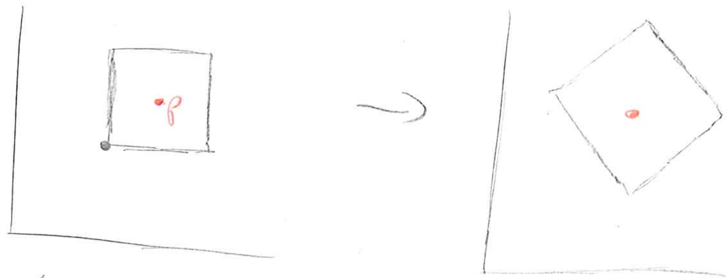
14.1 ~~13.2~~

Composing 2D Affine Transformations

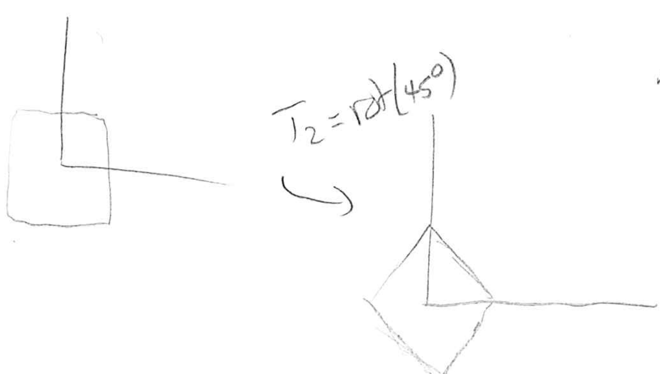
Matrix mutiplication - Just Works

Example: Rotate around a non-origin point P



$T_1 = \text{translate}(-p)$

$T_3 = \text{translate}(p)$



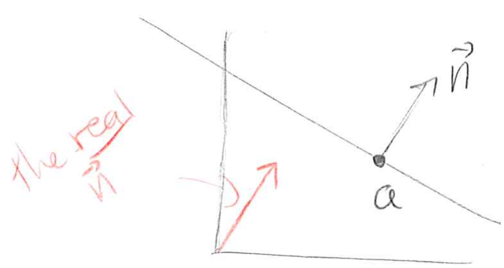
$$\boxed{T = T_3 T_2 T_1}$$

right-to-left!

Points vs Directions

13.3

Both 3-vectors, but one is a place, the other is a direction.



Plane through pt $\vec{a}$ w/ normal vector $\vec{n}$

Points get moved by translations

Directions don't

$$\vec{v} = \vec{p} - \vec{q}$$

$$\begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} - \begin{bmatrix} q_x \\ q_y \\ 1 \end{bmatrix}$$ neat!

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ 1 \end{bmatrix} = \begin{bmatrix} a_x + 1 \\ a_y + 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ 0 \end{bmatrix} = \begin{bmatrix} n_x \\ n_y \\ 0 \end{bmatrix}$$

turn off translation for vectors that indicate directions

14.2 ~~13.4~~

Affine Composition - Similarity Transforms:

1. Transform to a frame where it's easy
2. Do the easy thing
3. Transform back.

Let $F = \begin{bmatrix} \vec{u} & \vec{v} & \vec{p} \\ 0 & 0 & 1 \end{bmatrix}$, frame where desired transformation T is easy to write.

This is frame-to-canonical!

Canonical-to-frame is F^{-1}, so whole transform is

$$\overset{③}{F}\ \overset{②}{T}\ \overset{①}{F^{-1}}$$

Inverses?

Can compute algebraically, but sometimes it's easy:

- $\text{translate}(\vec{t})^{-1} = \text{translate}(-\vec{t})$
- $\text{Rotate}(\theta)^{-1} = \text{rotate}(-\theta)$
- $\text{Scale}(s_x, s_y)^{-1} = \text{Scale}(1/s_x, 1/s_y)$
- inverse of frame w/ orthonormal basis (rigid t_x)

$$\begin{array}{cc} 2\times2 & 2\times1 \\ \left[\begin{matrix} Q & \vec{u} \\ 0 & 1 \end{matrix}\right. & \left.\right]^{-1} \\ 1\times2 & 1\times1 \end{array} = \begin{bmatrix} Q^T & -Q^T\vec{u} \\ 0 & 1 \end{bmatrix}$$

(Math fact:) Inverse of an orthogonal matrix is its transpose.

Rotations in 3D

Euler angles - composition of 3 axis-aligned rotations.

Example $R_z R_x R_z$

- 12 possible sequences of 3 axis rotations!
- Also: can choose intrinsic vs extrinsic

intrinsic ↙ axes of rotation stick with object as it rotates

extrinsic ↘ axes of rotation are fixed to the global coordinate system

Yikes! See wikipedia. Pick a convention and stick with it.

Axis - angle

$\vec{a}$, θ Rotate θ about axis $\vec{a}$

How? Change coordinate frame so $\vec{a}$ is the "z" axis, rotate around z, change coordinate frame back.] Similarity transform!

Basis for frame: $\vec{w} \leftarrow \dfrac{\vec{a}}{\|\vec{a}\|}$

Pick a $\vec{t}$ not collinear with $\vec{w}$ $\quad \vec{t}$ = w, but with smallest component set to 1

$u \leftarrow \dfrac{\vec{t} \times \vec{w}}{\|\vec{t} \times \vec{w}\|}$

$v \leftarrow \vec{w} \times \vec{u}$