

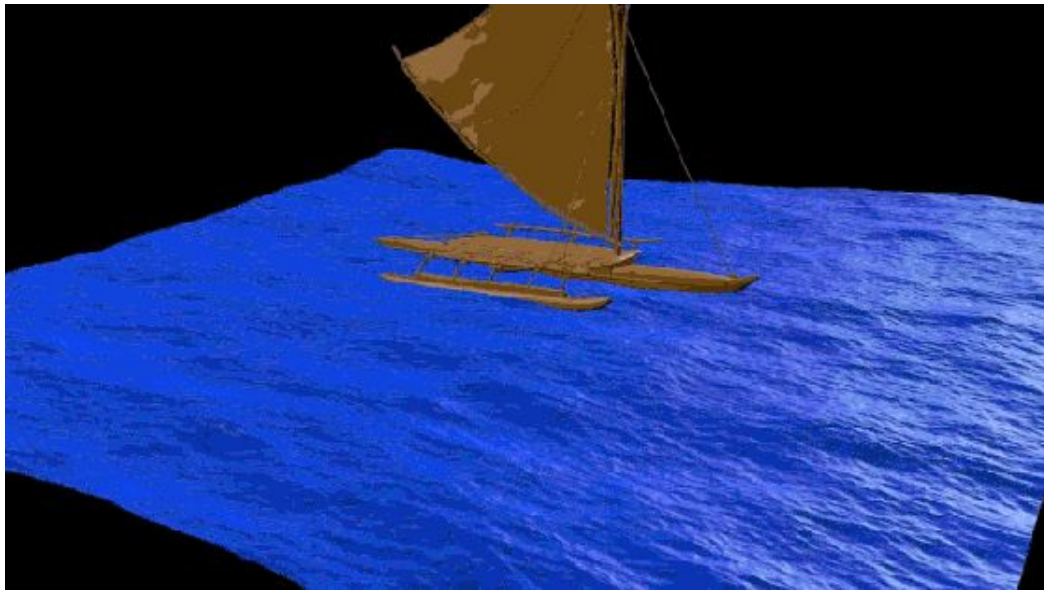
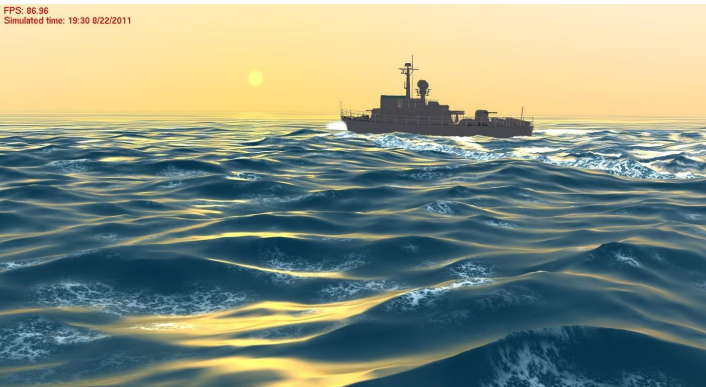


# Ocean Wave Simulation

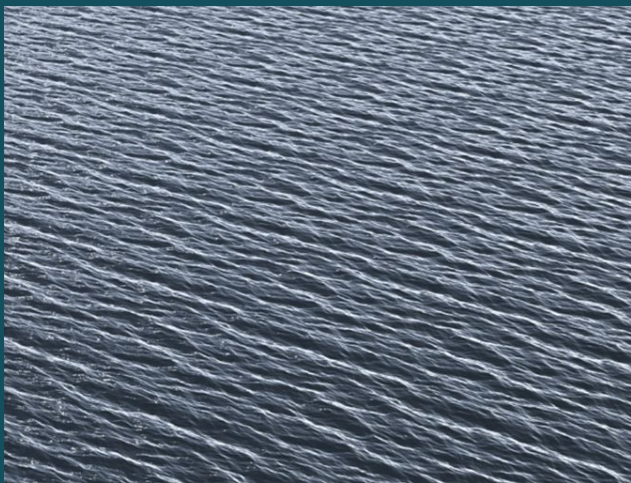
Robin Cosbey and Chloe Dawson



FPS: 86.96  
Simulated time: 19:30 8/22/2011



+

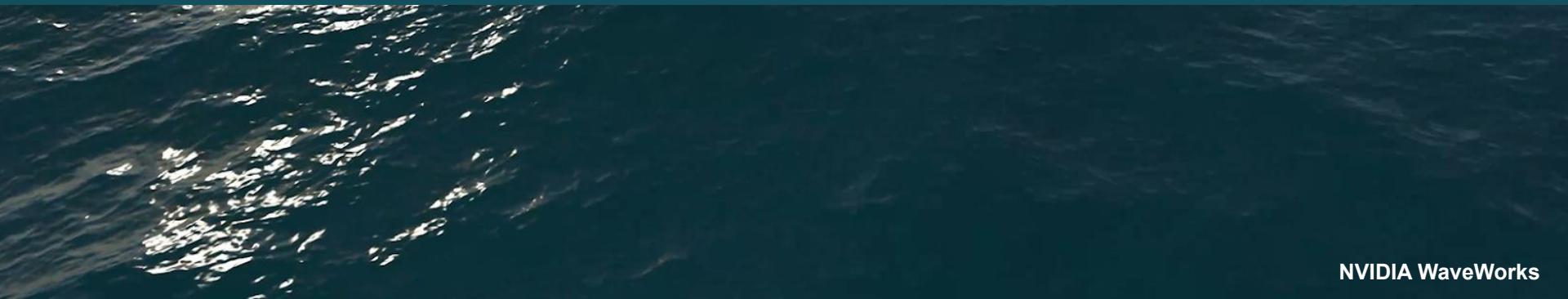


-

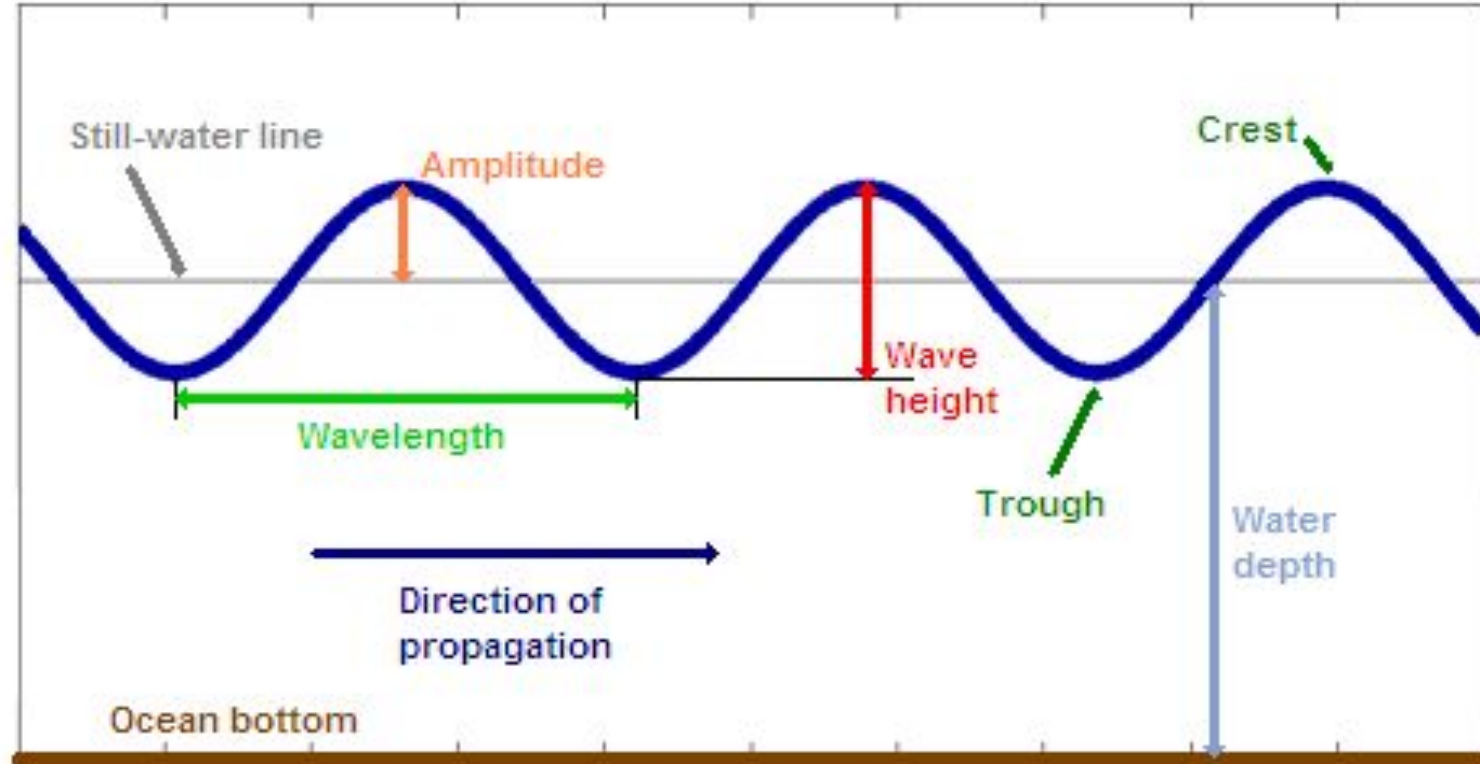




# Wave Properties



# Wave Dynamics



# Wave Dynamics

## Deep-water

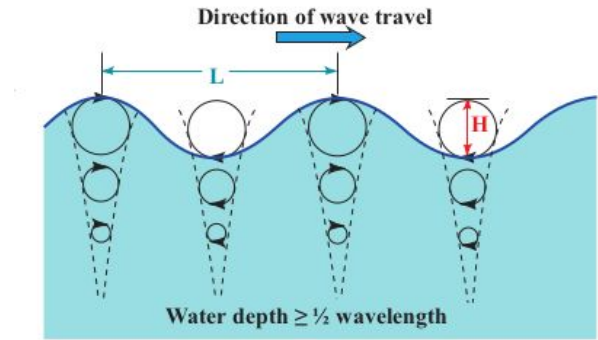
$$\text{Depth} \geq 1/2 L$$

## Intermediate

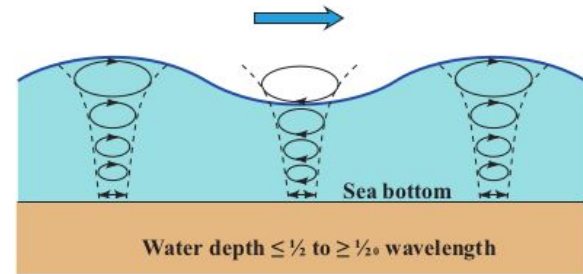
$$1/2 L > \text{Depth} > 1/20 L$$

## Shallow-water

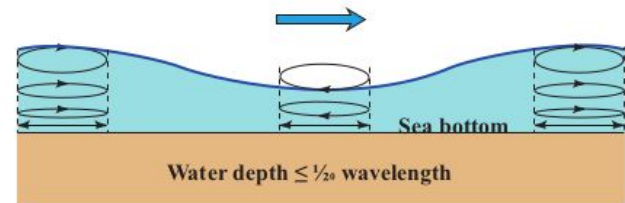
$$\text{Depth} \leq 1/20 L$$



(a) Deep-water wave



(b) Intermediate wave



(c) Shallow-water wave

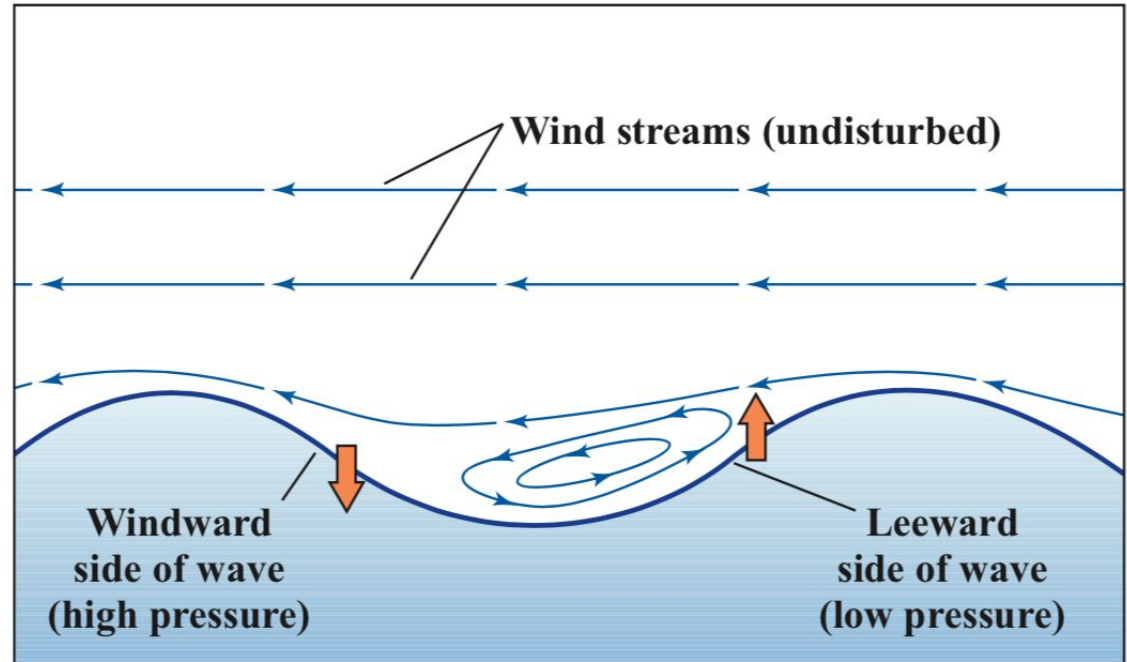
# Wave Generation

**Question: What causes a wave?**

# Wave Generation

**Question: What causes a wave?**

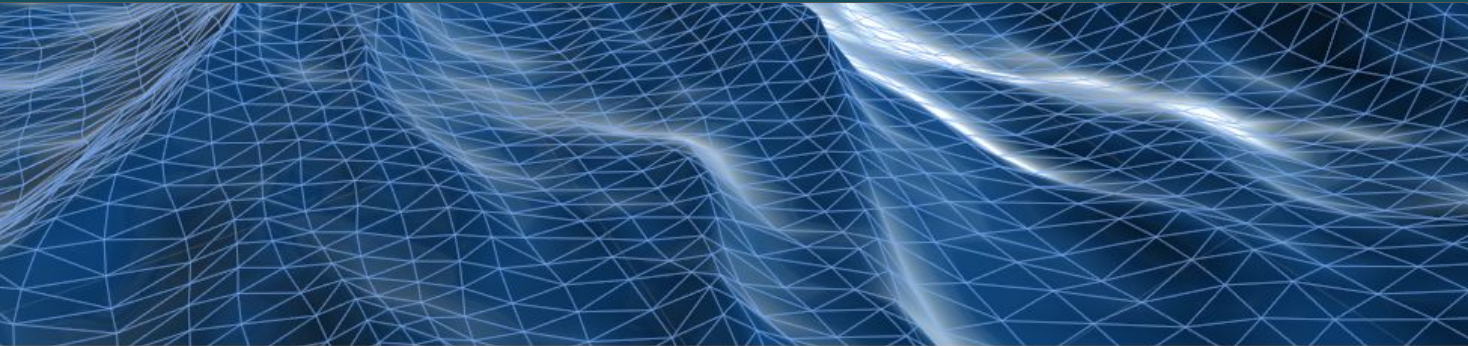
- > Wind
- > Boats
- > Earthquakes
- > Gravitational pull







# Wave Functions



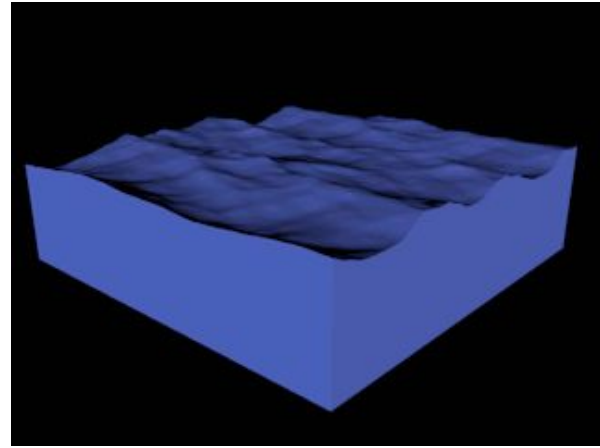
# Models

## Elevation Model

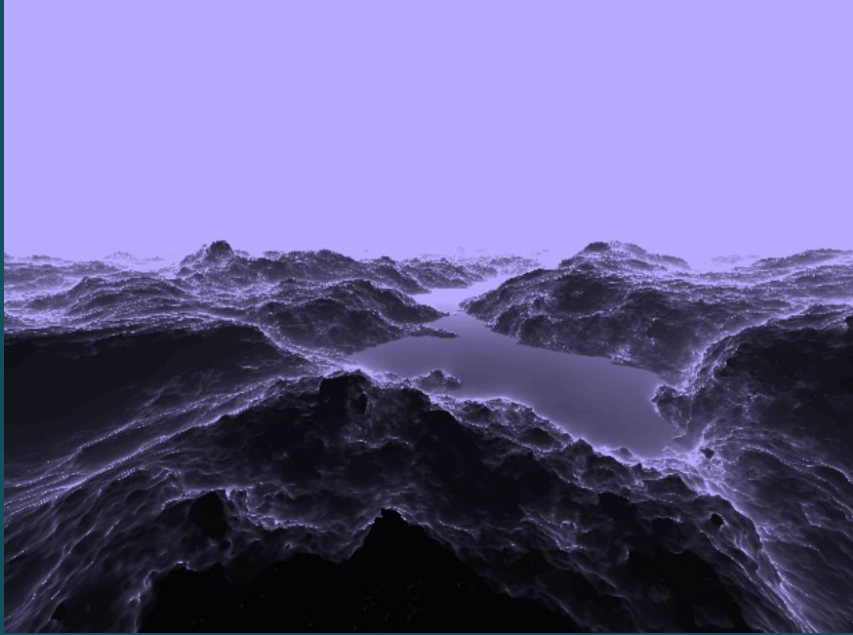
Heightmap - 2D regular grid, each cell represents the surface height

## Volumetric Model

Voxel - 3D regular grid, each cell is assigned a material index



# Simplex Noise



Similar to Perlin noise

Generates a height map which can be applied to other wave functions to provide a more realistic texture

# Navier-Stokes

Multiple equations describing motion of viscous fluids

Often used to capture breaking waves near shore



$$\frac{\delta \mathbf{u}}{\delta t} = \nu \nabla \cdot (\nabla \mathbf{u}) - (\mathbf{u} \cdot \nabla) \mathbf{u} + \mathbf{F}_{body} - \frac{1}{\rho} \nabla p$$

viscous drag      convection      gravity      pressure

mass conservation      velocity      density      pressure

$$\nabla \cdot \mathbf{u} = 0$$

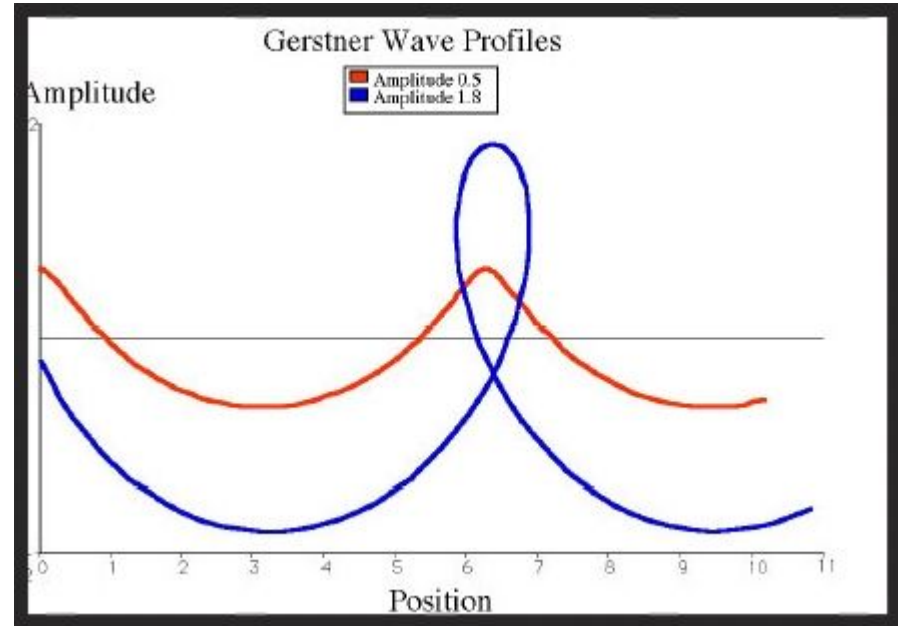
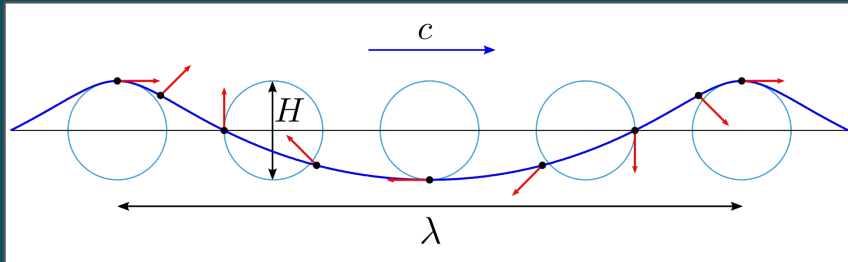
viscosity

Detailed description: The diagram shows the Navier-Stokes equation for fluid motion. The equation is  $\frac{\delta \mathbf{u}}{\delta t} = \nu \nabla \cdot (\nabla \mathbf{u}) - (\mathbf{u} \cdot \nabla) \mathbf{u} + \mathbf{F}_{body} - \frac{1}{\rho} \nabla p$ . Each term is enclosed in a red box. Arrows point from descriptive labels to these terms: 'viscous drag' points to  $\nu \nabla \cdot (\nabla \mathbf{u})$ ; 'convection' points to  $(\mathbf{u} \cdot \nabla) \mathbf{u}$ ; 'gravity' points to  $\mathbf{F}_{body}$ ; 'pressure' points to  $\frac{1}{\rho} \nabla p$ . Below the equation, 'mass conservation' points to the continuity equation  $\nabla \cdot \mathbf{u} = 0$ , which is also in a red box. 'viscosity' points to the  $\nu$  coefficient. 'velocity' points to the  $\mathbf{u}$  in the convection term. 'density' points to the  $\rho$  in the pressure term. 'pressure' points to the  $p$  in the pressure term.

# Gerstner Waves

Modified sine wave with sharper peaks and flatter valleys

Controlled by manipulating amplitude, direction and speed

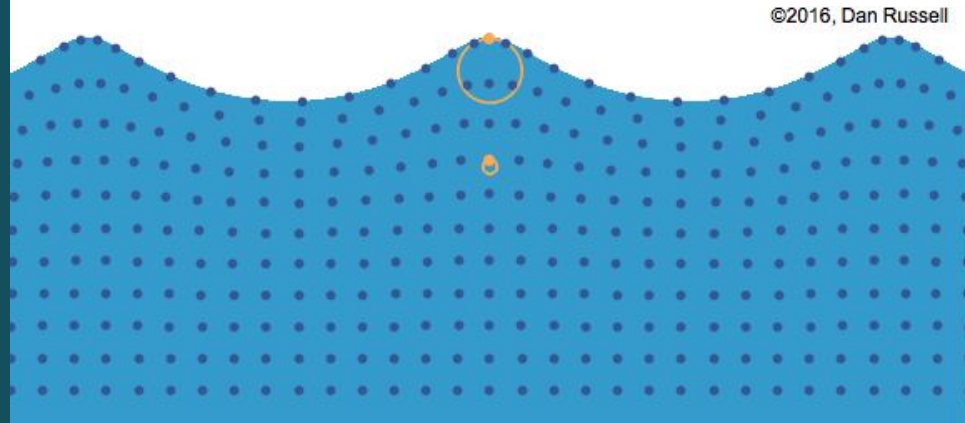
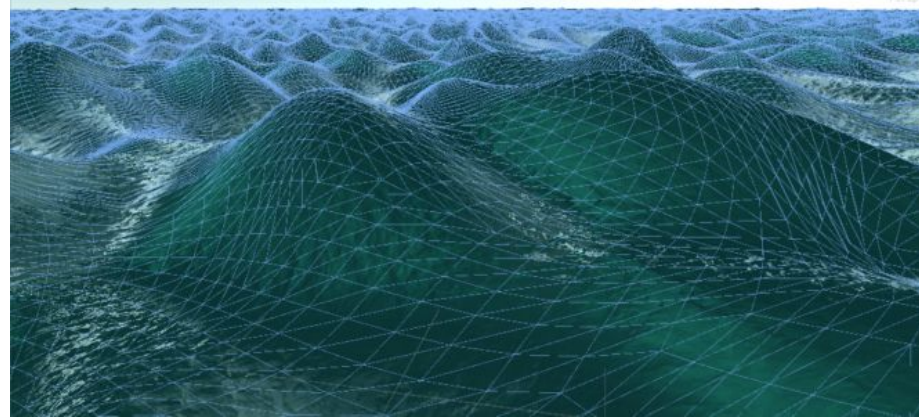


$$P(x, y, t) = \begin{pmatrix} x + \sum (Q_i A_i \times D_i \cdot x \times \cos(w_i D_i \cdot (x, y) + \varphi_i t)), \\ y + \sum (Q_i A_i \times D_i \cdot y \times \cos(w_i D_i \cdot (x, y) + \varphi_i t)), \\ \sum (A_i \sin(w_i D_i \cdot (x, y) + \varphi_i t)) \end{pmatrix}$$

# Gerstner Waves

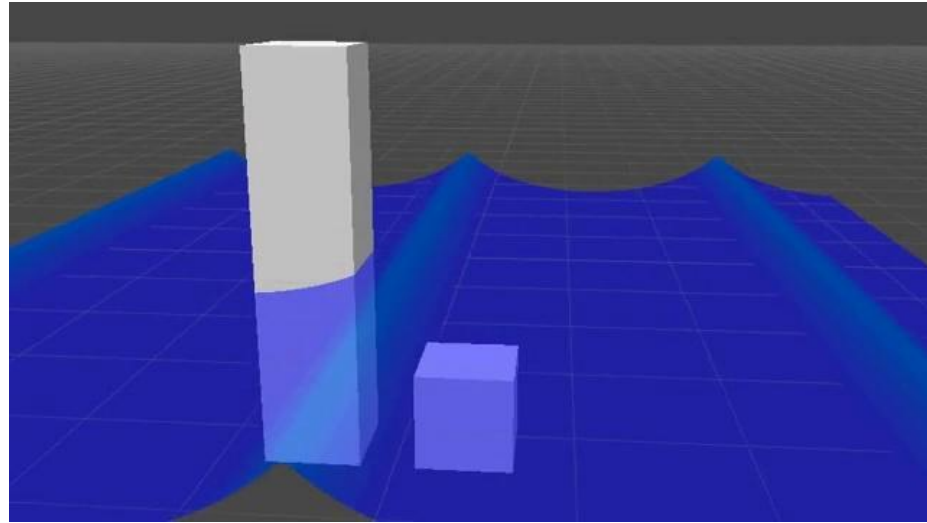
Particles of water are described with circular/elliptical stationary orbits

Moving vertices toward each crest forms sharper crests



# Gerstner Waves

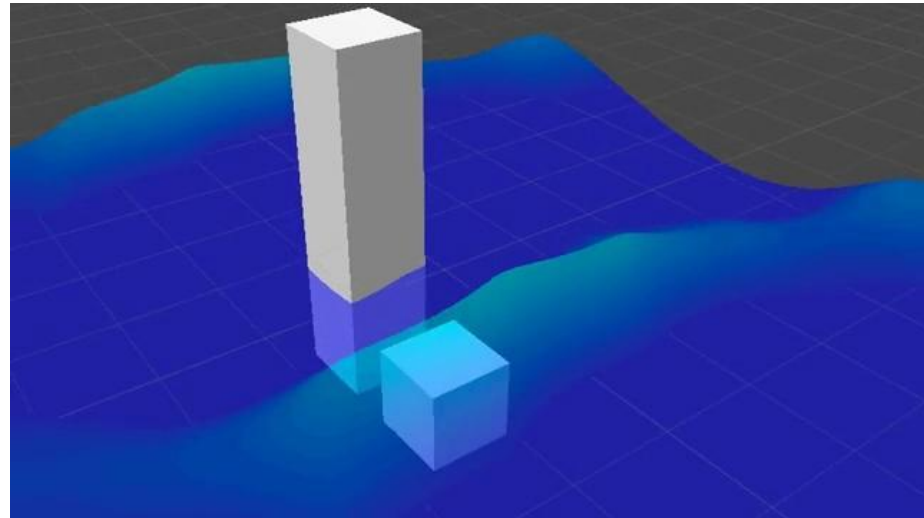
Question: How can we make this more realistic?



# Gerstner Waves

Question: How can we make this more realistic?

> Add waves with varying properties



4 waves!



# Flags and Waves - SIGGRAPH 1986



“A Simple Model of Ocean Waves”

Based on Gerstner Waves

A parametric surface made up of position at rest ( $x_0$ ,  $y_0$ ) and time

Incorporates effects of wave direction, depth and shore distance

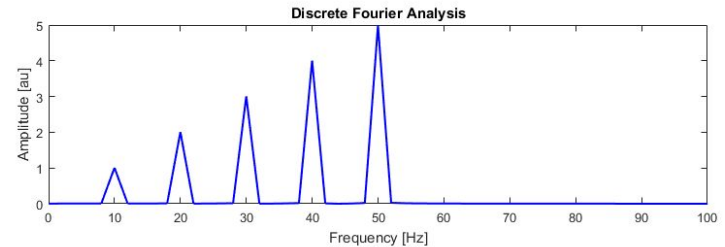
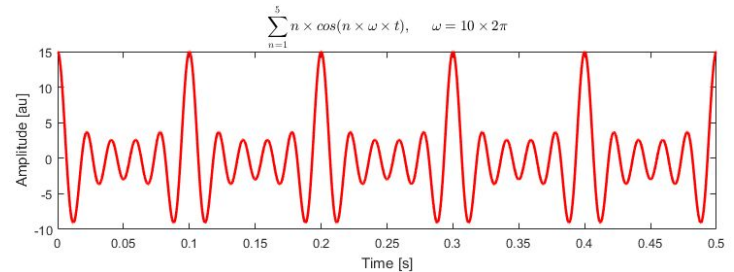


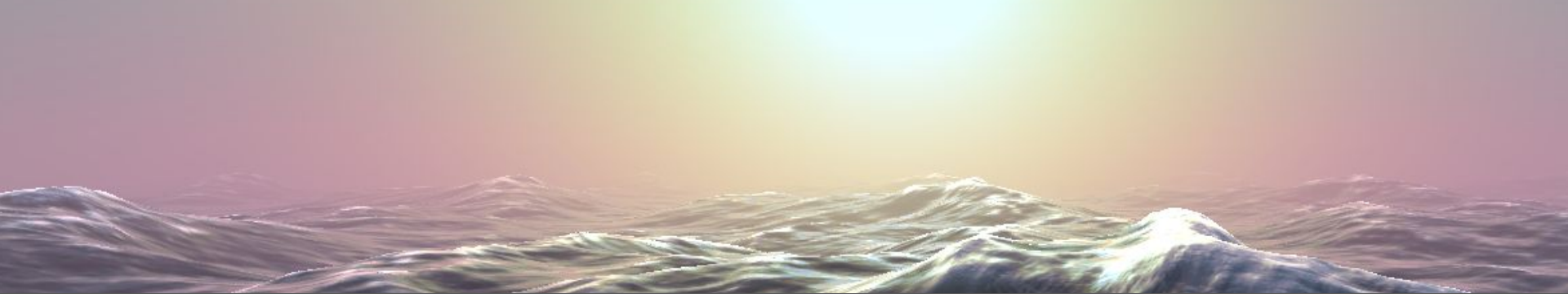
# Fast Fourier Transforms



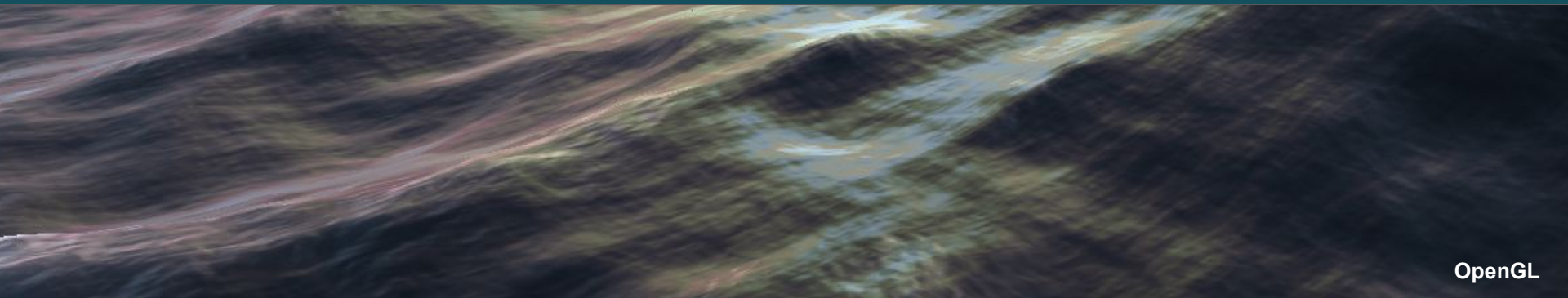
Statistical model where wave height is considered a random variable of horizontal position and time,  $h(x, t)$

Reasonable representation of naturally occurring wind-waves





# Ocean Phenomena



# Foam and Spray

## 1. Semi-transparent textures

Foam spawns near tops of big waves

For a given **vertex**:

Compute **difference** in slope  
between vertex and neighbors

If **difference** < **threshold**:

    Increase foam amount

Otherwise

    Decrease foam amount



**Figure 3-13.** Water spray generated when two waves of opposite direction meets.

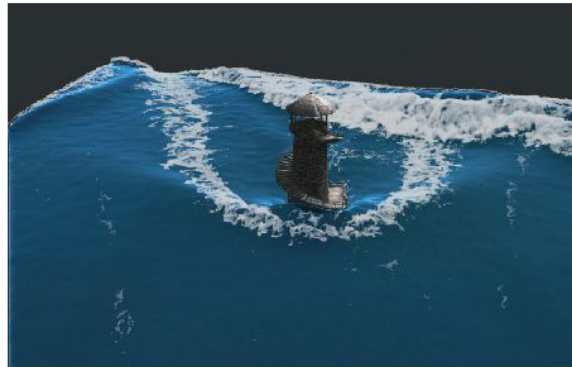
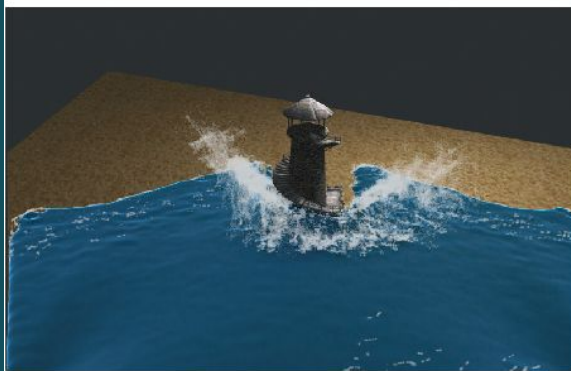
# Foam and Spray

## 2. Particle systems

More realistic but more costly

Spray subsystems are generated based on velocity of water particles and wave amplitude

Result is rendered as cloud points



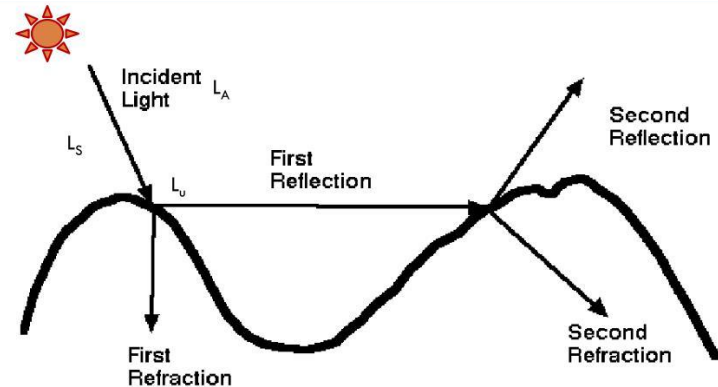
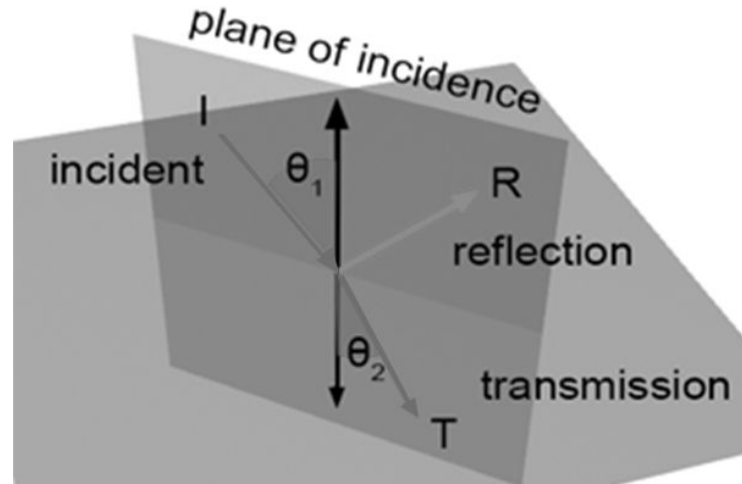
# Radiosity

$$L_{\text{above}} = \text{reflection}(L_{\text{sun}} + L_{\text{sky}}) + \text{transmission}(L_{\text{under}})$$

**Specular reflection** from direct sunlight and skylight

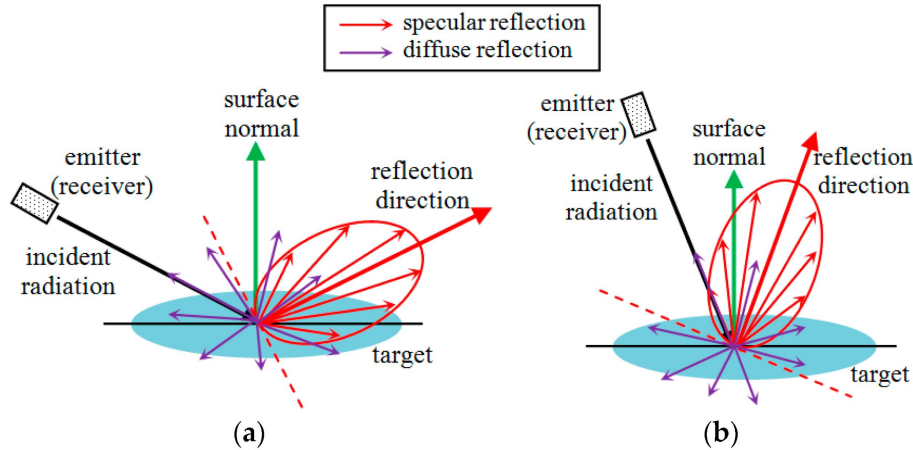
**Transmission** of incident light through surface

(More reflection / refraction from previously reflected / refracted light)



# Specular Reflection

For a ray intersecting the surface, the direction of the reflected ray depends only on the **incident direction** and the **surface normal**

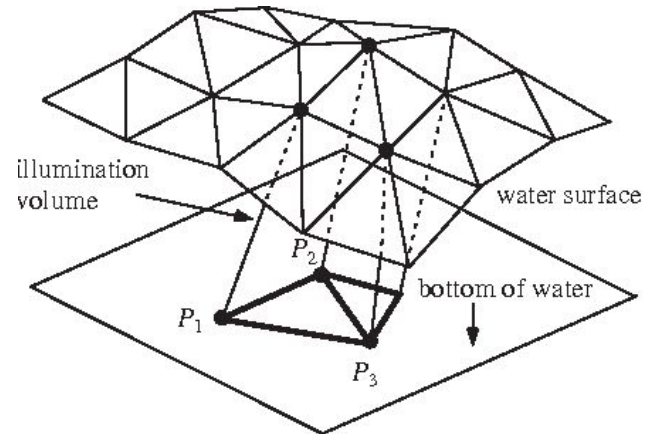
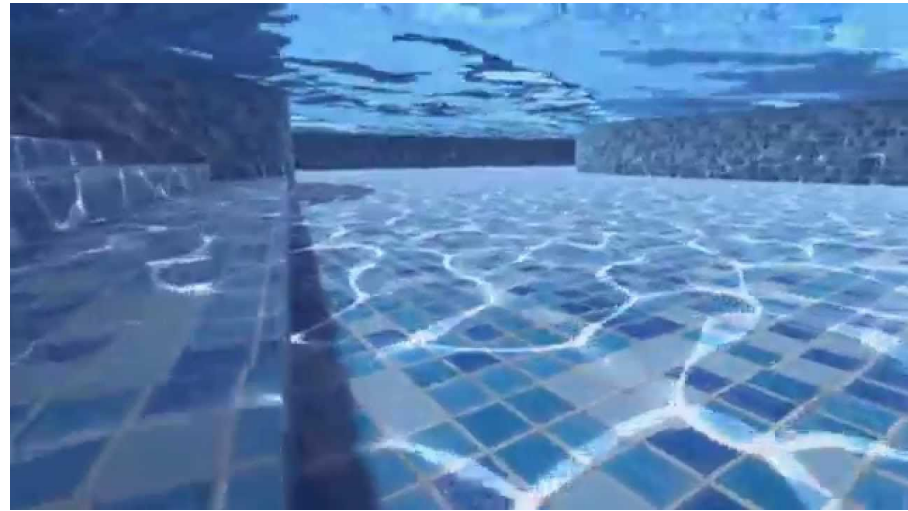


# Caustics

Snell's Law determines how refracted light will bend the trajectory

As light advances deeper into the water the intensity attenuates

Eventually light strikes the ocean floor





# Color

Based on reflected and refracted light

## Opacity of water

Function of depth

Viewing angle

## Tint of reflection

Environment mapping



An aerial photograph of the ocean with a dark teal banner across the middle. The banner contains the text "Thank you! Questions?". The ocean below shows various shades of blue and green, with white foam from a ship's wake visible in the lower right.

**Thank you! Questions?**