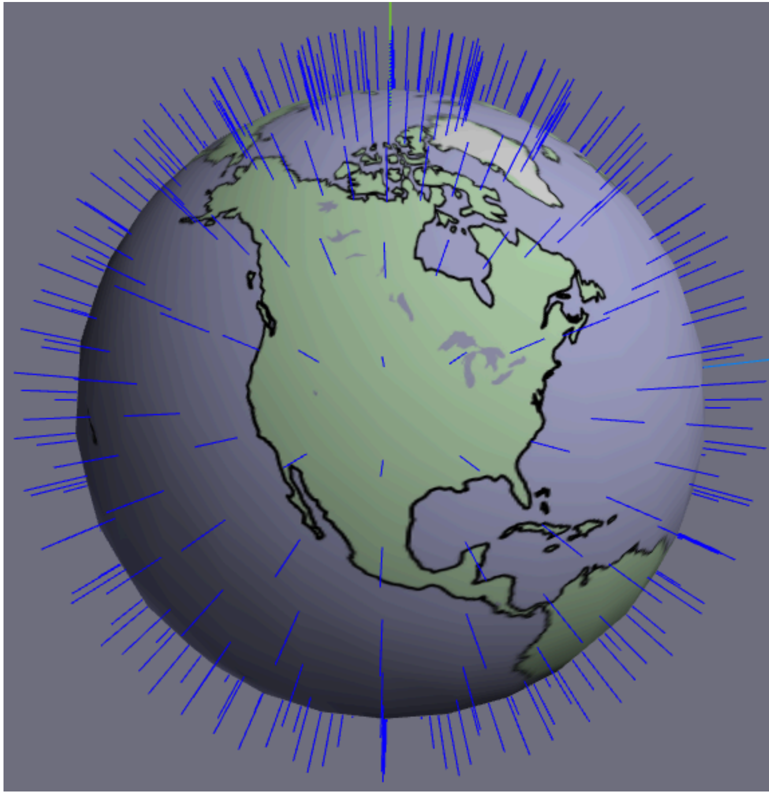
Computer Graphics



Lecture 4

Implicit and Parametric Representations
Triangle Meshes: Texture Coordinates

Big Math Idea:

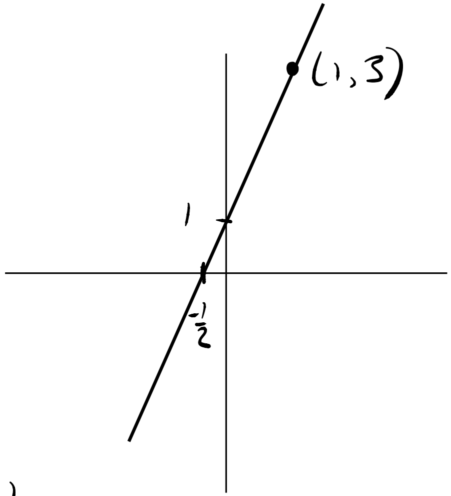
Implicit vs Parametric Representations

Implicit: a **property** that's **true** at all points

Parametric: a **recipe** for **generating** all points

Implicit vs Parametric: Lines

Implicit:



$y = mx+b$

$y = 2x+1$

$3 = 2\cdot 1 + 1$ ✓

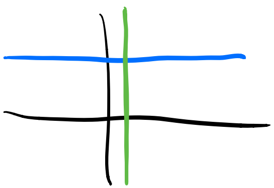
(x,y) is on the line iff $y=mx+b$

known

Alternatively: $ax+by+c=0$

$y=2x+1$

$\underset{a}{-2x}+\underset{b}{y}\underset{c}{-1}=0$



$y=4$

$0x+y-4=0$

$x=1$

$1x+0y-1=0$

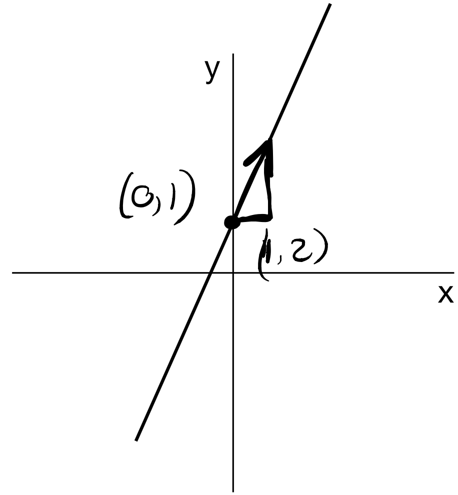
Implicit vs Parametric: Lines

Parametric:

parameter

$$x = 0 + 1 \cdot t$$
$$y = 1 + 2t$$

pick any t, $(x, y)(t)$ lies on the line



Alternatively:

$$\vec{p} + t\vec{d}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Works in 3D too!

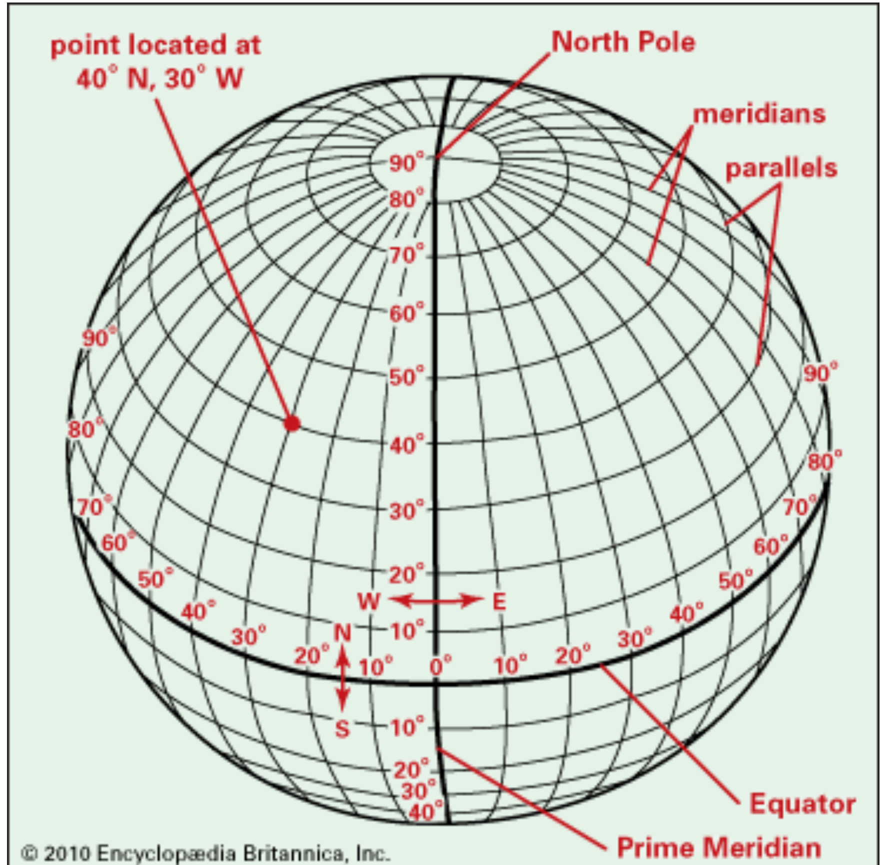
Parametric Surfaces

- Sometimes it's useful to have **2D** coordinates for positions on a **3D** surface.
- This is called *parameterizing* the surface.
- Examples:
  - Cartesian coordinates on a 3D plane
  - Latitude and longitude on Earth's surface
  - Spherical coordinates (θ, ϕ) on a sphere
  - Cylindrical coordinates (θ, y) on a cylinder

Example: Earth

Two coordinates (lat, lon) identify a position in 3D space.

This is possible because the earth is a 2D surface (manifold)



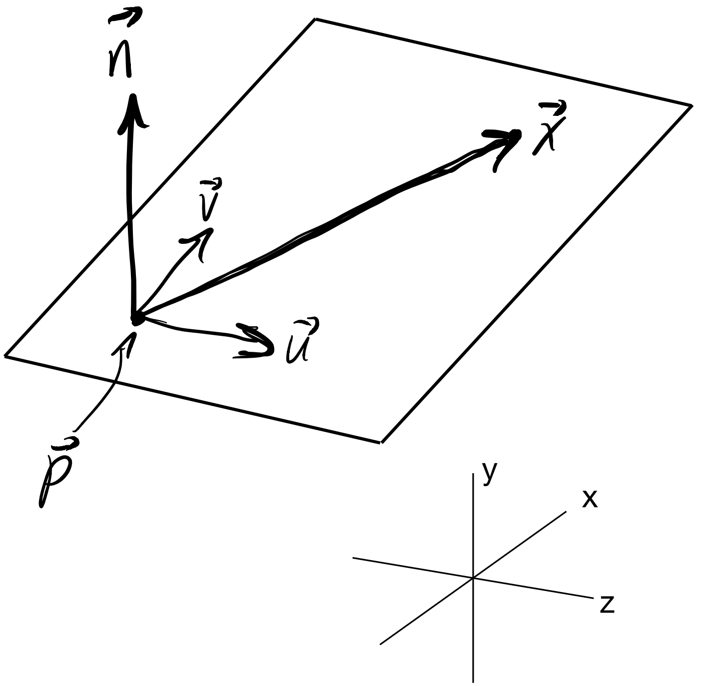
Implicit vs Parametric: Planes

Parametric:

$$\vec{p} + s\,\vec{u} + t\,\vec{v}$$

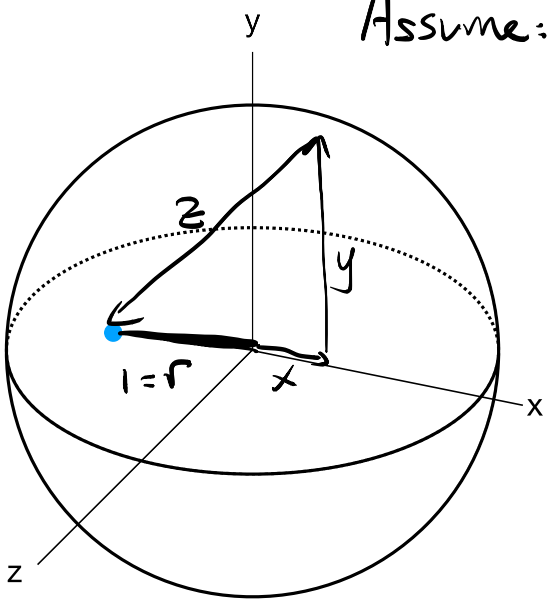
Implicit:

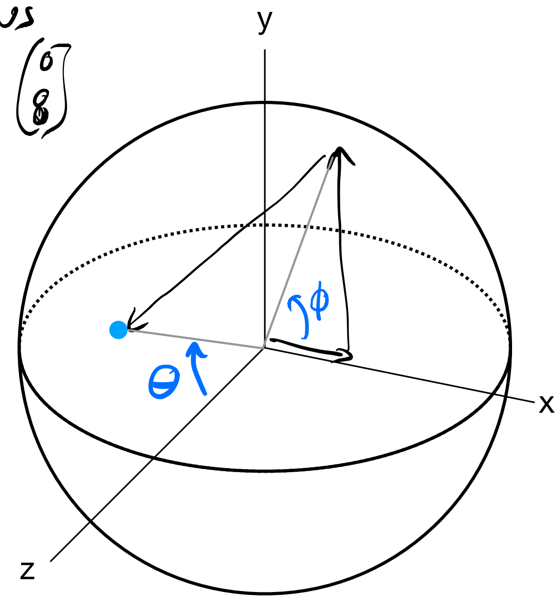
$$\vec{n} \cdot (\vec{x} - \vec{p}) = 0$$



Implicit vs Parametric: Sphere

Assume: unit radius
centered at $\begin{bmatrix}0\\0\\0\end{bmatrix}$





Implicit:

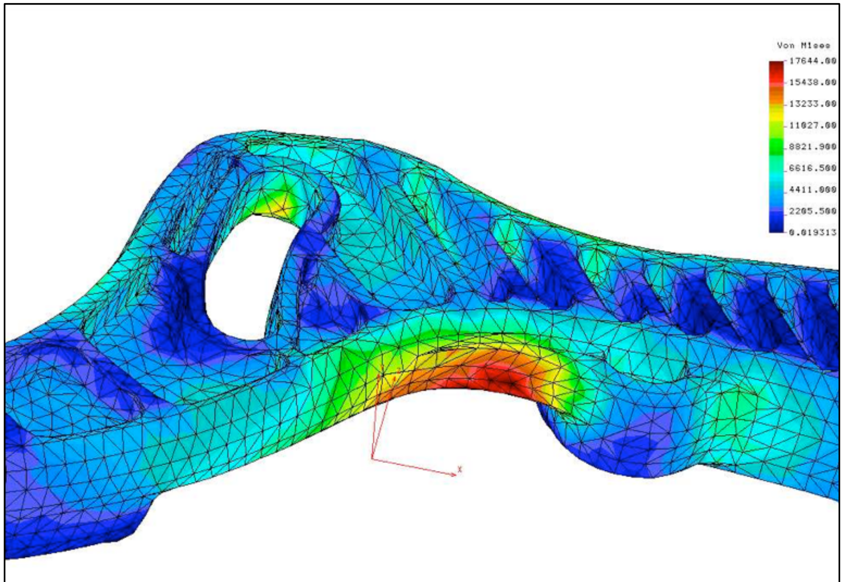
$$\rightarrow x^2 + y^2 + z^2 = 1^2$$

$$x^2 + y^2 + z^2 - 1 = 0$$

$$x = r \cos\phi \sin\theta$$
$$y = r \sin\phi \sin\theta$$
$$z = r \cos\theta$$

Last time: data on Meshes

- Often we need more than just geometry.
- Many properties vary continuously over a smooth surface.



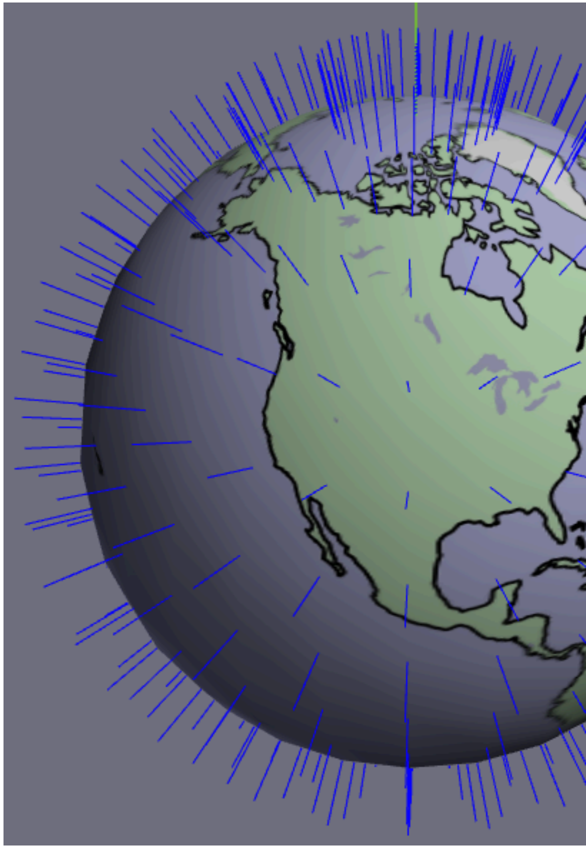
Data on Meshes

- What do we need to store at vertices?

  - **Surface Normals**
    to more accurately portray geometry

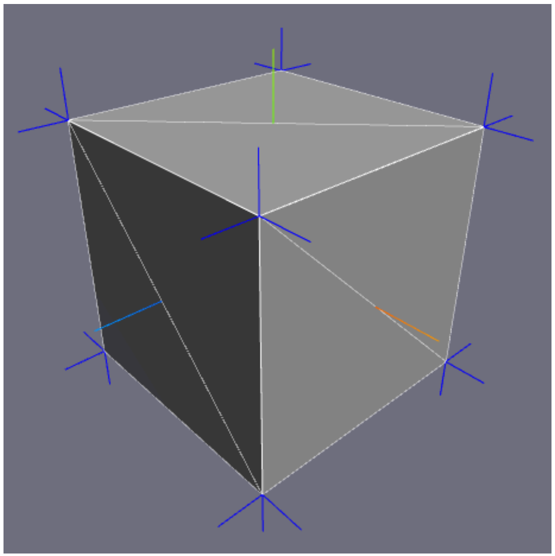
  - **Texture Coordinates**
    to paste image data onto surfaces

  - **Positions!?**
    just another piece of per-vertex data!

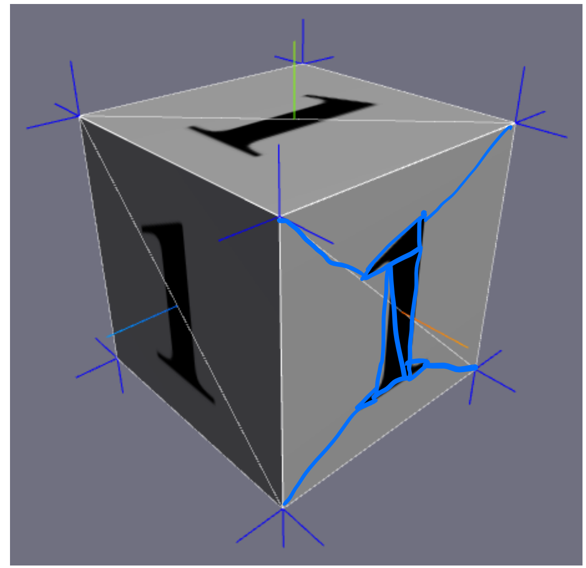


Textures

You are here:



You wish to be here:

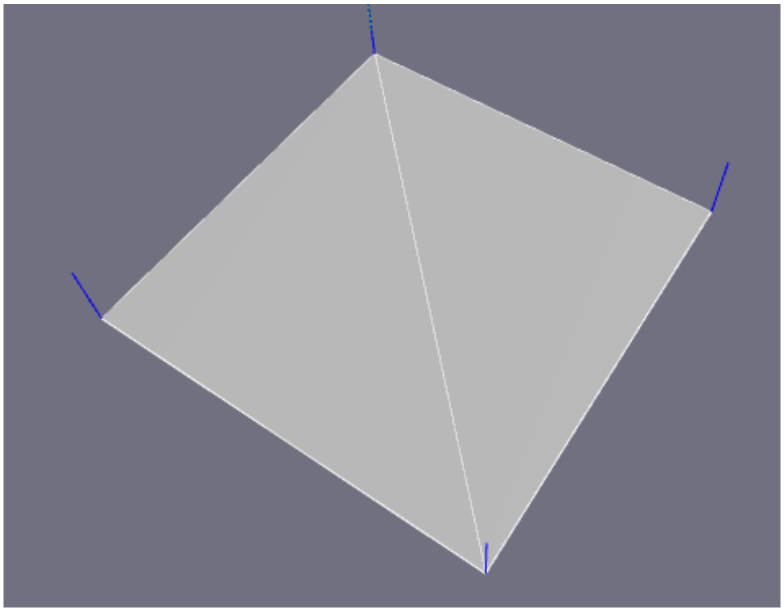


Using current machinery: store a color at each vertex and interpolate between them.

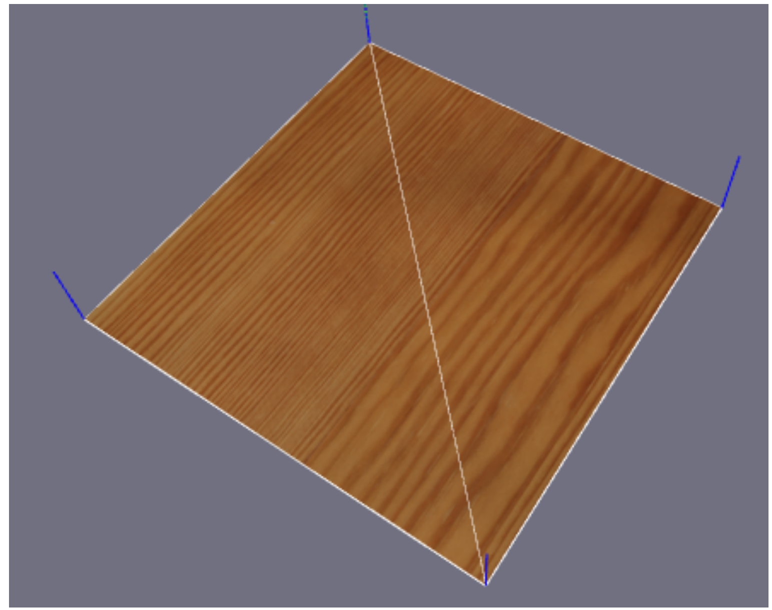
We'd need a bunch more triangles.

Textures

You are here:



You wish to be here:



We'd need a **bunch** more triangles.

Textures

- Store **spatially varying surface properties:**
  - color is an intuitive example, but many other things too;

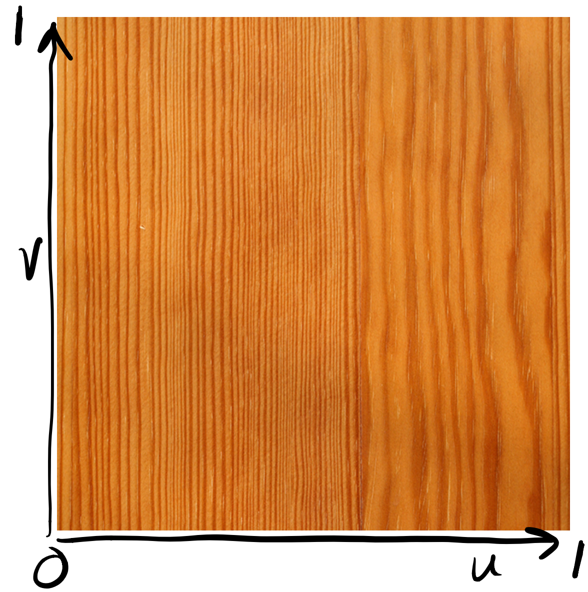
    anything that changes over the surface but doesn't affect geometry (much)
  - roughness, faked lighting effects, normals(!?), bumps

What is a texture?

- A texture is basically a 2D image that stores some **spatially-varying surface property**.

(use color for intuition, but keep in mind it's more general)

2D grid of values ("texels")
u, v coordinates in [0, 1]

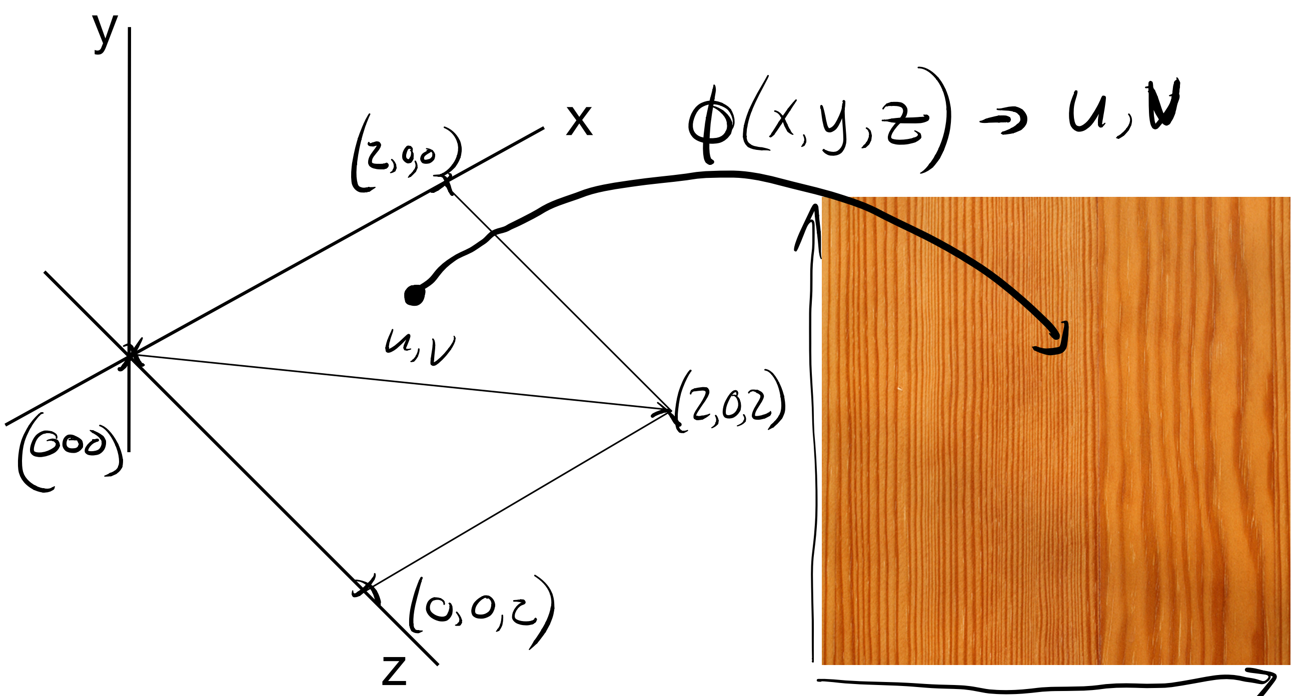


Texture Mapping

- To use this, we need a **mapping** (function)
  - from the surface we're modeling/rendering
  - to (u,v) **texture coordinates**

- **Simplest possible example:**
  a 2x2 tabletop in the xz plane
- When rendering, non-vertex points get colors via **interpolated** (u,v) coordinates.

Texture Mapping Function



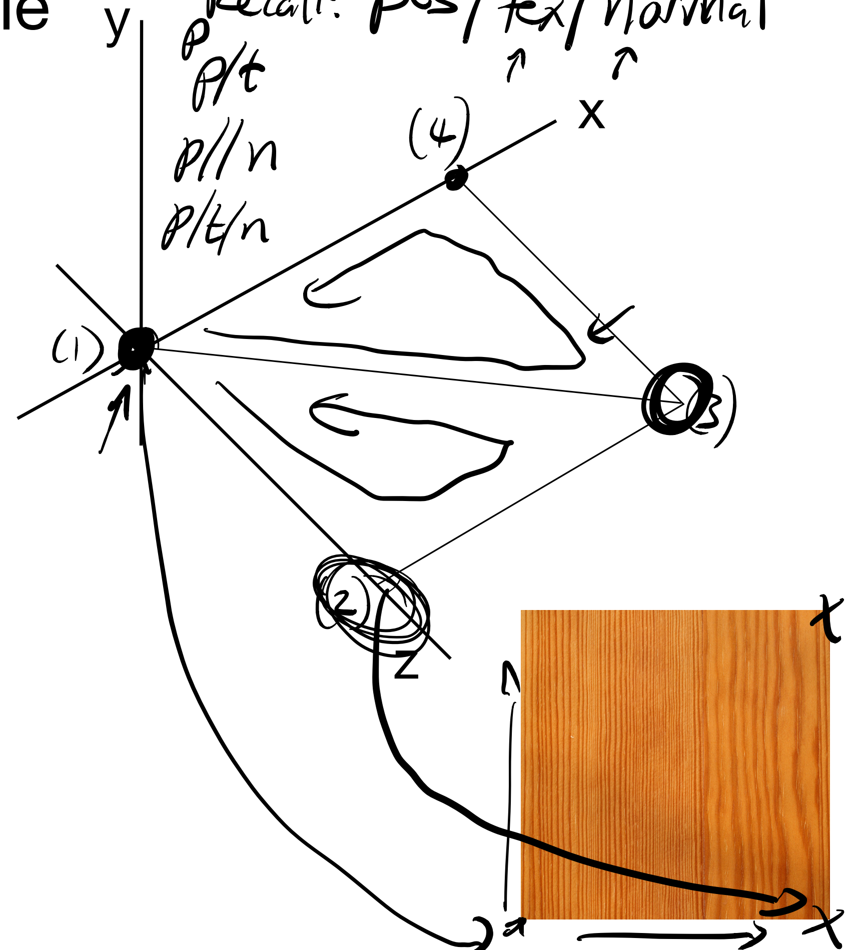
Modeling the Tabletop

Let's write an OBJ file

Recall: pos/tex/normal

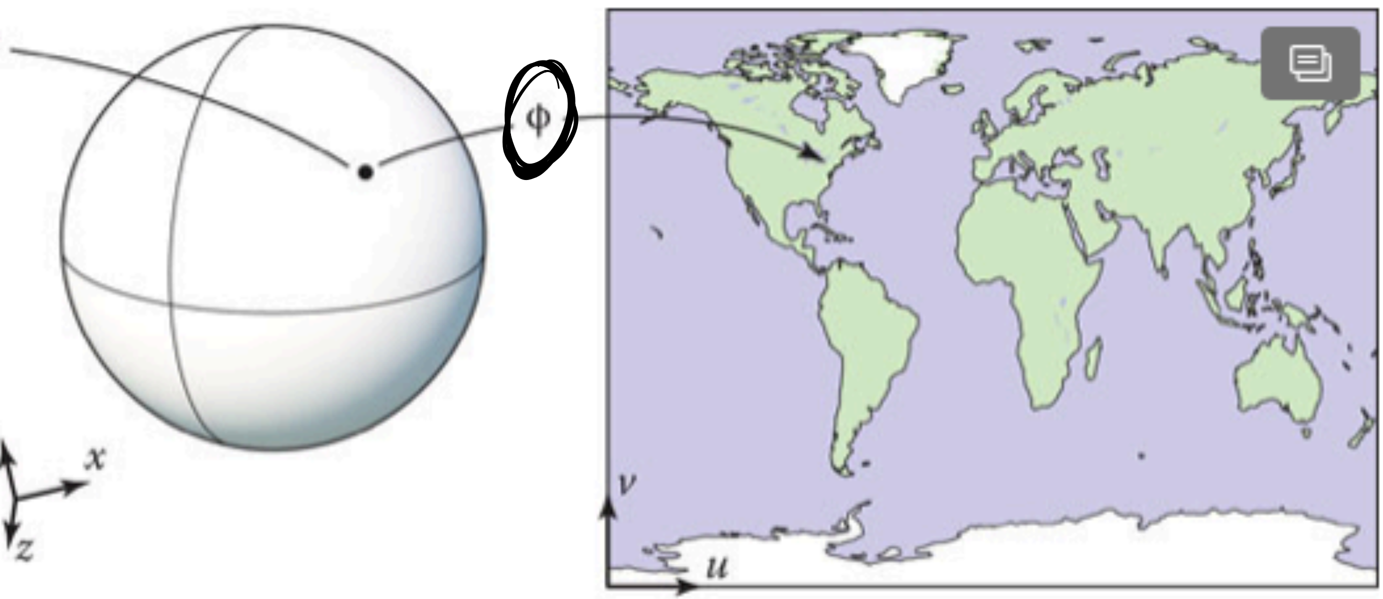
p
p/t
p//n
p/t/n

```
(1) v 0 0 0
(2) v 0 0 2
(3) v 2 0 2
(4) v 2 0 0

t 0 0
t 1 0
t 1 1
t 0 1

f 1/1 2/2 3/3
f 1/1 3/3 4/4
```



Texture Mapping: nontrivial surfaces

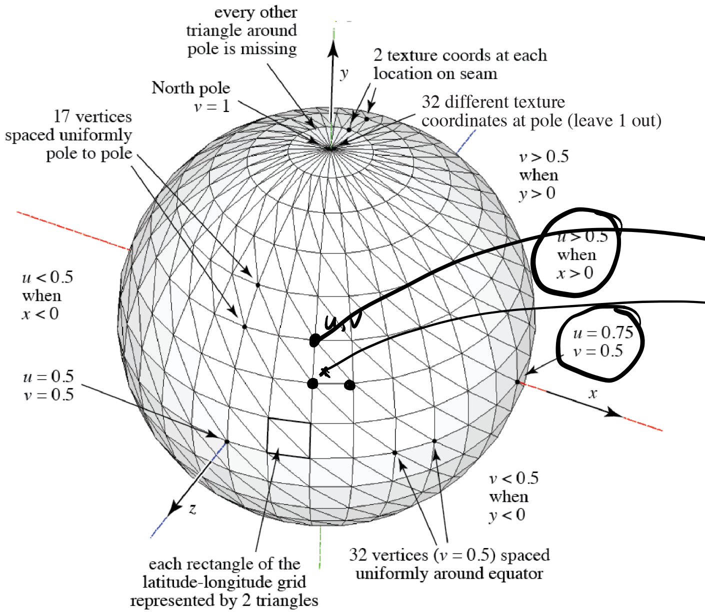
Map from point on sphere to point in (u,v)

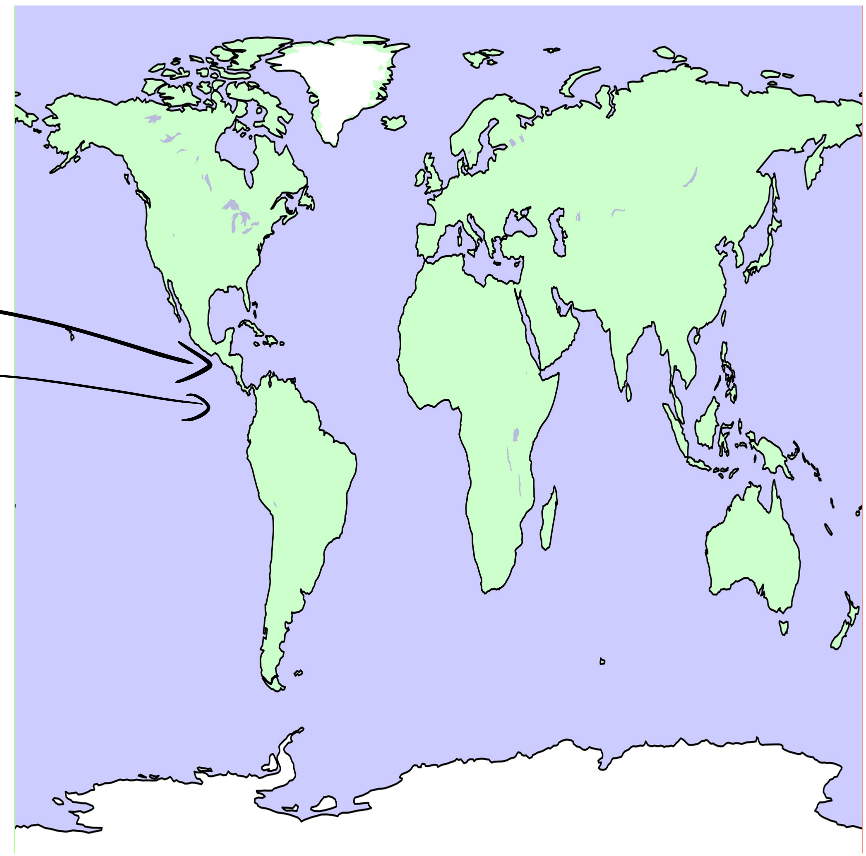


Surface S in world space

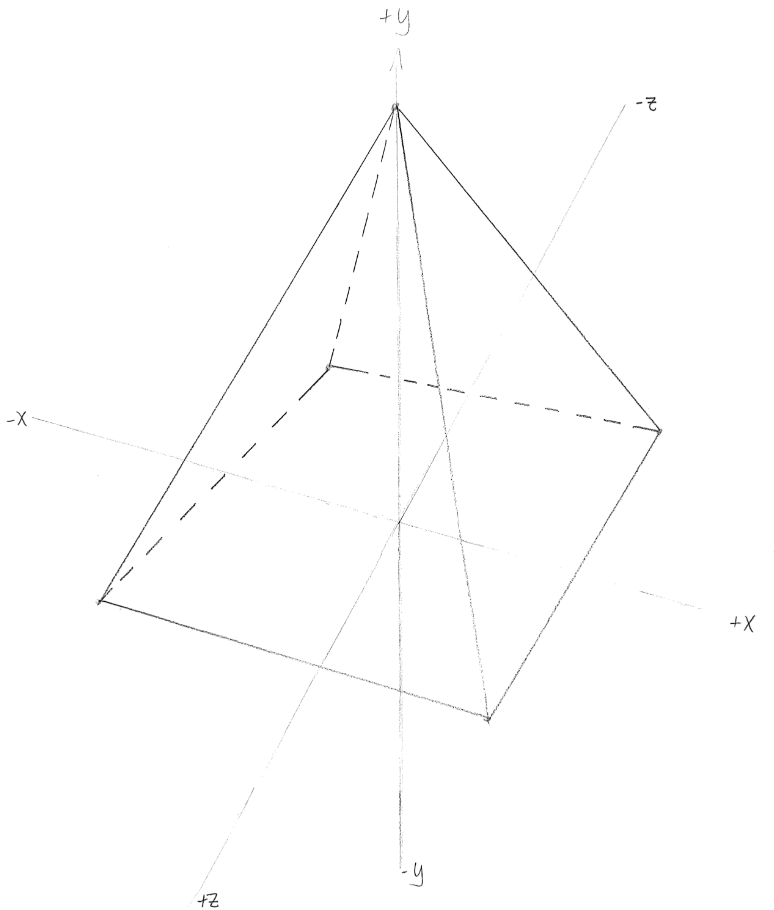
Texture space, T

A1 sphere - demo

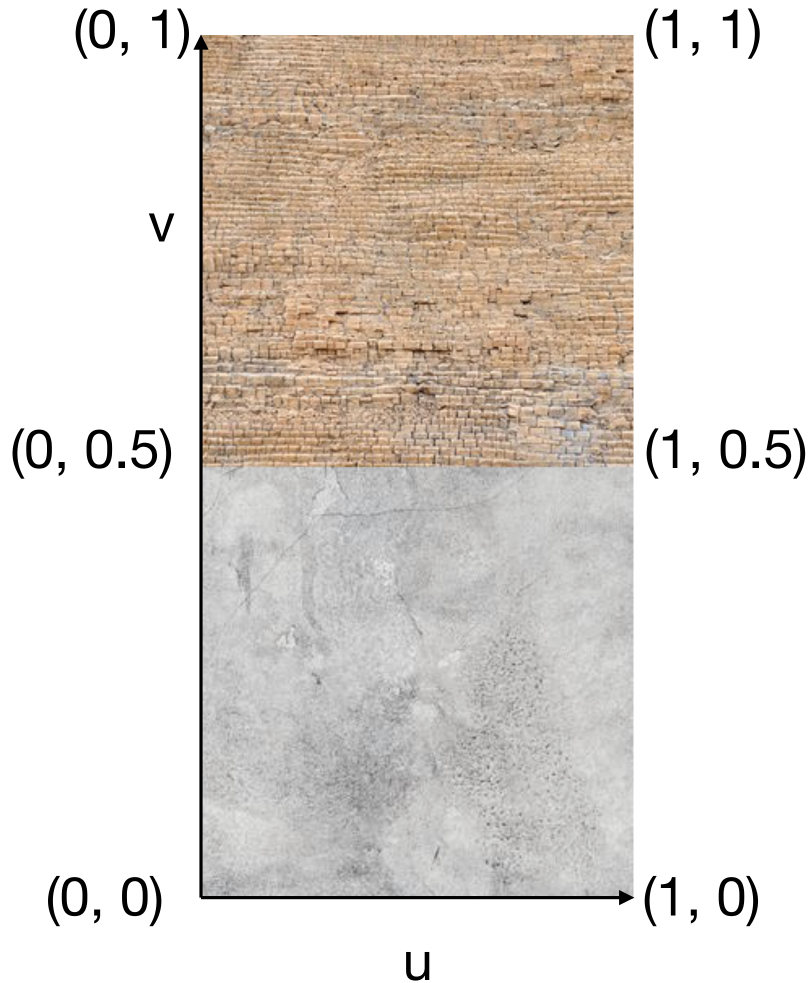




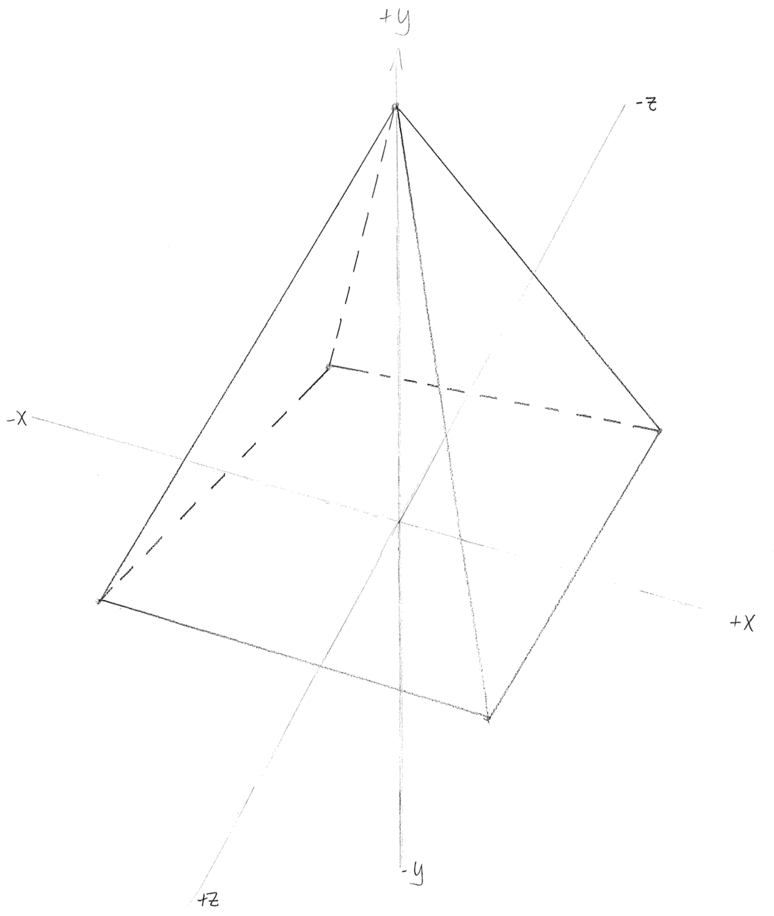
Texturing the Pyramid: The Texture

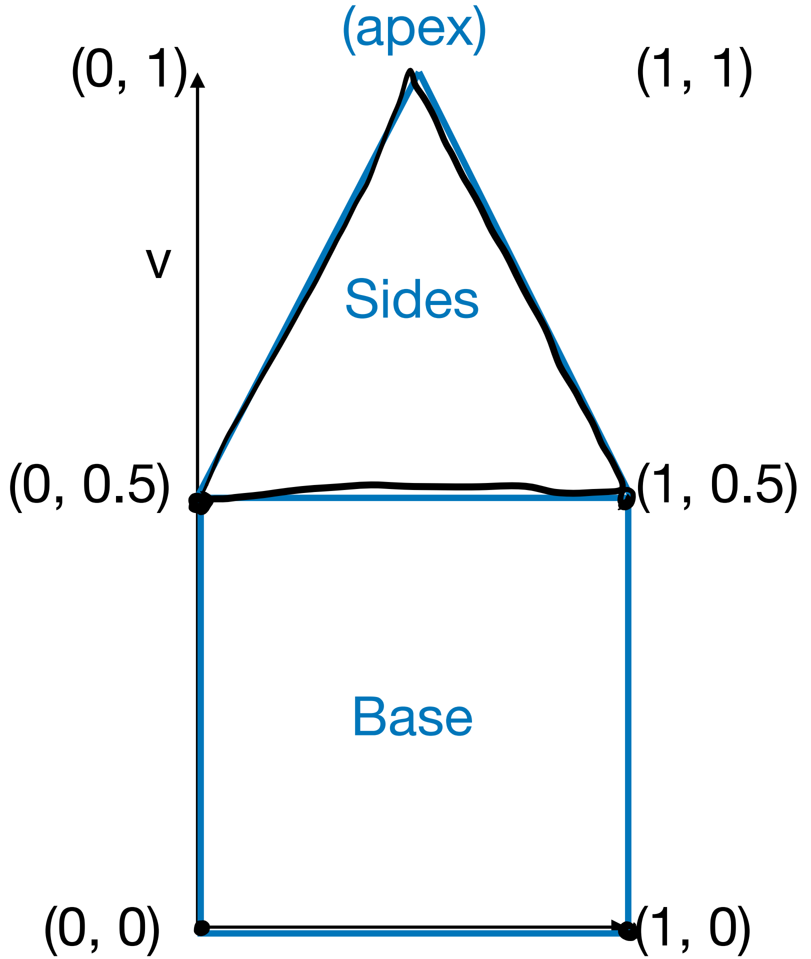


Textures aren't necessarily square - still [0, 1]



Texturing the Pyramid: The Texture Mapping Function





Texturing the Pyramid: The Texture Mapping Function