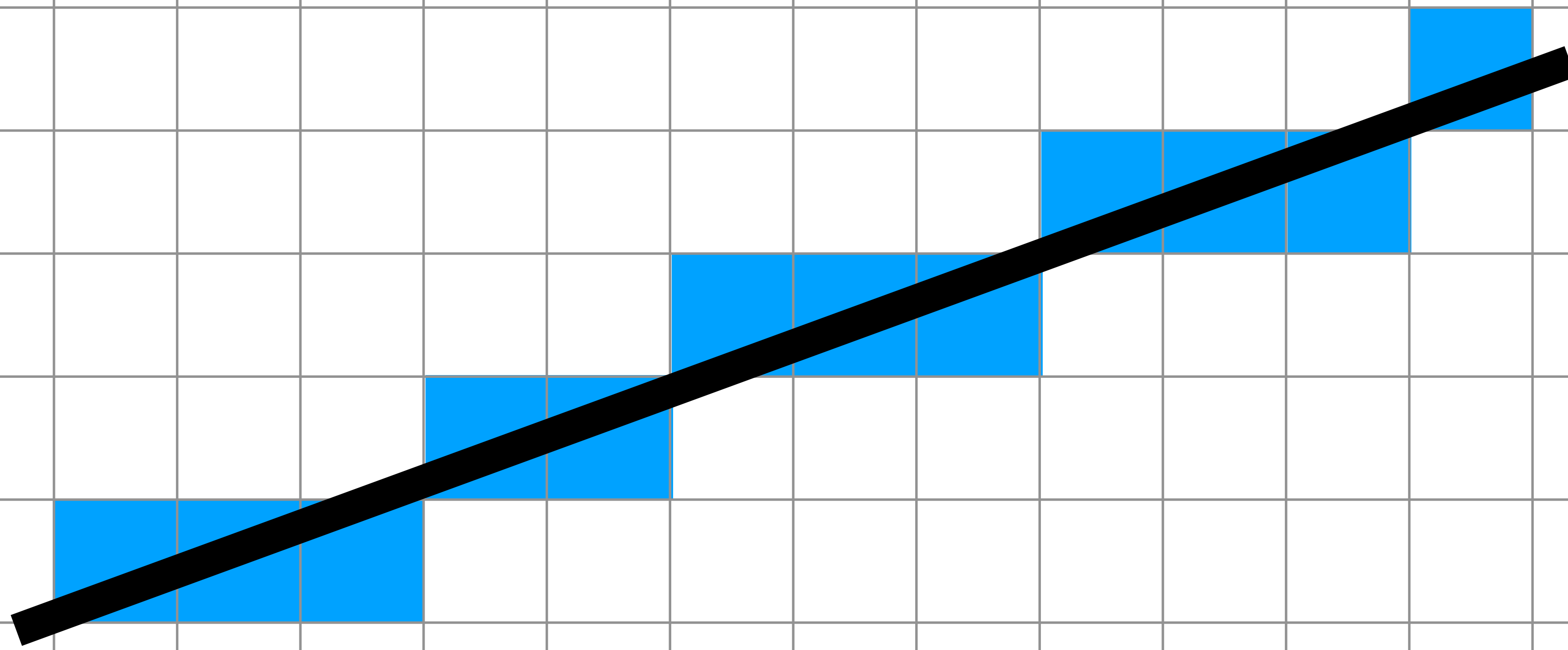


Computer Graphics

Lecture 23

Rasterization:
Lines and Triangles

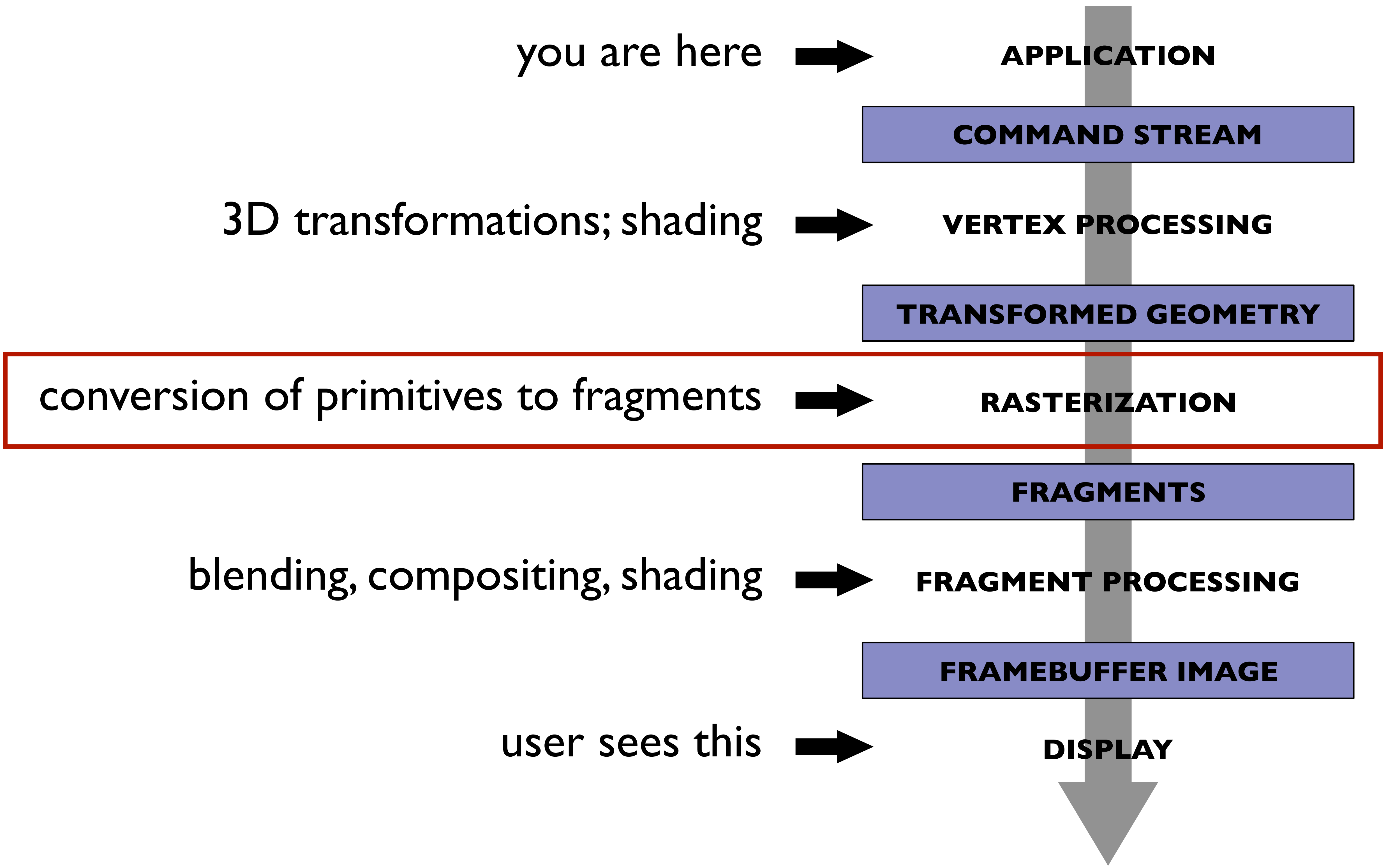
Announcements

- Midterm due tonight.
  One typo was fixed midday Saturday.
  A couple clarifications are on Piazza.

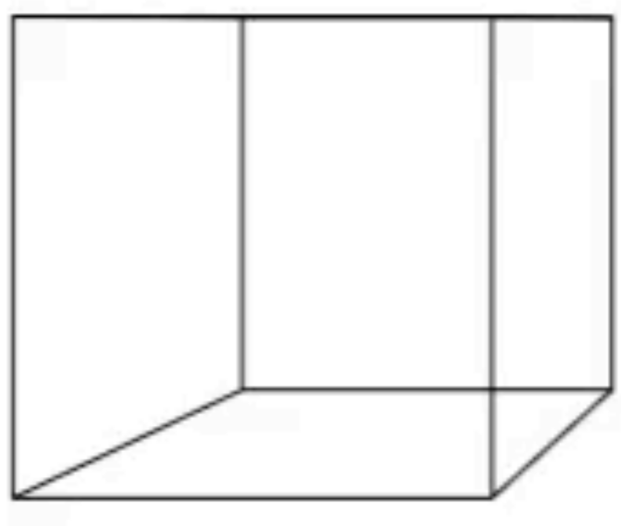
- Final project feedback out by tonight.

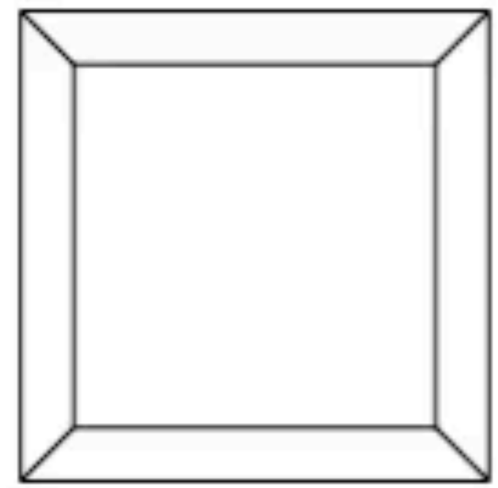
- Grad lectures next week - planning to schedule tonight.

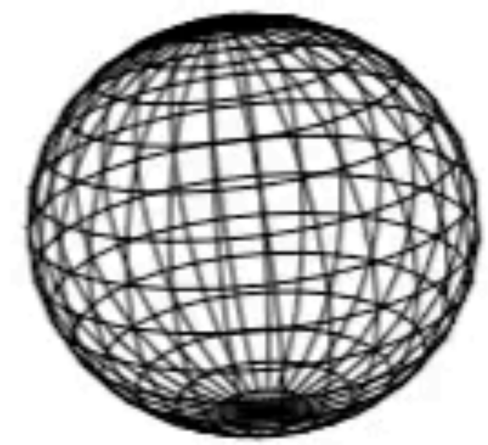
Graphics Pipeline: Overview



Recall: Wireframe Rendering







$M = M_{vp}\ M_{proj}\ M_{view}\ M_{model}$

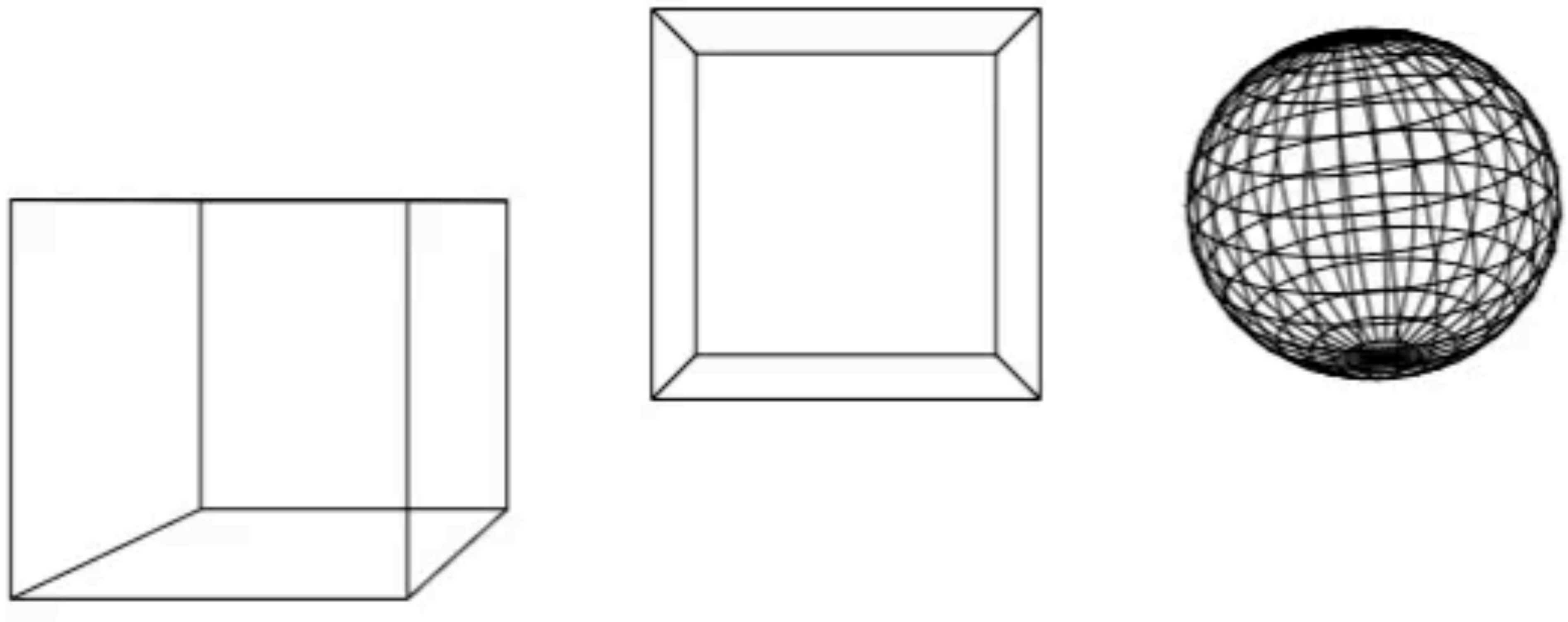
for each line segment $\mathbf{a}_i$, $\mathbf{b}_i$

    $\mathbf{p} = \mathbf{M}\ \mathbf{a}_i$

    $\mathbf{q} = \mathbf{M}\ \mathbf{b}_i$

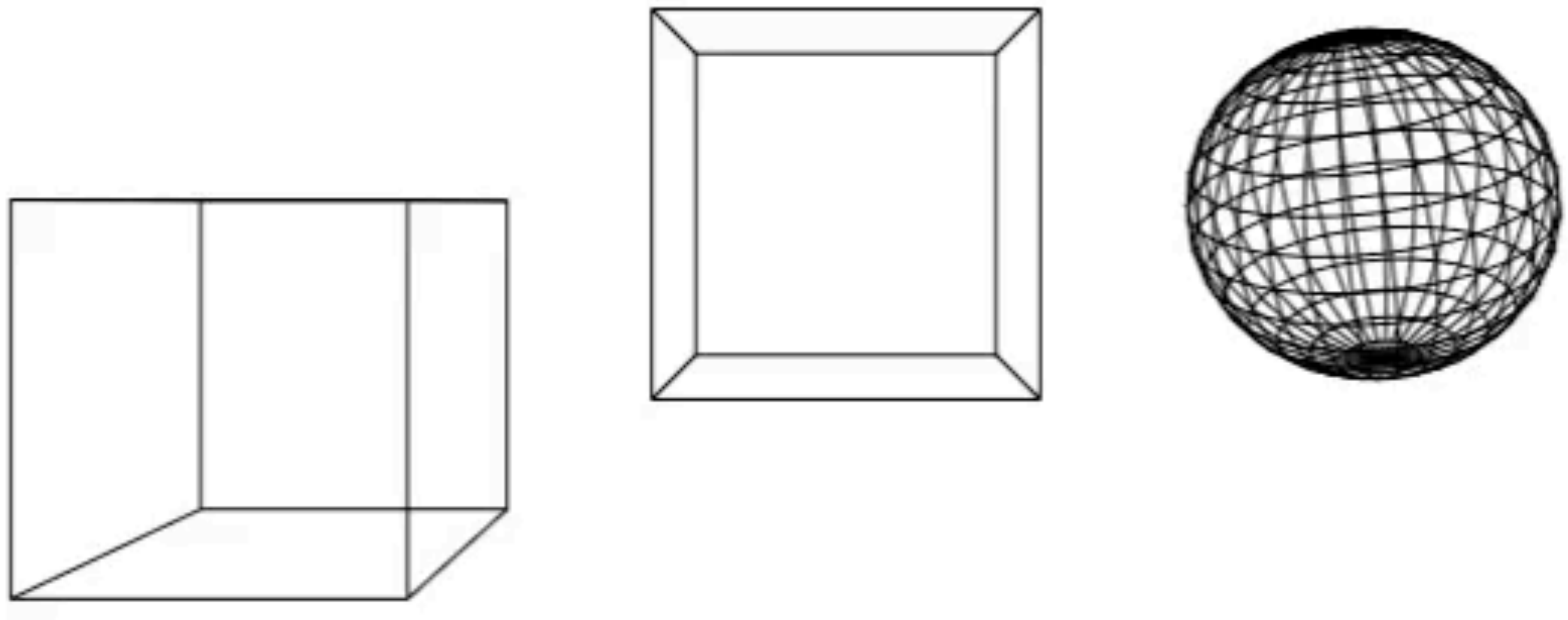
    draw_line(**p**, **q**)

Recall: Wireframe Rendering



```
M = M_vp M_proj M_view M_model
for each line segment a_i, b_i
  p = M a_i
  q = M b_i
  draw_line(p, q)
```

Recall: Wireframe Rendering



```
M = Mvp Mproj Mview Mmodel
for each line segment ai, bi
  p = M ai
  q = M bi
  draw_line(p, q)
```

How do we do this?

Line Drawing

- This is a rasterization problem: given a primitive (line segment), generate fragments (pixels)

$$
\begin{aligned}
&\texttt{M = M}_{\texttt{vp}}\ \texttt{M}_{\texttt{proj}}\ \texttt{M}_{\texttt{view}}\ \texttt{M}_{\texttt{model}}\\
&\texttt{for each line segment } \mathbf{a}_{\texttt{i}}\texttt{, } \mathbf{b}_{\texttt{i}}\\
&\quad \mathbf{p}\ \texttt{=}\ \mathbf{M}\ \mathbf{a}_{\texttt{i}}\\
&\quad \mathbf{q}\ \texttt{=}\ \mathbf{M}\ \mathbf{b}_{\texttt{i}}\\
&\quad \texttt{draw\_line(}\mathbf{p}\texttt{, }\mathbf{q}\texttt{)}
\end{aligned}
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for each line segment a_i, b_i
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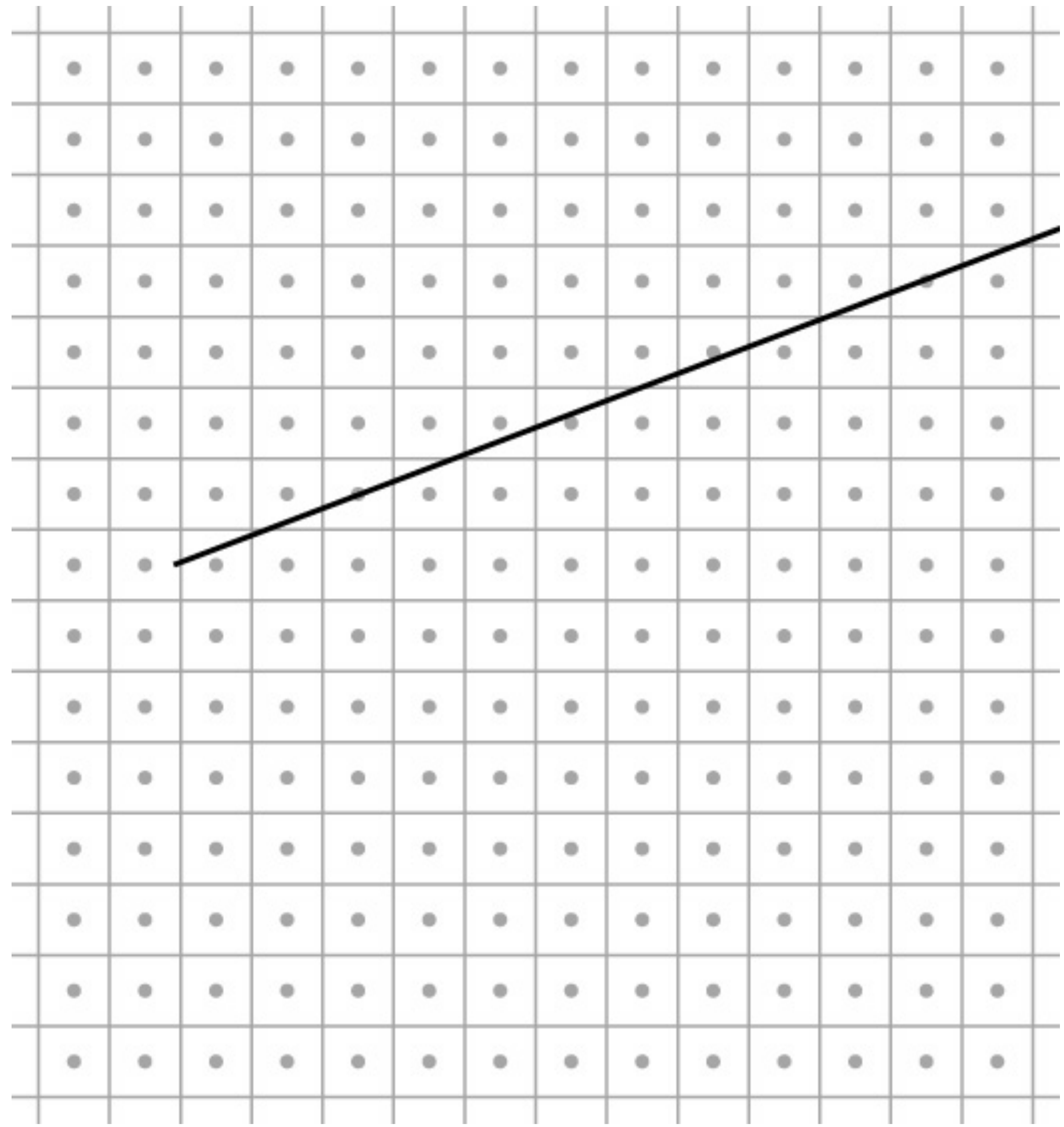
How do we do this?

Rasterizing lines - possible definition

- Define line as a rectangle
- Specify by two endpoints
- Ideal image: black inside, white outside



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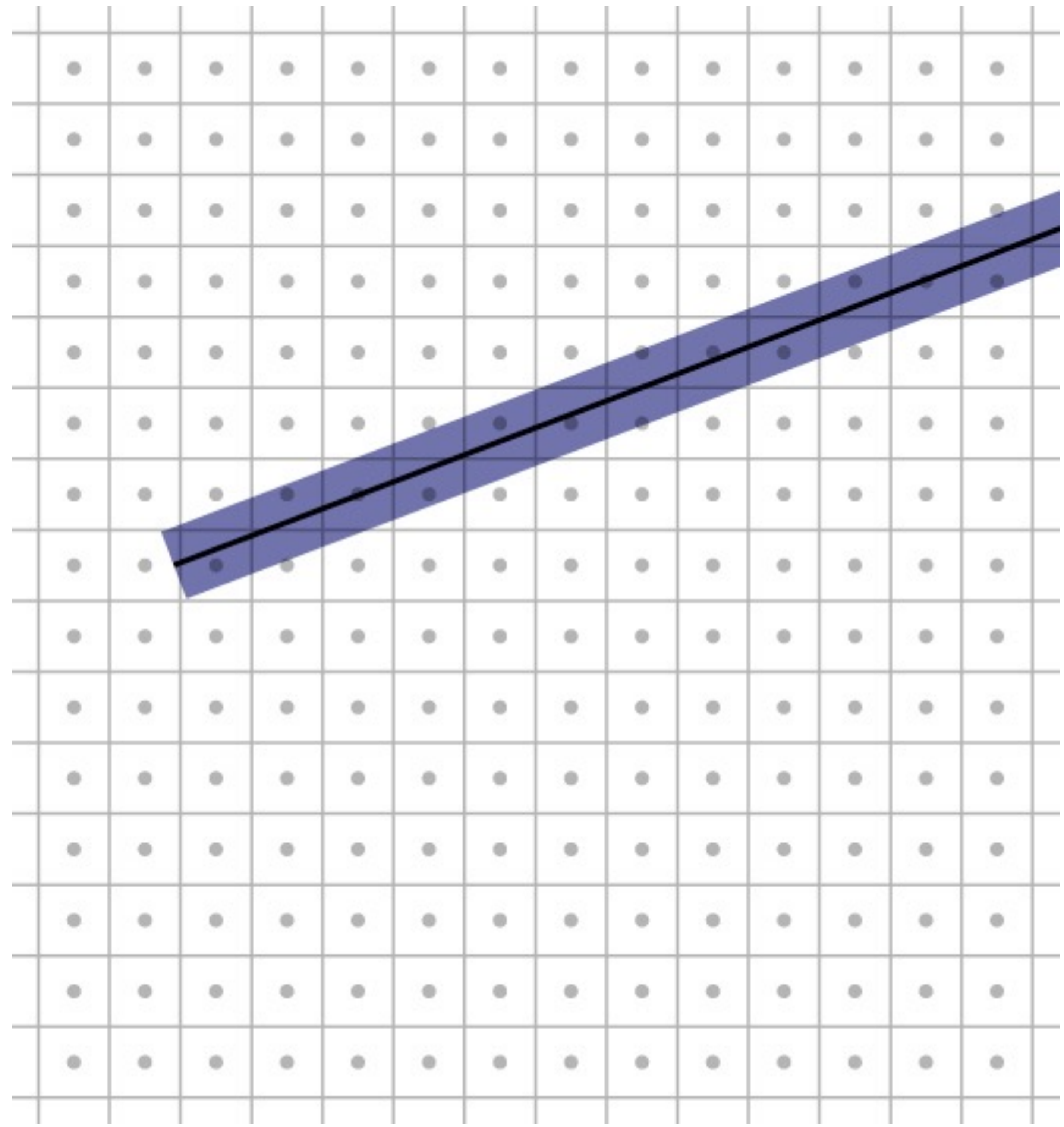
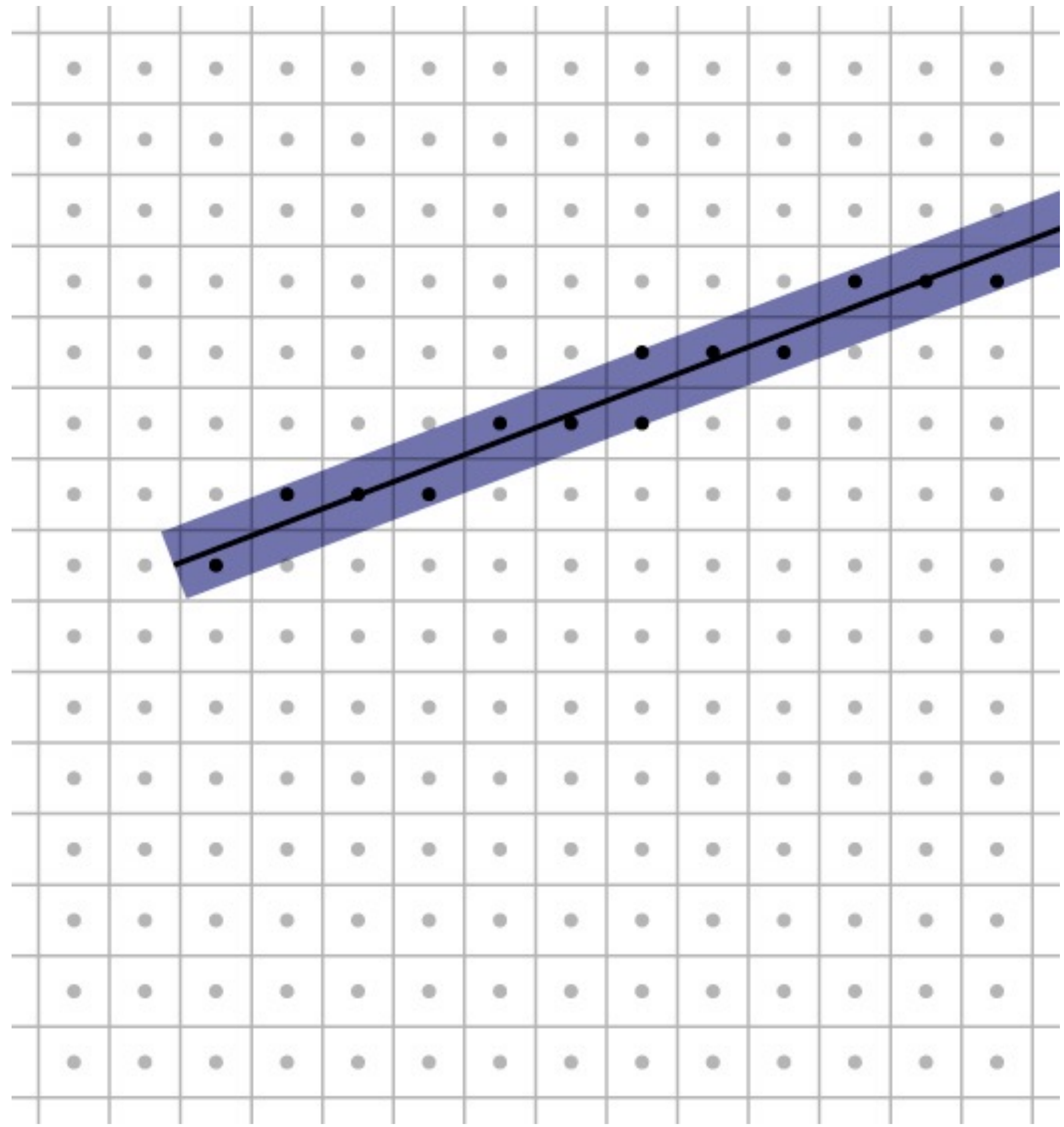


Point sampling

- Approximate rectangle by drawing all pixels whose centers fall within the line
- Problem: sometimes turns on adjacent pixels



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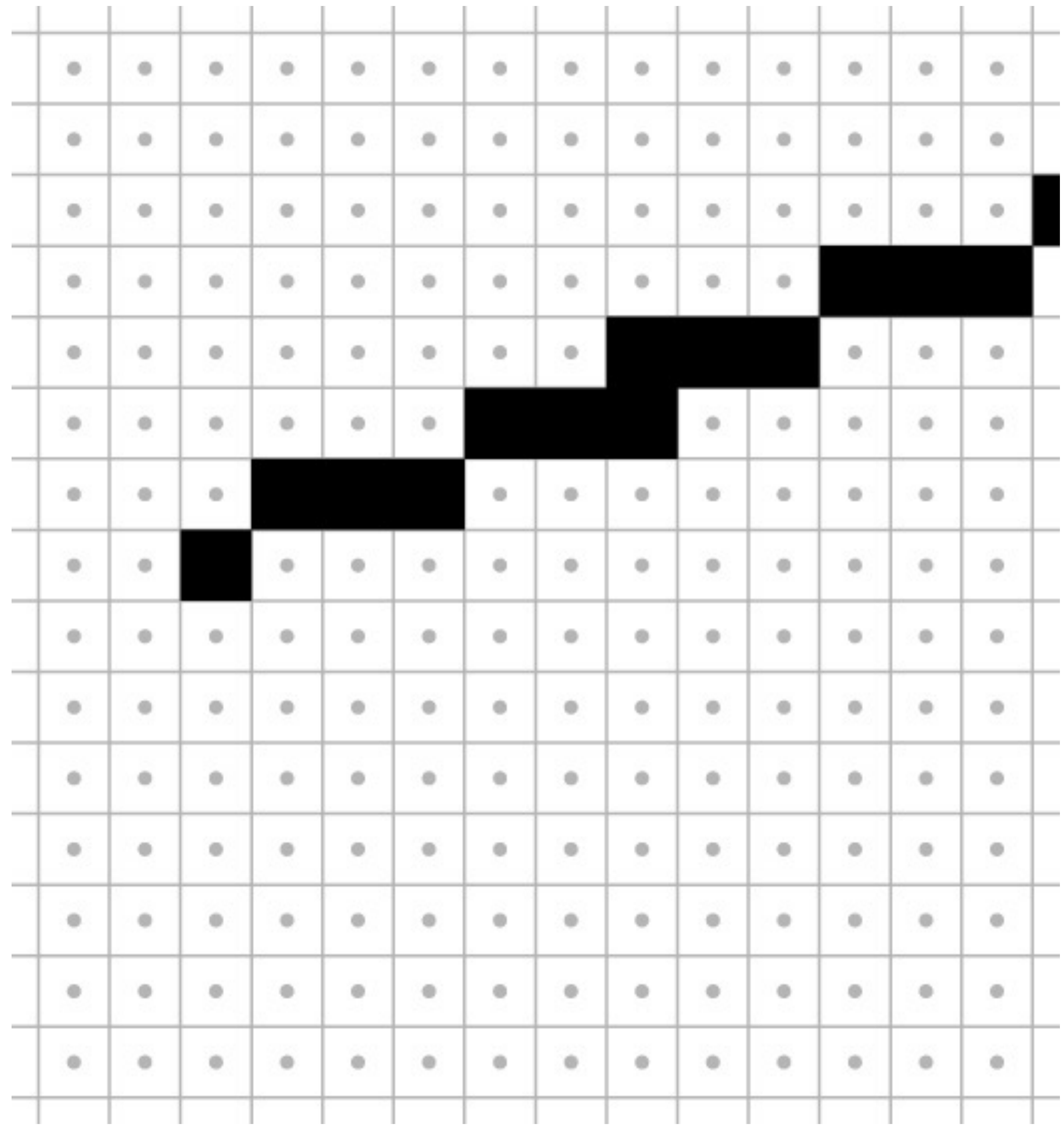
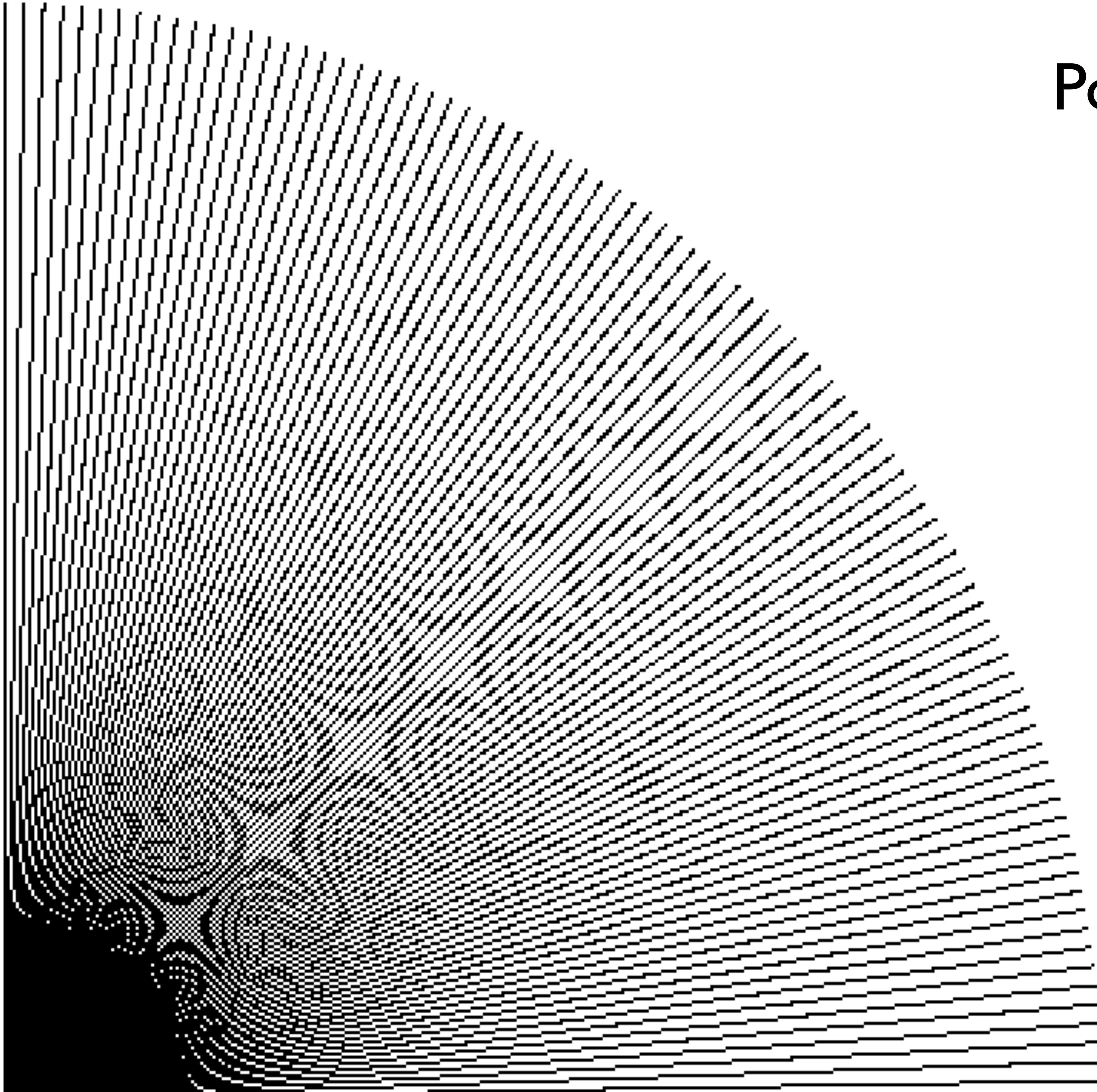
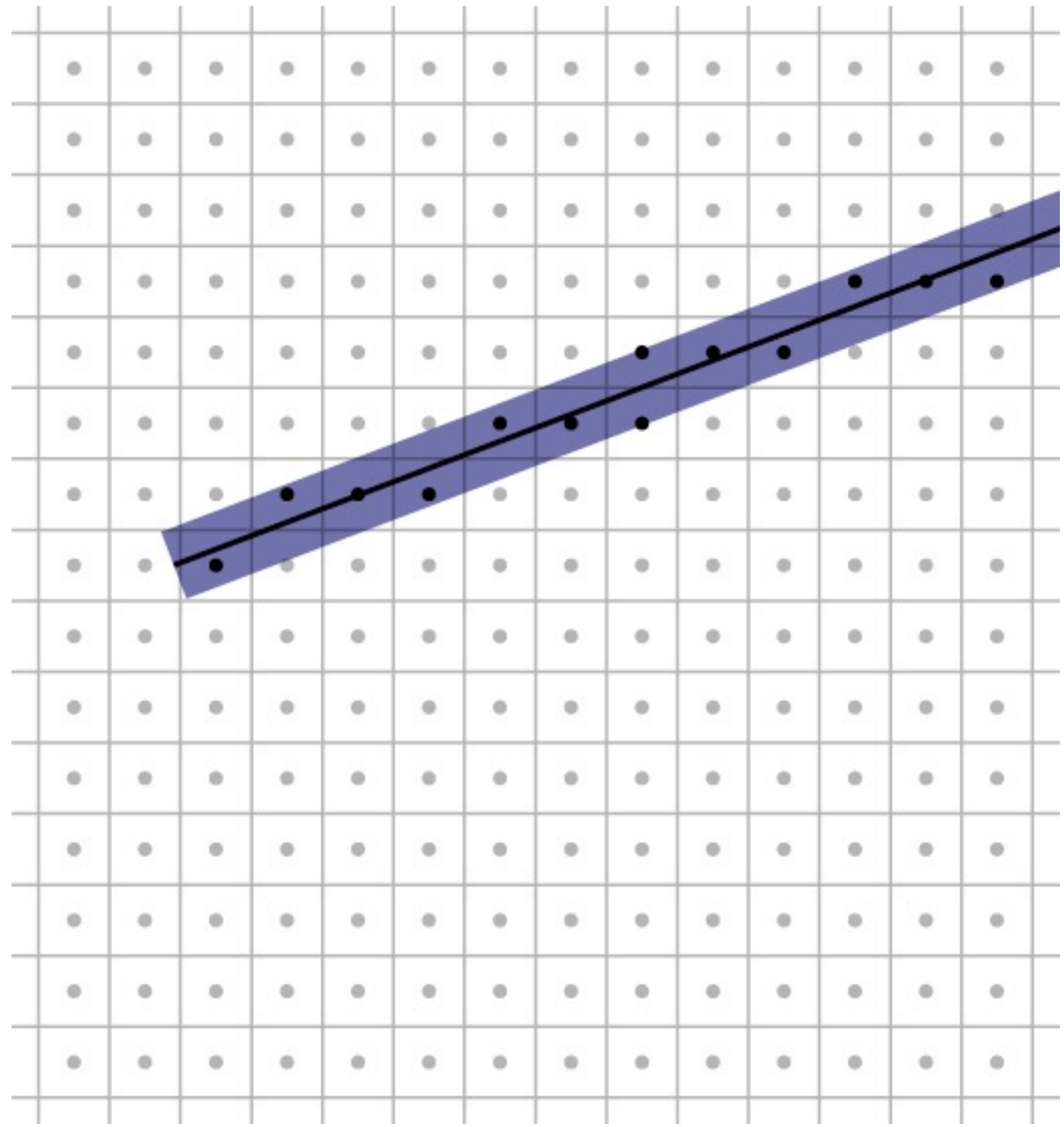




Point sampling in action

Bresenham lines (midpoint alg.)

- Point sampling unit width rectangle leads to uneven line width
- Define line width parallel to pixel grid
- That is, turn on the single nearest pixel in each column
- Note that 45° lines are now thinner



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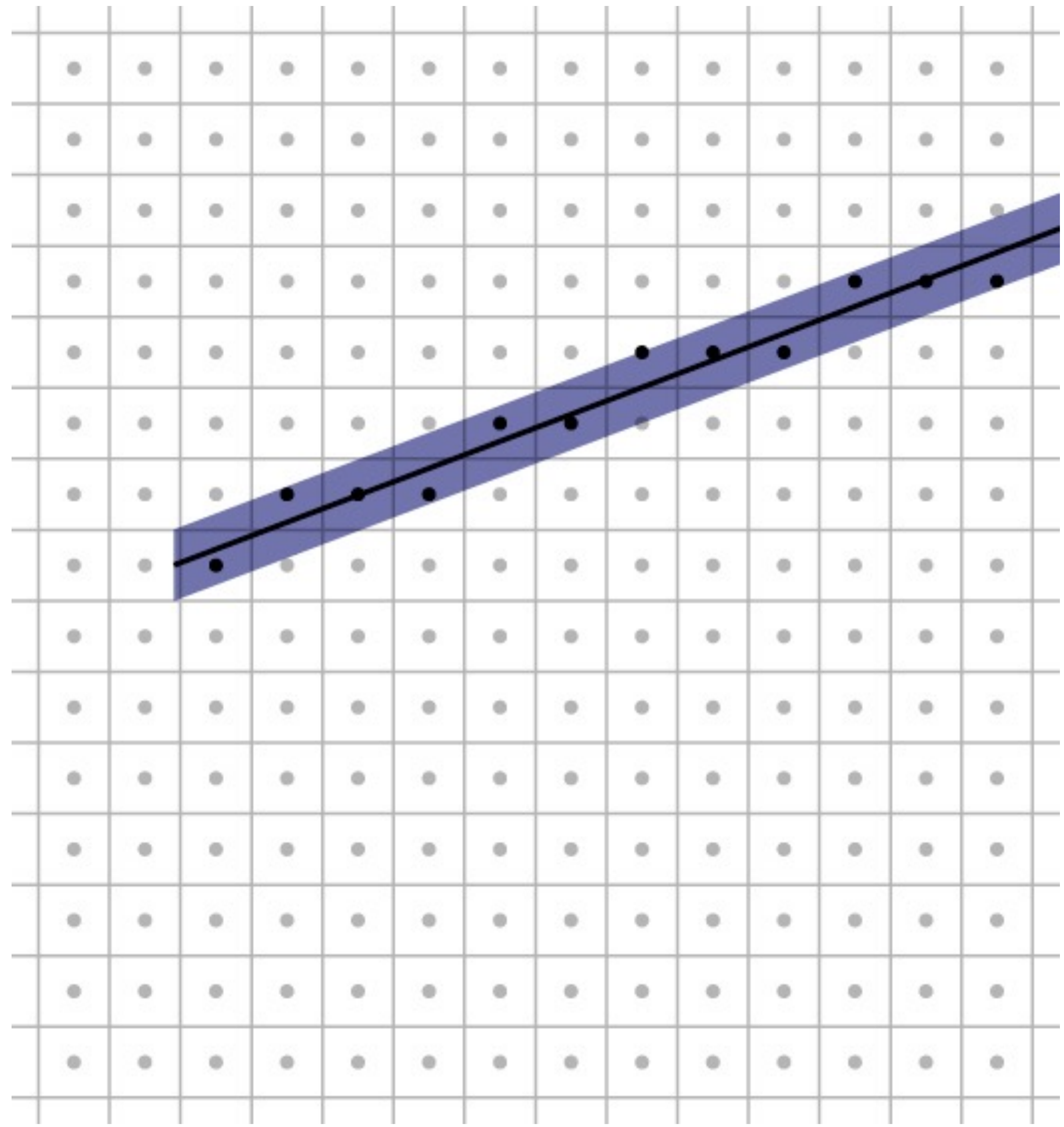

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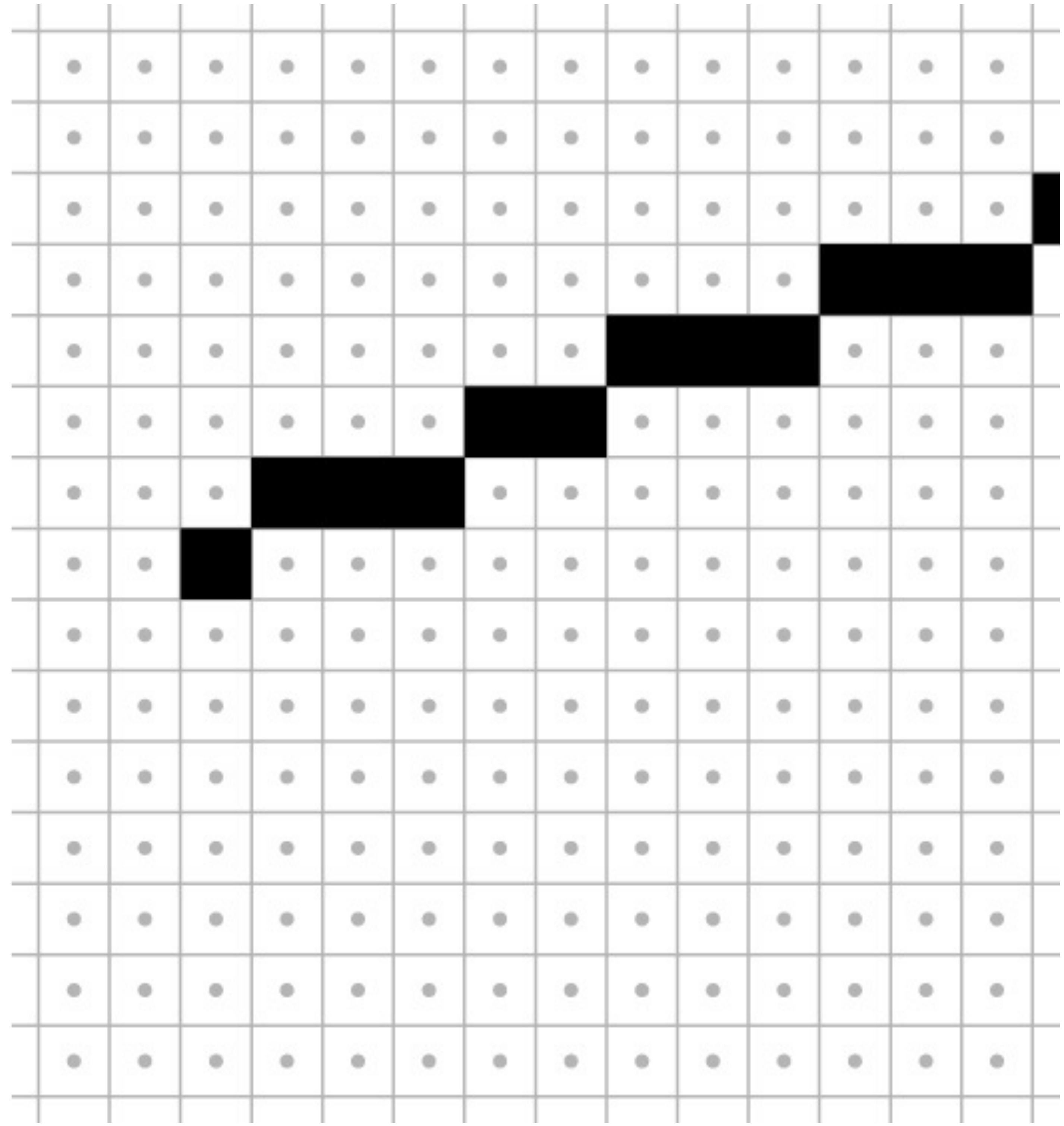
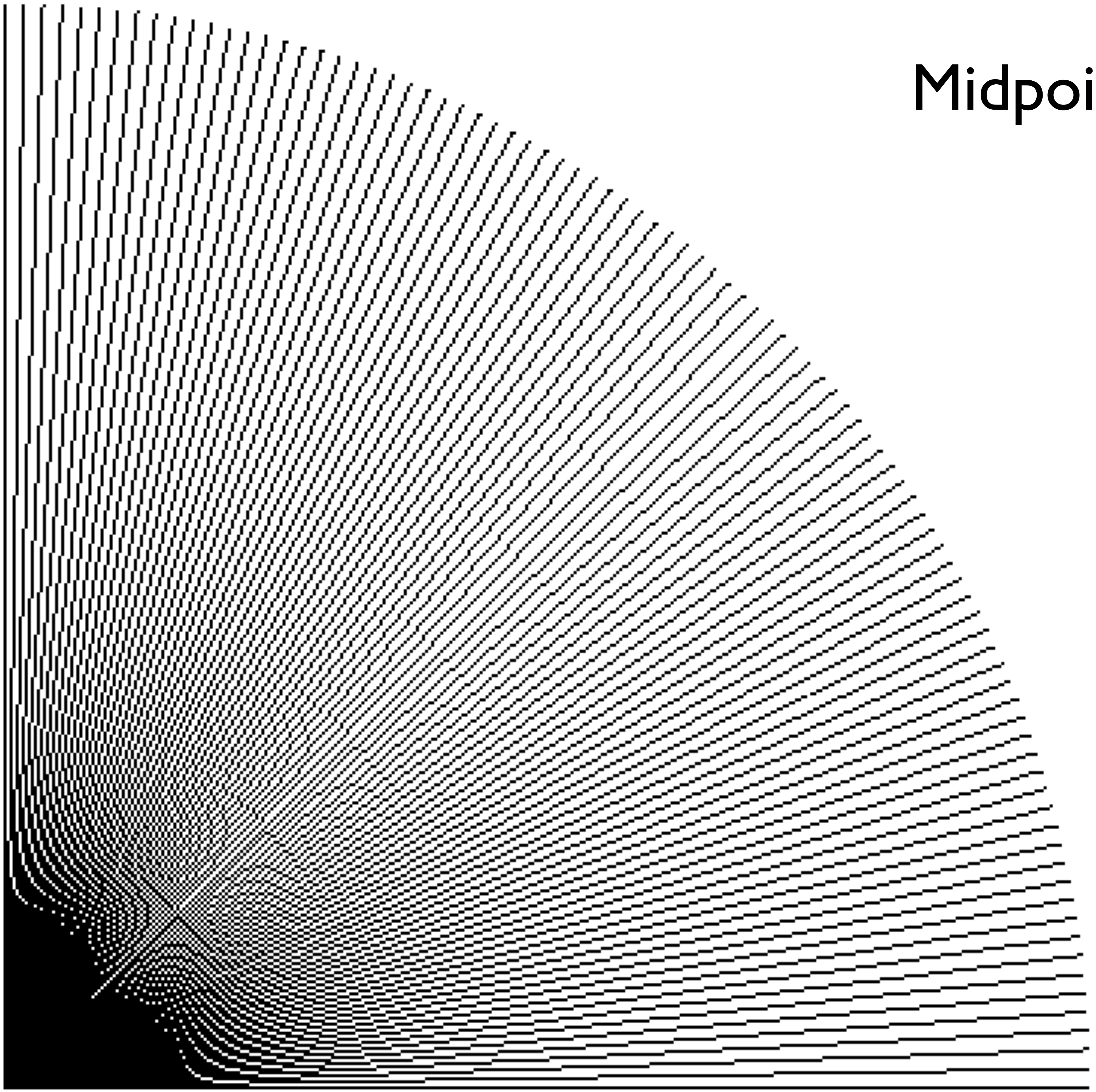


Midpoint algorithm in action



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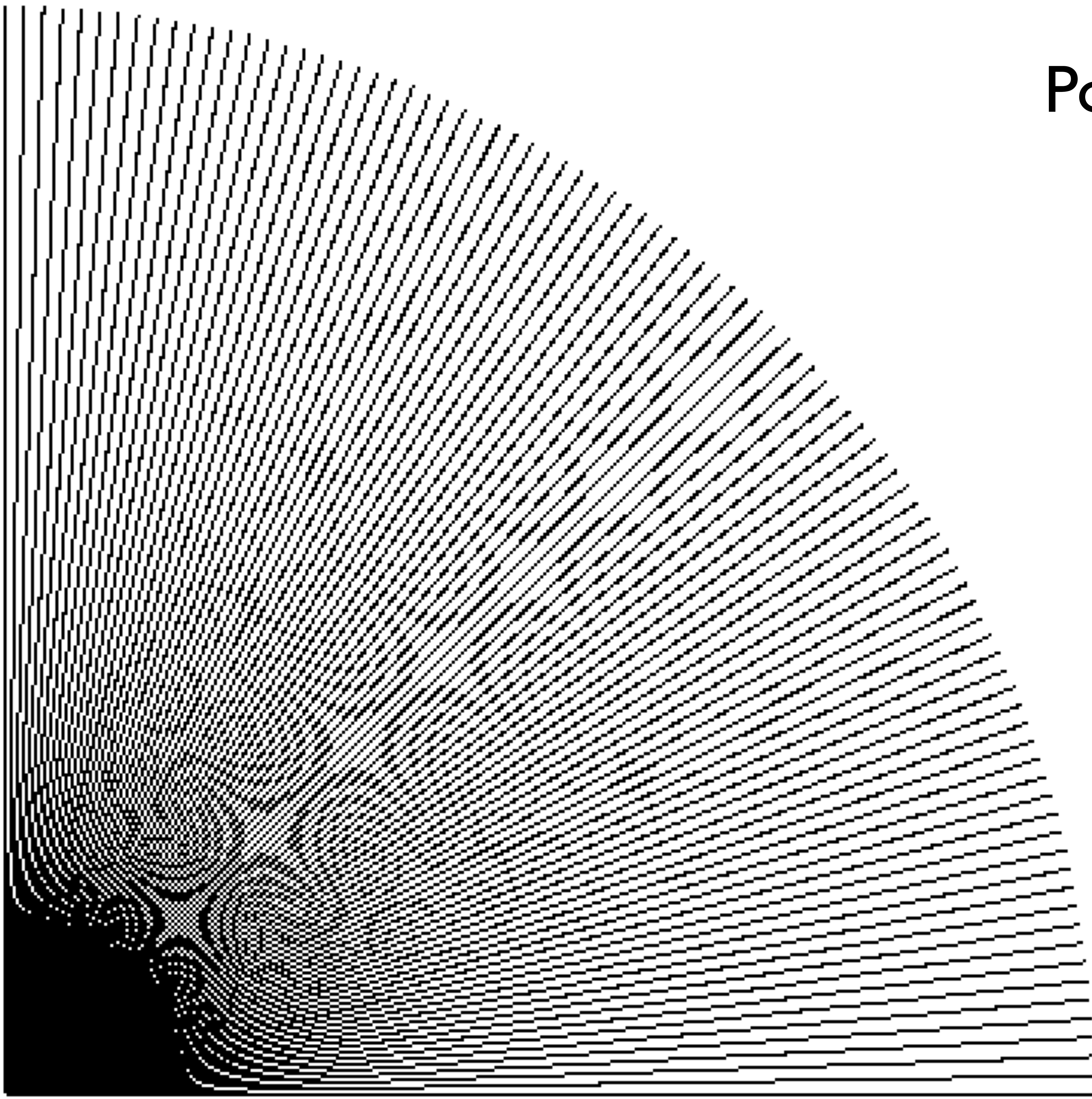
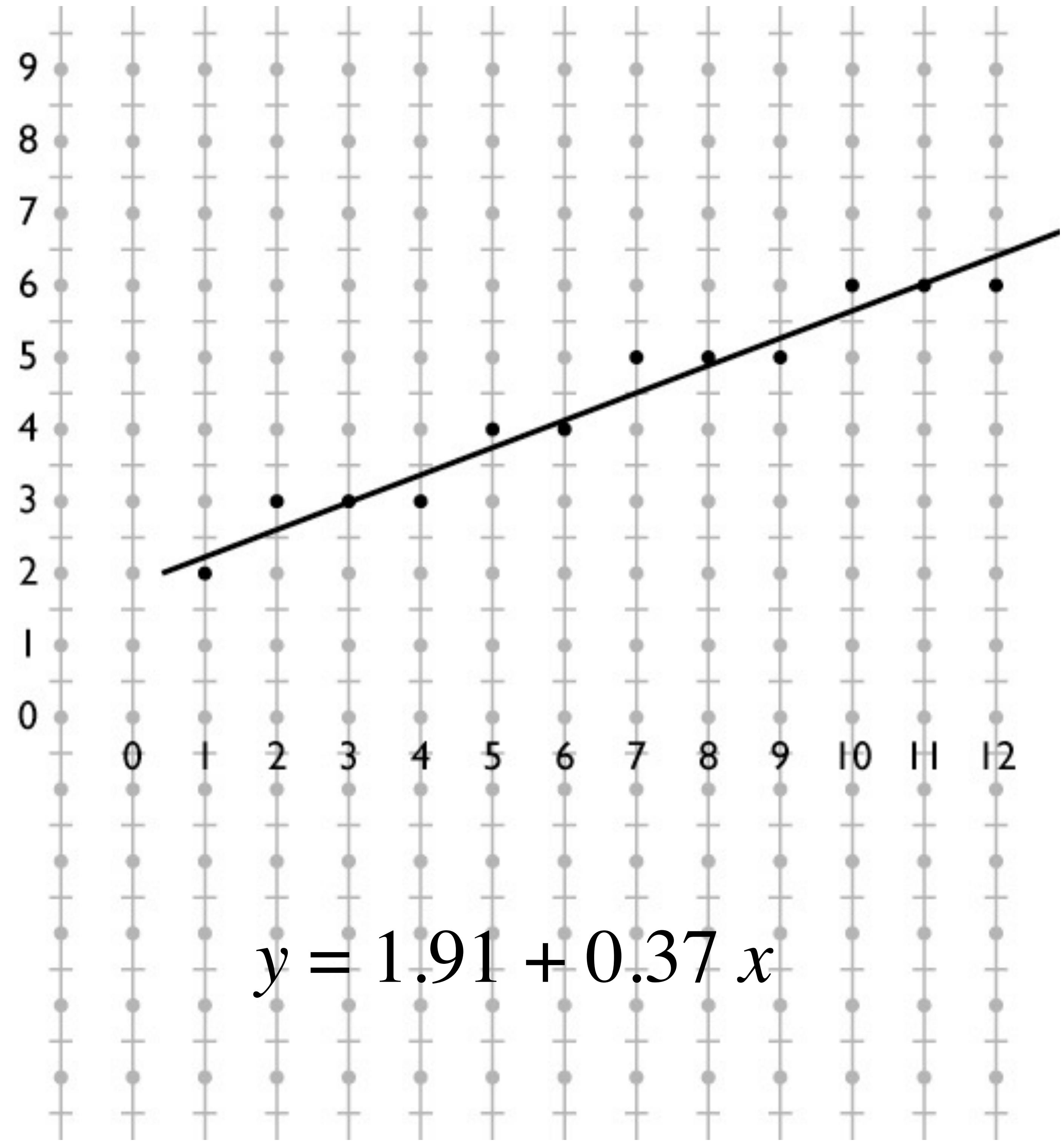


Algorithms for drawing lines

- line equation:
  $y = b + m\ x$
- Simple algorithm:
  evaluate line equation per column
- W.l.o.g. $x_0 < x_1$;
  $0 \leq m \leq 1$



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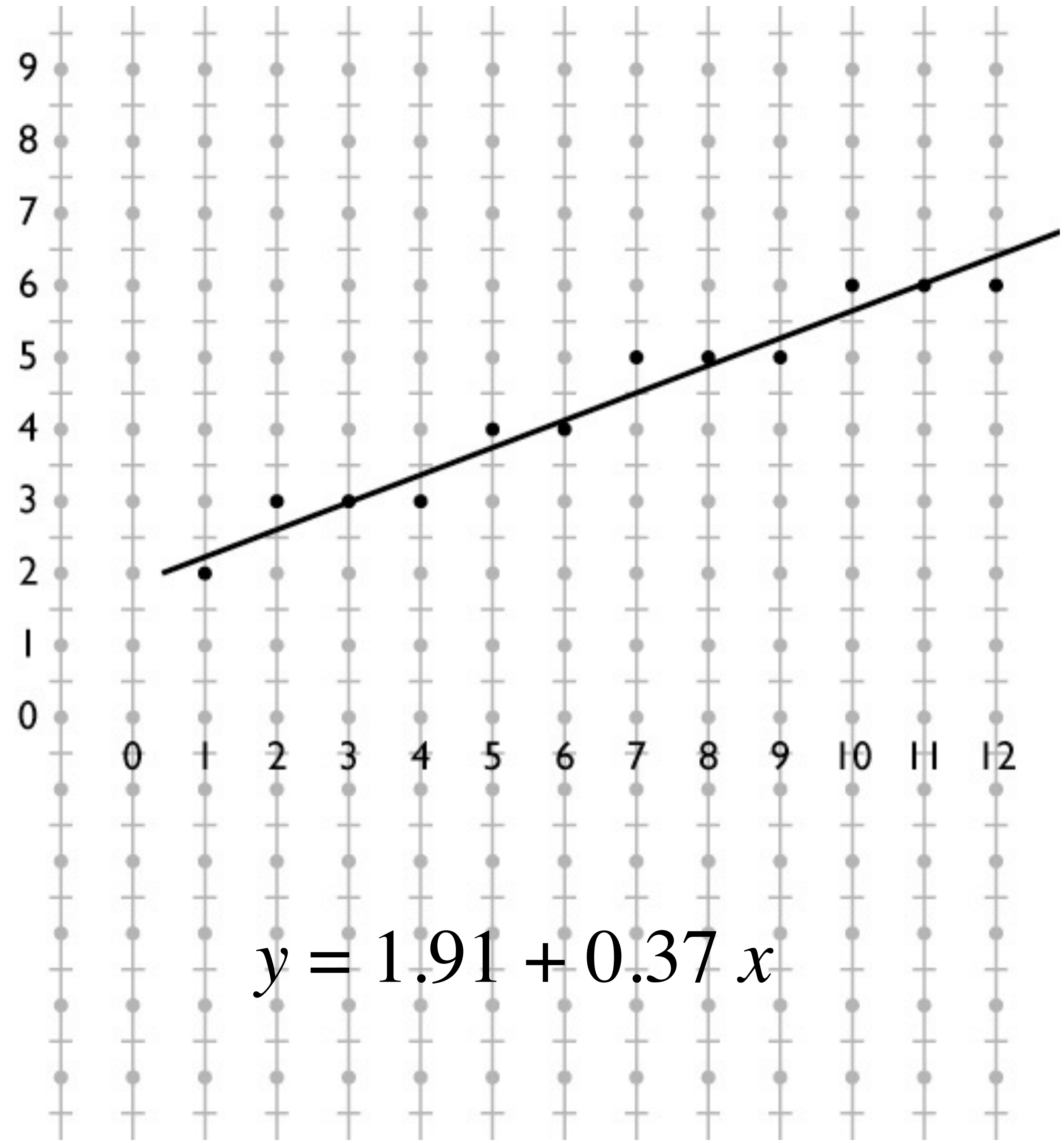

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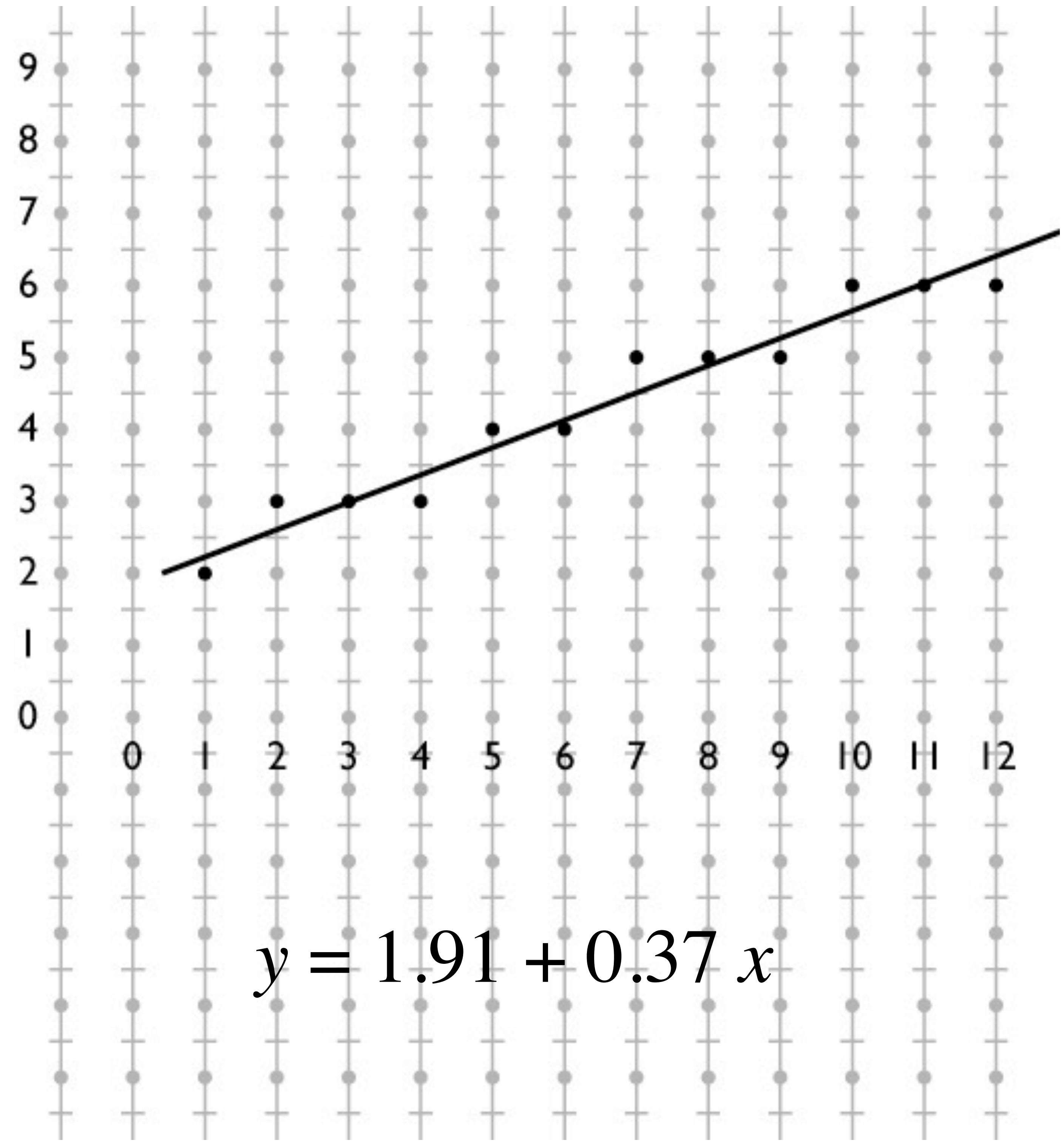

Algorithm:

```
// compute m, b

for x = ceil(x0) to floor(x1)
    y = b + m*x
   // Ex: what goes here?
```

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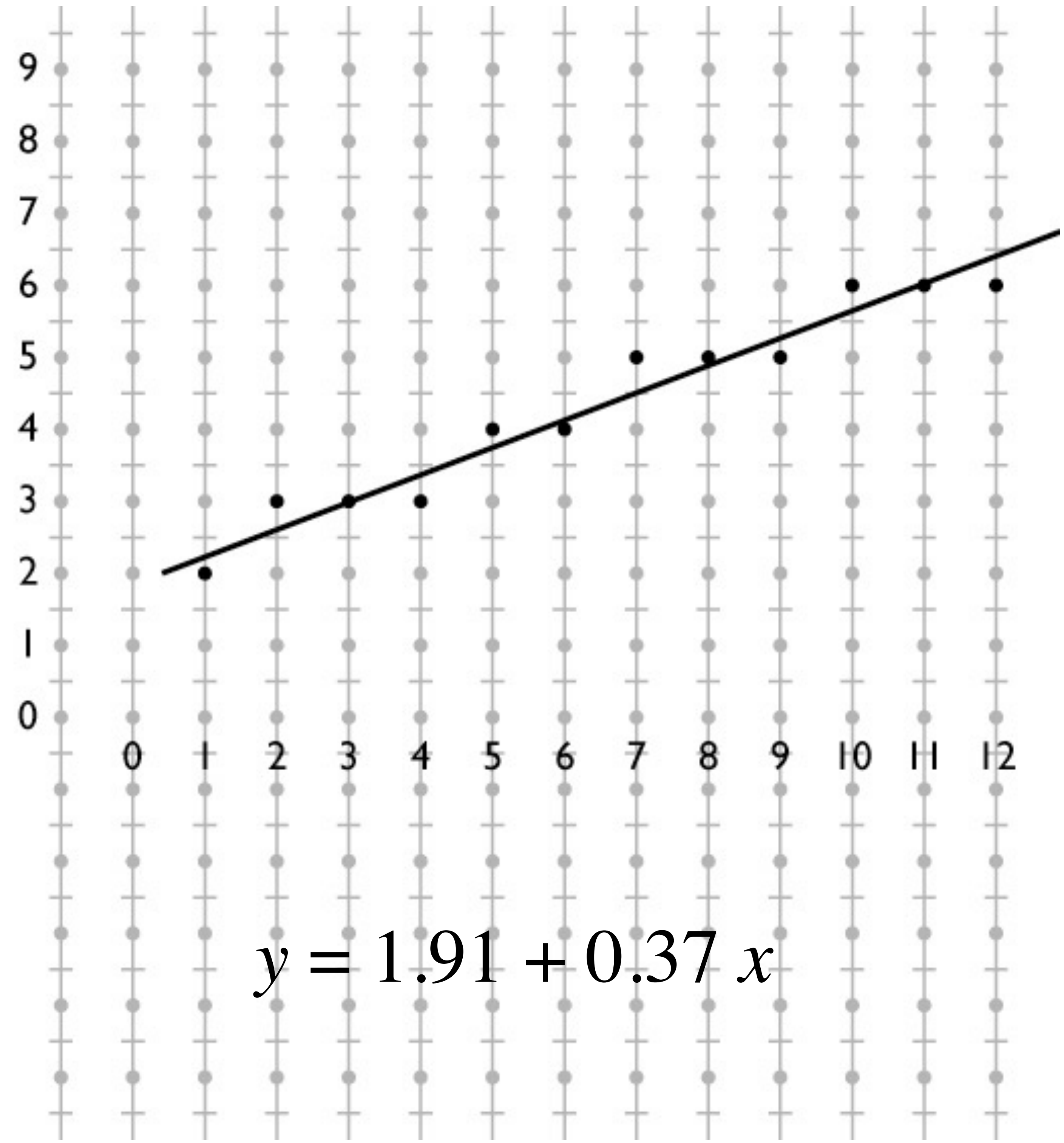


Algorithm:

```
// compute m, b

for x = ceil(x0) to floor(x1)
    y = b + m*x
    draw(x, round(y))
```

Optimizing Line Drawing

Can we take stuff out of the inner loop?

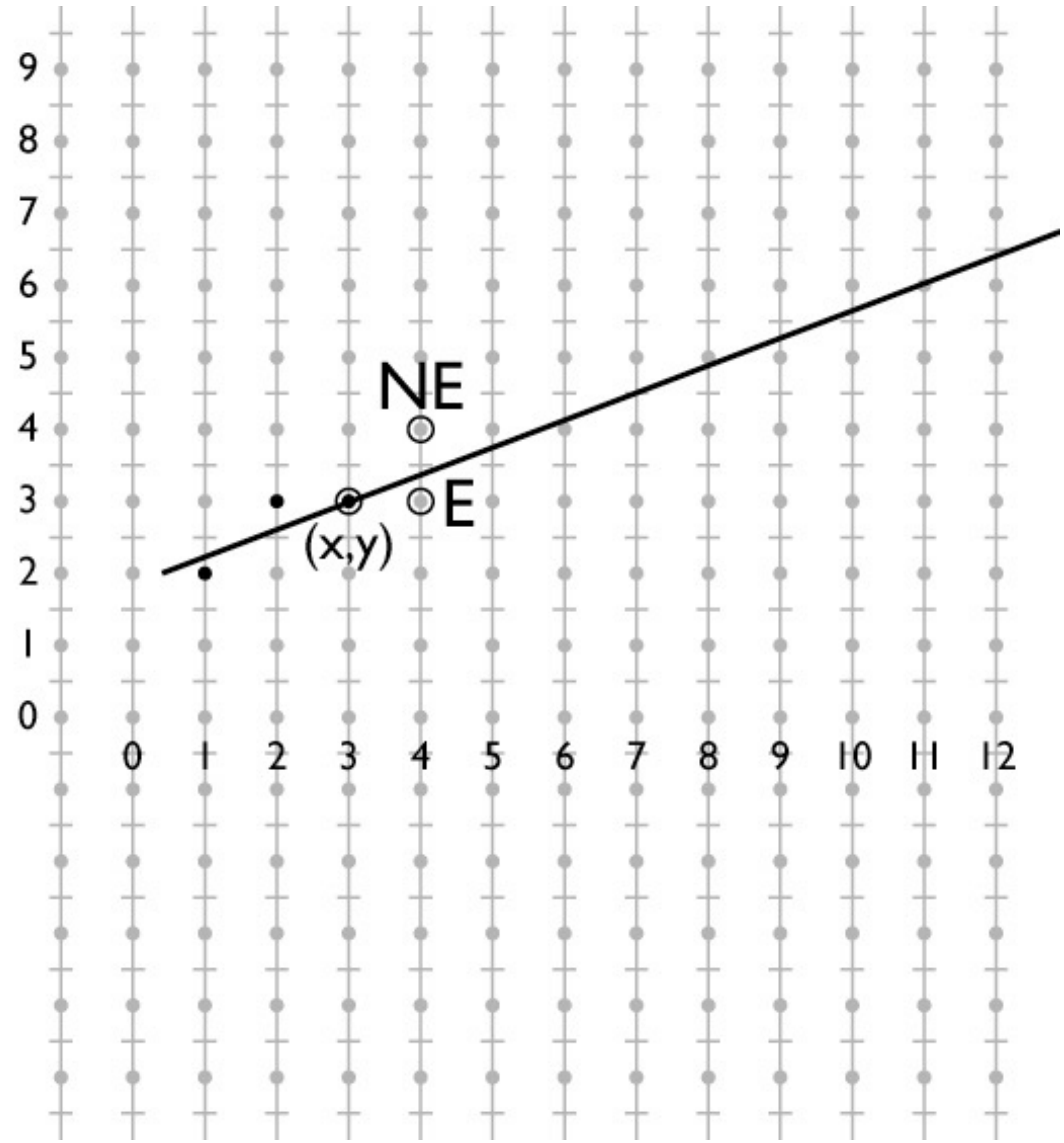
Exercise: optimize this

```
function slow_line(p1, p2):
  // compute m, b
  for x = ceil(x0) to floor(x1)
      y = b + m*x
      draw(x, round(y))
```

```
function fast_line(p1, p2):
  // compute m, b



  for x = ceil(x0) to floor(x1)


      draw(x, round(y))
```

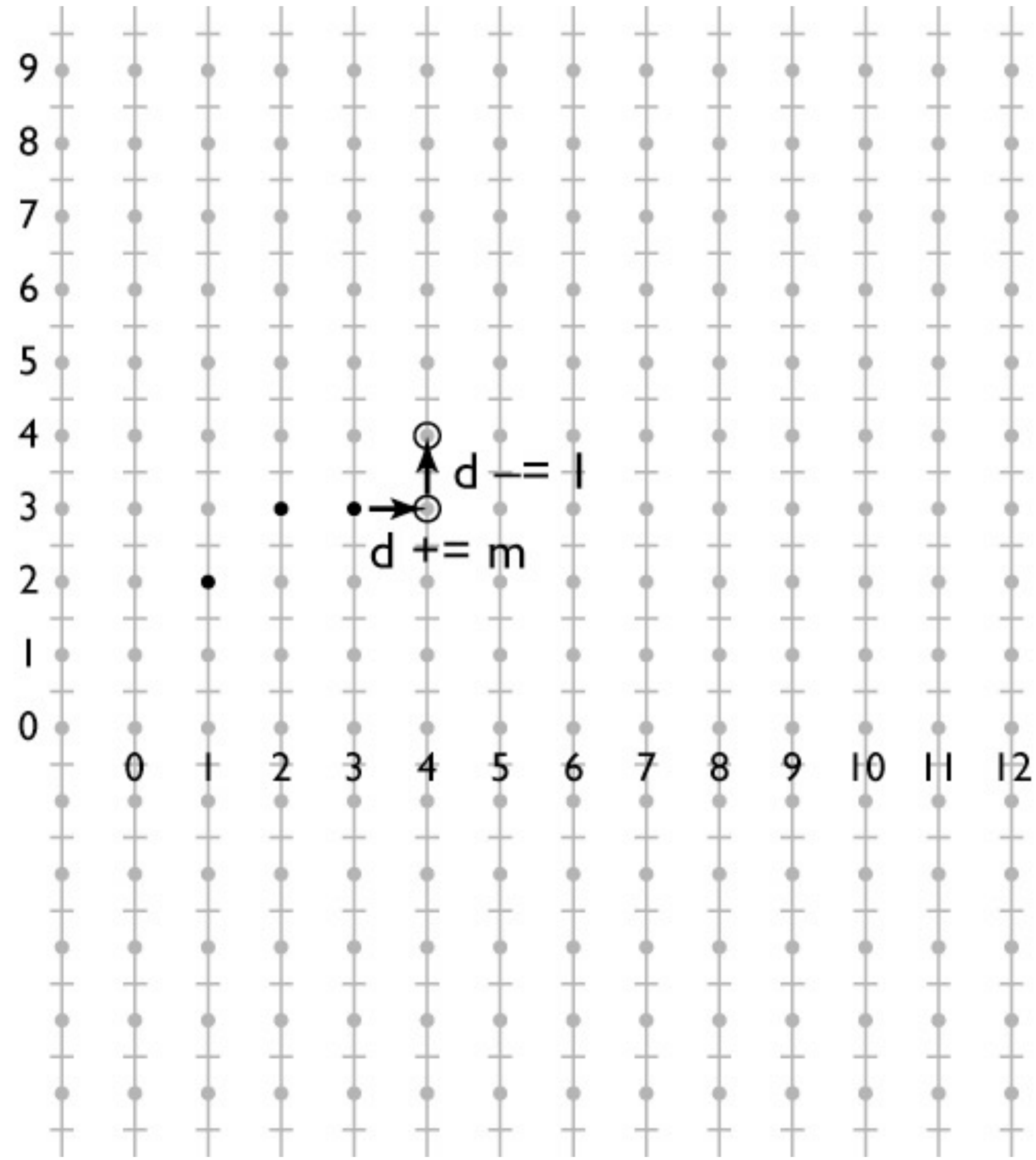
Optimizing Line Drawing Even More

- Rounding is slow too
- At each pixel the only options are E and NE
- Track distance to line:
  - $d = m(x + 1) + b - y$
  - $d > 0.5$ decides between E and NE



Optimizing Line Drawing Even More

- $d = m(x + 1) + b - y$
- Only need to update d for integer steps in x and y
- Do that with addition
- Known as "DDA" (digital differential analyzer)



Linear interpolation

- We often attach attributes to vertices
  – e.g. computed diffuse color of a hair being drawn using lines
  – want color to vary smoothly along a chain of line segments

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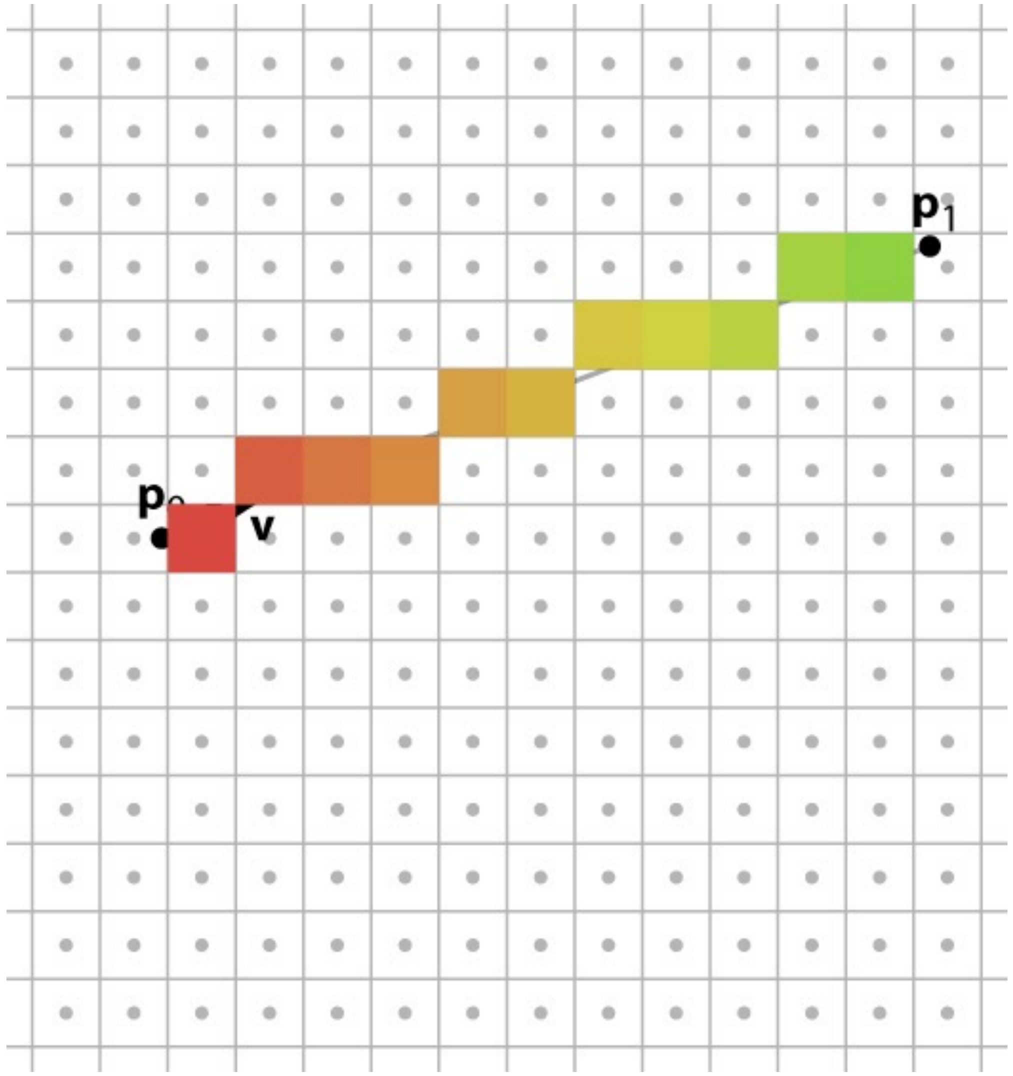

Linear interpolation

- We often attach attributes to vertices
  – e.g. computed diffuse color of a hair being drawn using lines
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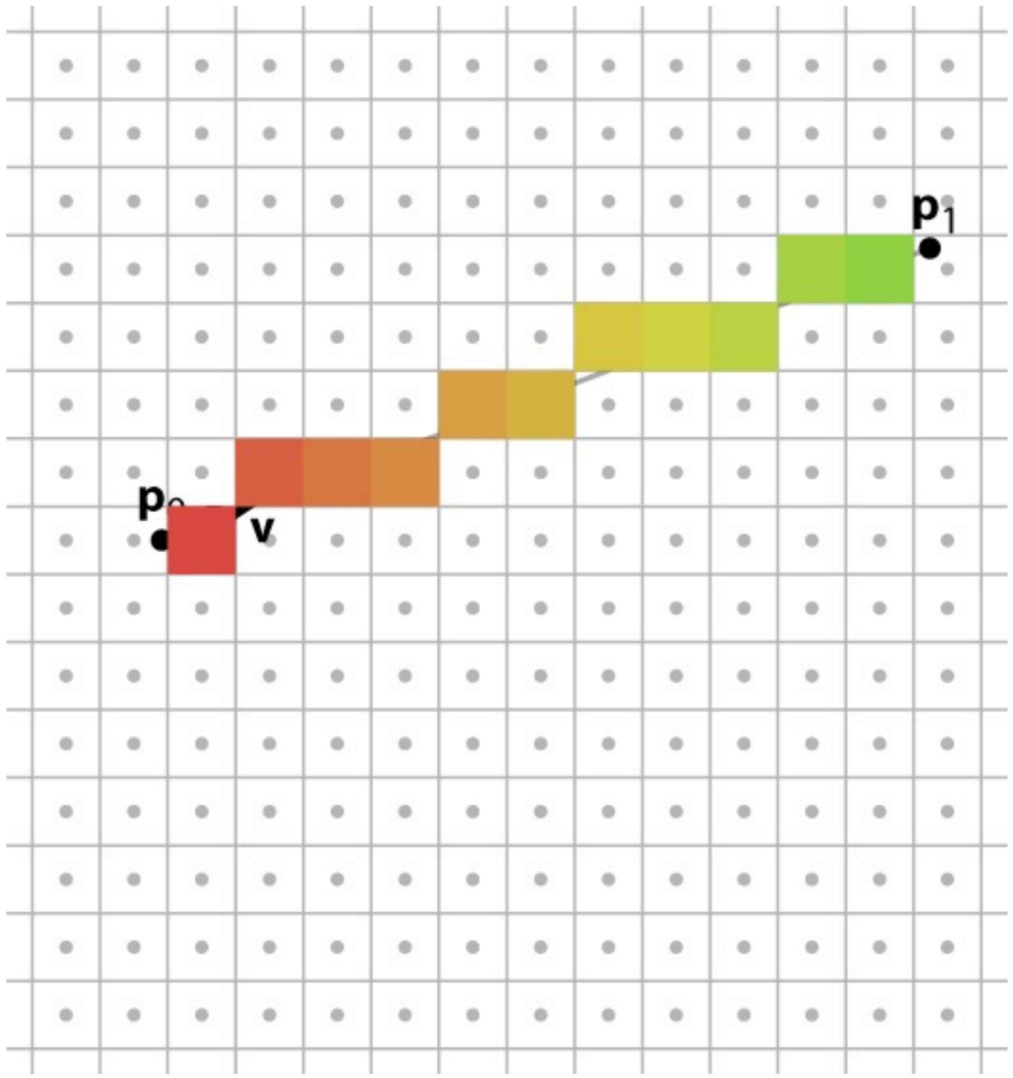
- Same machinery as we used for y works for other values!

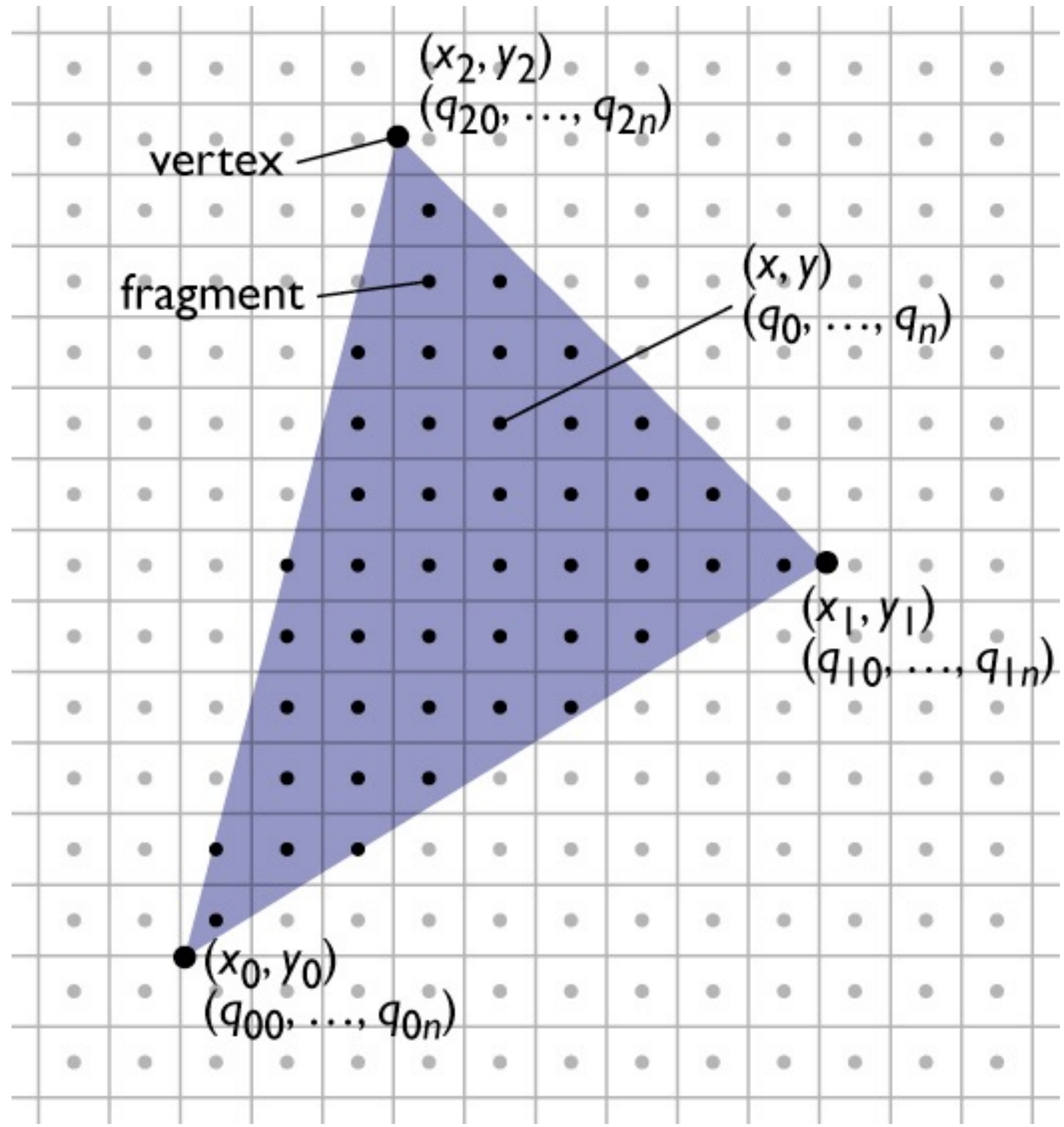


Rasterizing triangles

- Input:
  - three 2D points (the triangle's vertices in pixel space)
    - (x_0, y_0); (x_1, y_1); (x_2, y_2)
  - parameter values at each vertex
    - $q_{00}, \ldots, q_{0n}$; $q_{10}, \ldots, q_{1n}$; $q_{20}, \ldots, q_{2n}$
- Output: a list of fragments, each with
  - the integer pixel coordinates (x, y)
  - interpolated parameter values $q_0, \ldots, q_n$

Rasterizing triangles

- Summary
  1 evaluation of linear functions on pixel grid
  2 functions defined by parameter values at vertices
  3 using extra parameters to determine fragment set



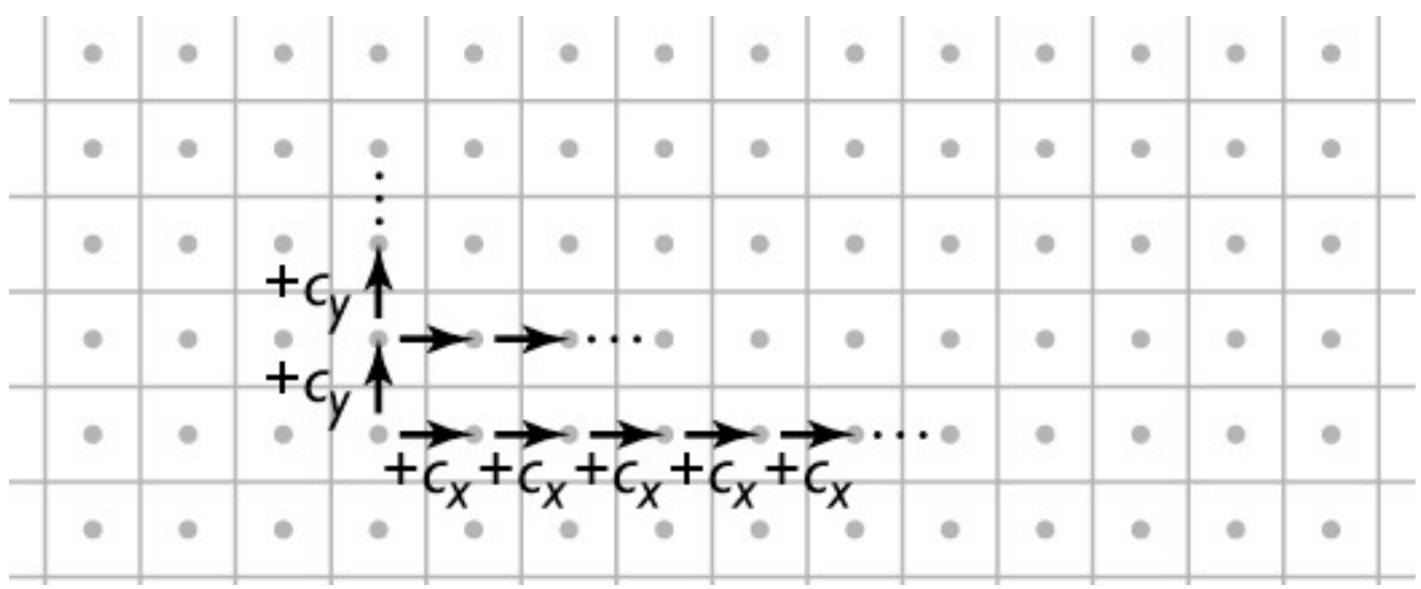
Incremental linear evaluation

- A linear (affine, really) function on the plane is:

$$q(x, y) = c_x x + c_y y + c_k$$

- Linear functions are efficient to evaluate on a grid:

$$q(x+1, y) = c_x(x+1) + c_y y + c_k = q(x, y) + c_x$$
$$q(x, y+1) = c_x x + c_y(y+1) + c_k = q(x, y) + c_y$$



Pixel-walk (Pineda) rasterization

- Conservatively visit a superset of the pixels you want
- Interpolate linear functions
  - barycentric coords (determines when to emit a fragment)
  - colors
  - normals
  - whatever else!