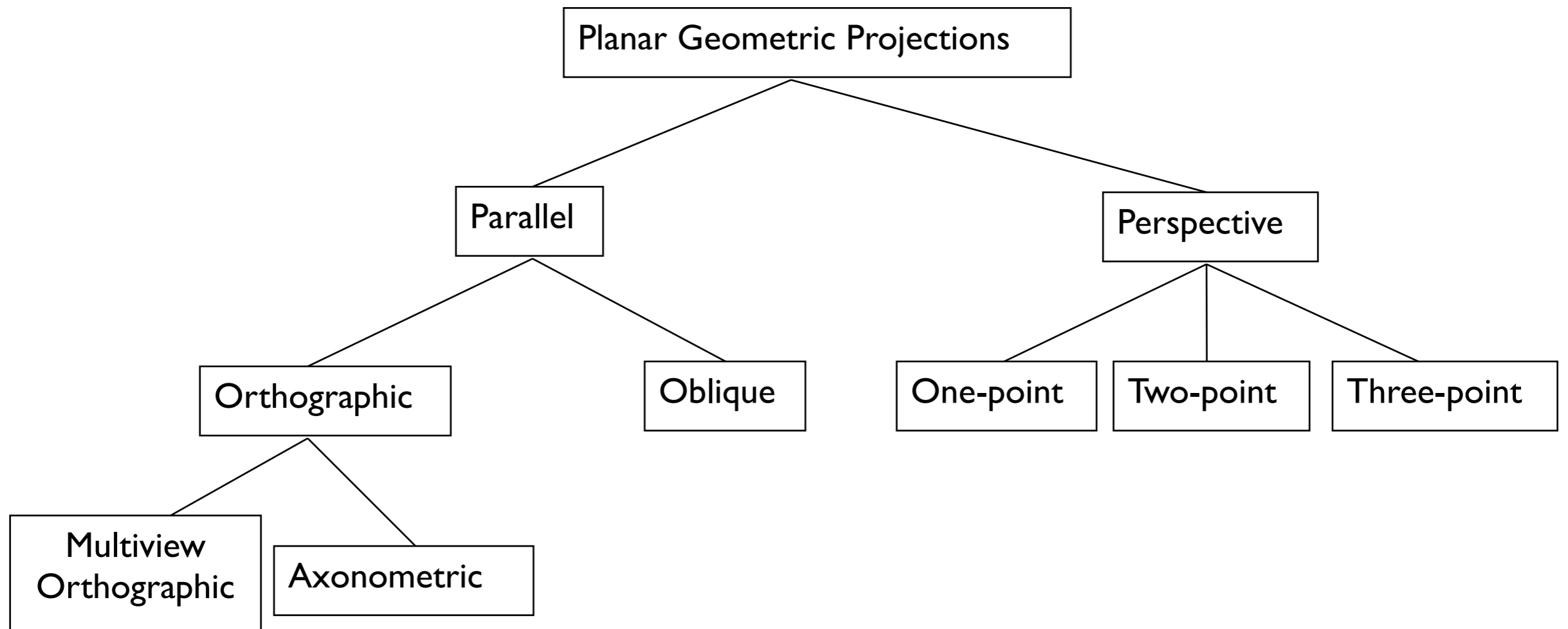
# Perspective Cameras: IR(ish)L

- Thin lens model

# Classical Projections: Taxonomy



# Classical Projections: Taxonomy

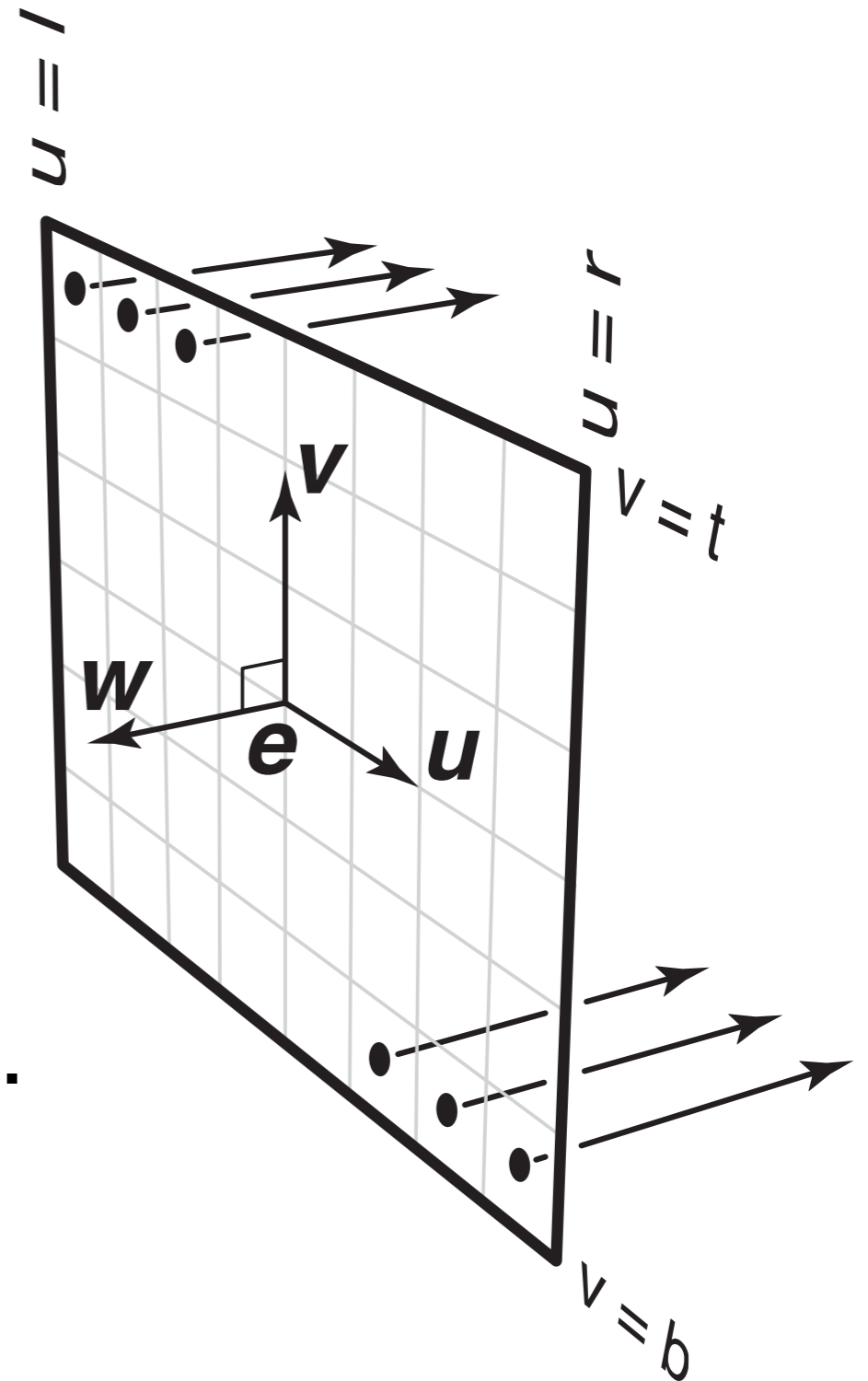


# Parallel Projections

- Parallel viewing rays
- Ray origins from pixels
- Camera origin (eye) is on the image plane

**Orthographic:** viewing rays are perpendicular to projection plane.

i.e., ray direction  $\mathbf{d} = -\mathbf{w}$

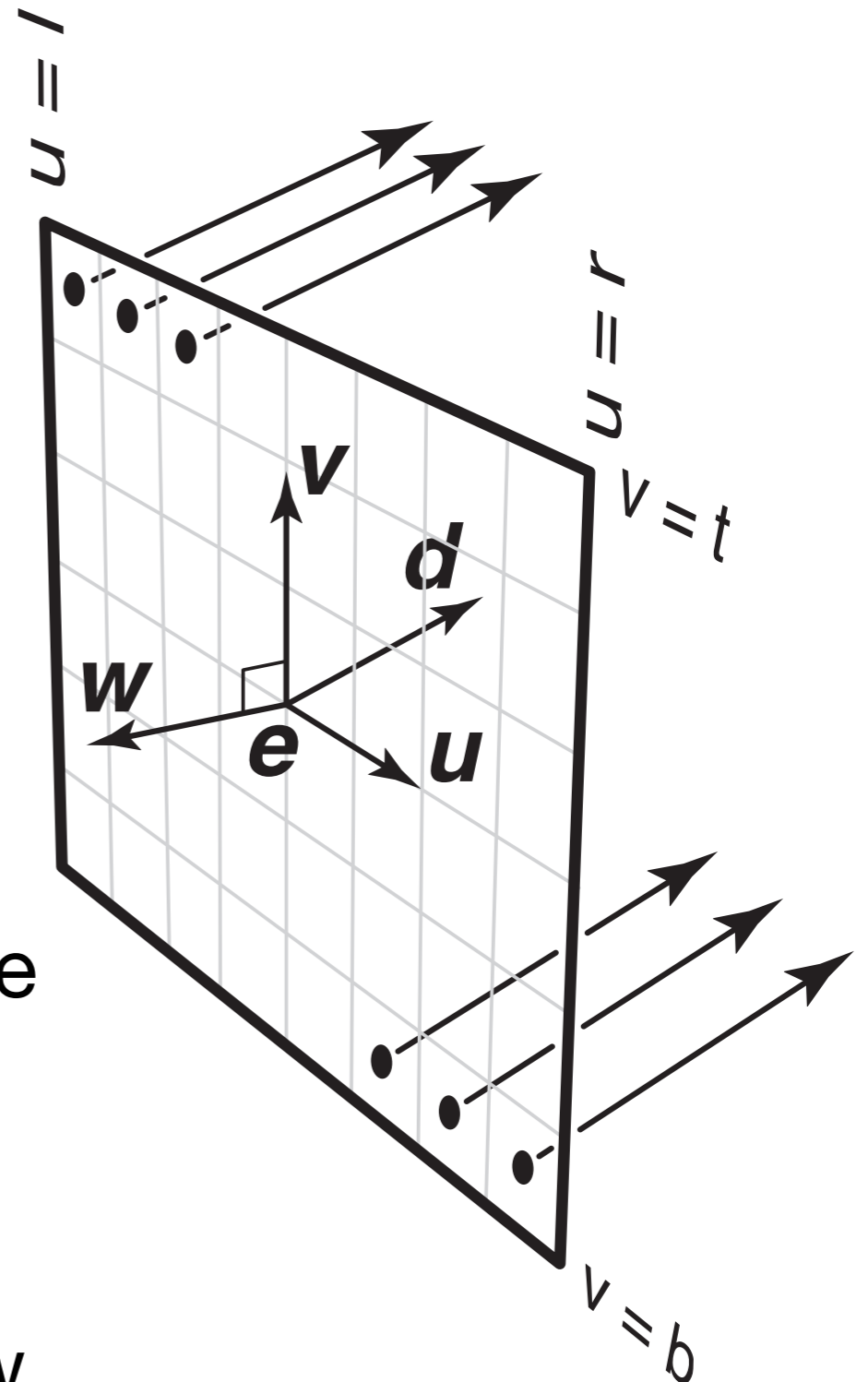


# Funky Parallel Projections

- Parallel viewing rays
- Ray origins from pixels
- Camera origin (eye) is on the image plane

**Oblique parallel:** viewing rays are *not* perpendicular to projection plane.

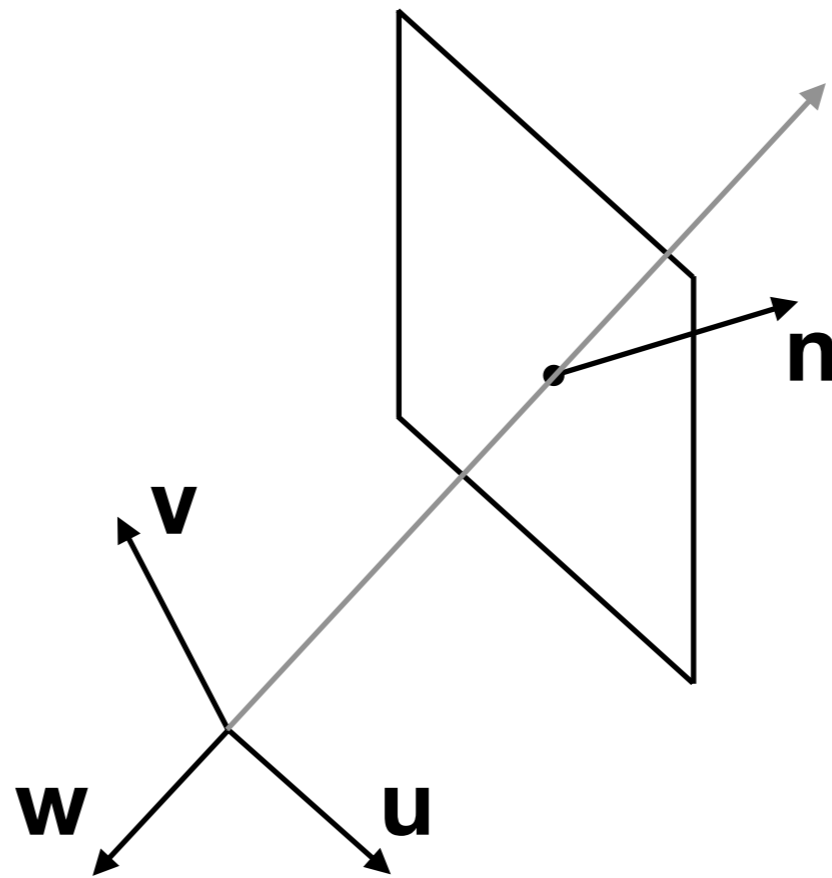
i.e., ray direction  $\mathbf{d}$  differs from  $-\mathbf{w}$





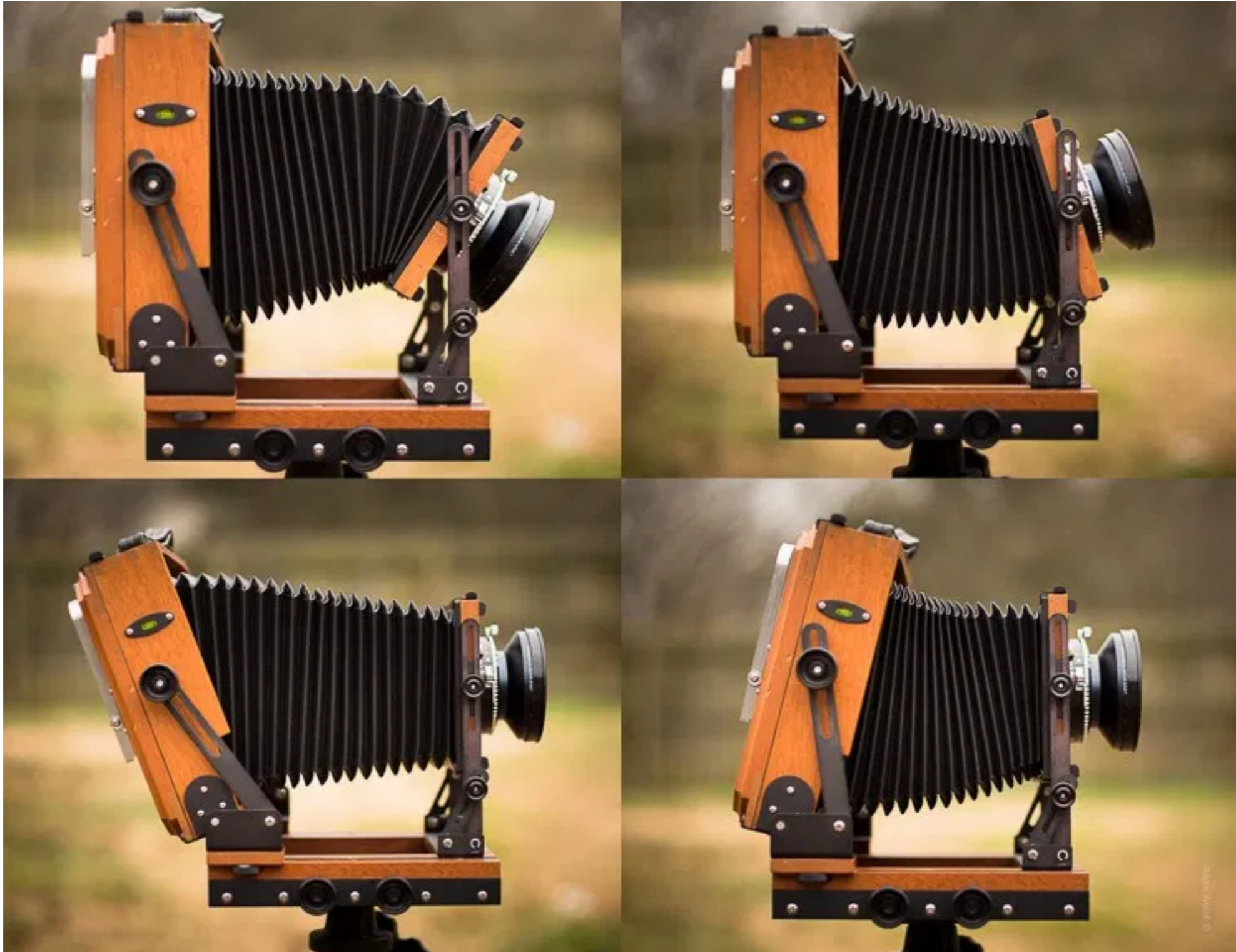
# Funky Perspective Projections

**Shifted perspective:** view direction **not** the same as the projection plane normal



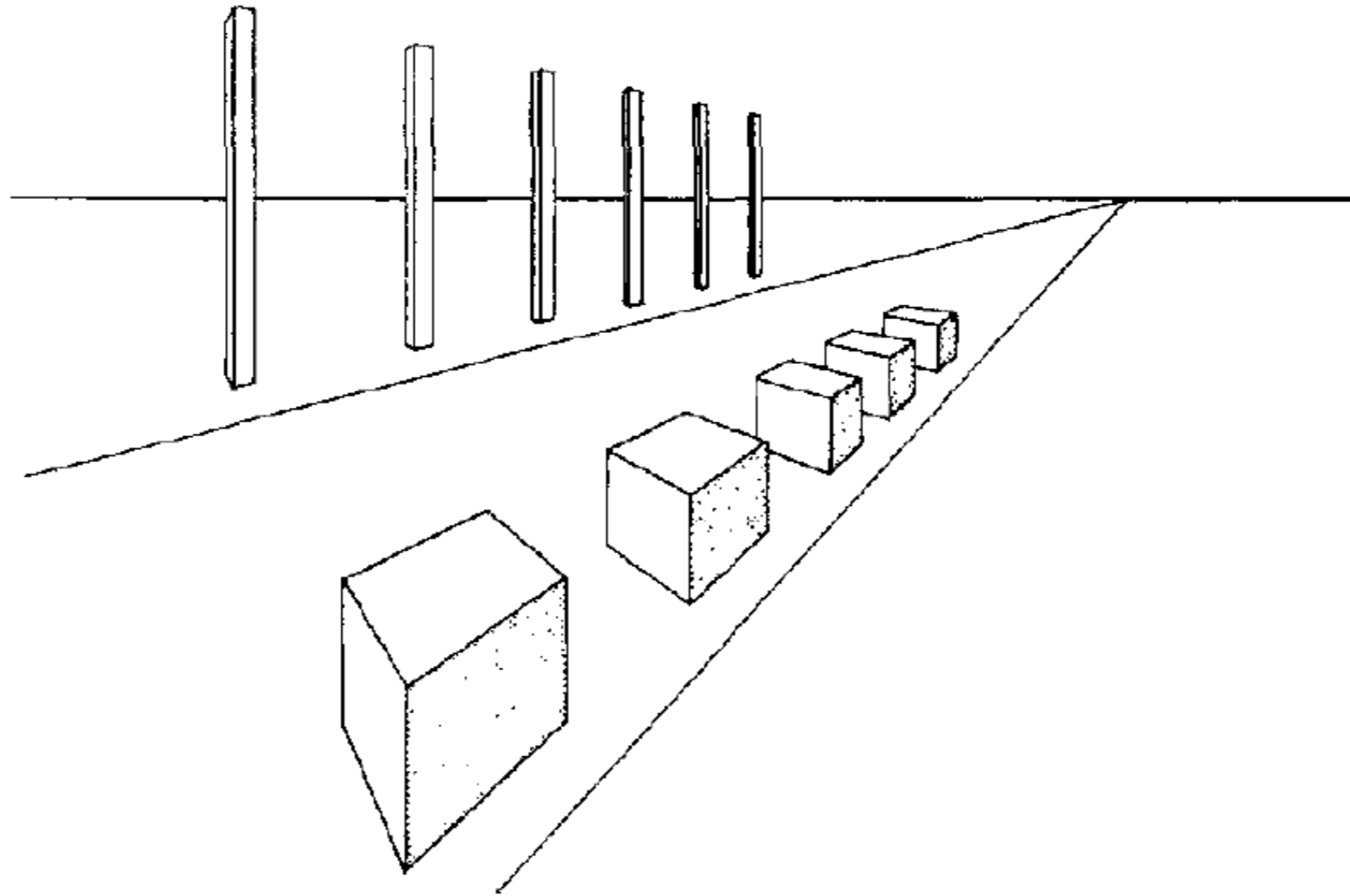
...why do we want this?

# Funky Perspective Projections: IRL



# Perspective distortions

- Lengths, length ratios



"foreshortening": object size is inversely related to depth



camera tilted up: converging vertical lines



lens shifted up: parallel vertical lines



# Problems

- Create a camera basis given an **at** point, i.e., a 3D point that the camera should be looking at, instead of a **view** direction.
- Generate viewing rays for an orthographic projection, given a basis **u, v, w**.
- The "view volume" associated with a projection is the volume of 3D space that projects onto the image plane/viewport. Describe (informally) the **shape** of the view volume for an orthographic camera and a perspective camera.