Translation - impossible using a 2x2 matrix!

Proof: $f(\vec{x}) = \vec{x} + \vec{t}$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$, regardless of a_{ij}'s.

Origin can't move!

Solution 1: Carry around a 2x2 matrix T and a translation $\vec{t}$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = T \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Composition gets annoying, though:

$$(T_1, t_1) \circ (T_2, t_2) \; ?$$

Solution 2: A Clever Math Hack

try w/ zeroes here first

$$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y + t_x \\ a_{21}x + a_{21}y + t_y \\ 1 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} x' + t_x \\ y' + t_y \\ 1 \end{bmatrix} \end{array} \begin{array}{c} x \\ y \\ w \end{array}$$

huh?

Homogeneous Coordinates: tack on an extra dimension, set point value to 1, and put translation in 3rd col of T.

HW2: prove it!

Fact: composition now Just Works$^{(TM)}$!

2D Affine Transformations - 2D transformations, represented with 3D matrices

Have the form:

linear component → translation component

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

fixed last row

Ex. 2: unit square to diamond

Change-of-basis view: affine tx let us change coordinate frame, not just basis

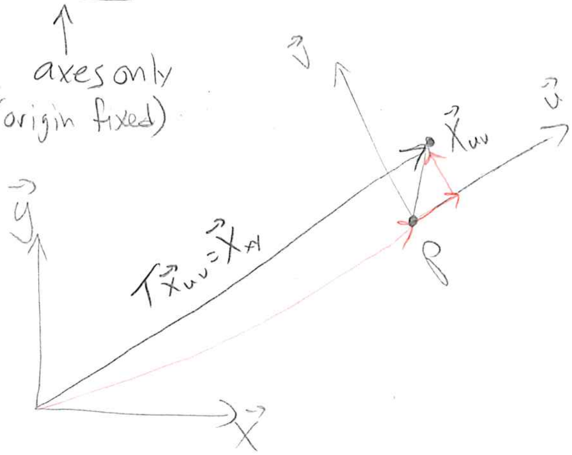
frame: origin + basis

basis: axes only (origin fixed)

"frame-to-canonical transform"

$$T = \begin{bmatrix} \vec{u} & \vec{v} & \vec{p} \\ 0 & 0 & 1 \end{bmatrix}$$

Goes from u,v,p frame to canonical frame.



If the basis is orthonormal (unit length, mutually orthogonal axes), this is a special affine tx called a Rigid transformation.

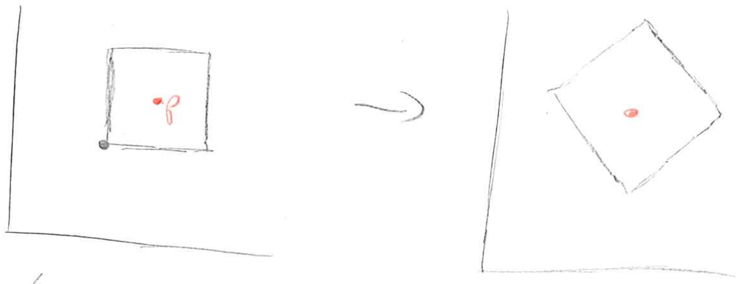
→ shape doesn't change - only rotation and translation

Composing 2D Affine Transformations

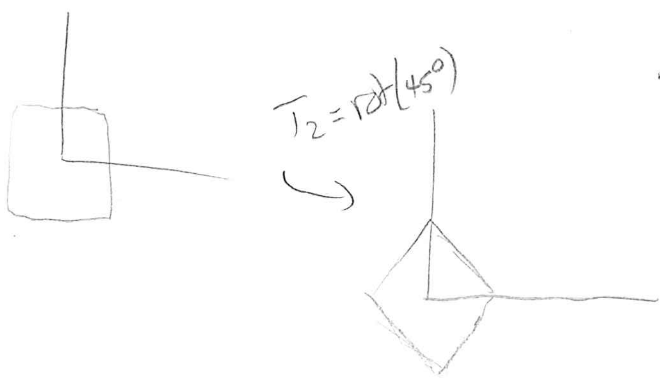
(14.1) ~~(13.2)~~

Matrix multiplication - Just Works

Example: Rotate around a non-origin point P



$T_1 = \text{translate}(-p)$



$T_3 = \text{translate}(p)$

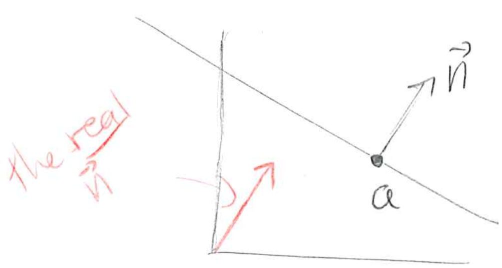
$$\boxed{T = T_3 T_2 T_1}$$

right-to-left!

Points vs Directions

(13.3)

Both 3-vectors, but one is a place, the other is a direction.



Plane through pt $\vec{a}$ w/ normal vector $\vec{n}$

Points get moved by translations

Directions don't

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ 1 \end{bmatrix} = \begin{bmatrix} a_x + 1 \\ a_y + 1 \\ 1 \end{bmatrix}$$

$$\vec{v} = \vec{p} - \vec{q}$$

$$\begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} - \begin{bmatrix} q_x \\ q_y \\ 1 \end{bmatrix}$$

neat!

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ 0 \end{bmatrix} = \begin{bmatrix} n_x \\ n_y \\ 0 \end{bmatrix}$$

turn off translation for vectors that indicate directions