Transforming objects in a scene: need a function (mapping) that describes new position in terms of old position.

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, \text{ in 2D. } \mathbb{R}^3 \text{ for 3D.}$$

Simple example: $T(\vec{x}) = \vec{x} + \vec{t}$ <u>translation</u>

But first!

Some Review on Matrices.

(for our purposes)

Matrix: 2D array of (real) numbers

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4.5 & 0.5 \end{bmatrix} \text{ is a } \underset{\text{rows}}{2}\text{-by-}\underset{\text{cols}}{3} \text{ matrix}$$

M-by N

a_{ij} = i'th row, j'th col, so $a_{21} = 3$.

Matrix-vector multiplication:

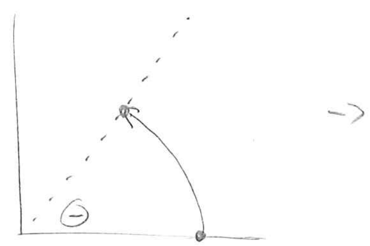
$$\begin{bmatrix} | \\ \vec{y} \\ | \end{bmatrix} = \begin{bmatrix} — \vec{r}_1 — \\ — \vec{r}_i — \\ \vdots \\ — \vec{r}_m — \end{bmatrix} \begin{bmatrix} | \\ x \\ | \end{bmatrix} \qquad y_i = \vec{r}_i \cdot \vec{x}$$

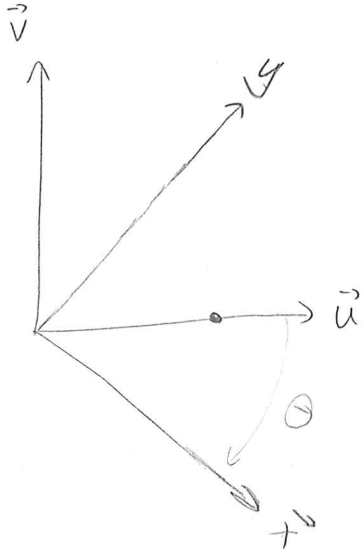
Equivalently:

$$\begin{bmatrix} | \\ \vec{y} \\ | \end{bmatrix} = \begin{bmatrix} | & | & & | \\ \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_m \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \vec{y} = x_1\vec{c}_1 + x_2\vec{c}_2 + \ldots + x_n\vec{c}_n$$

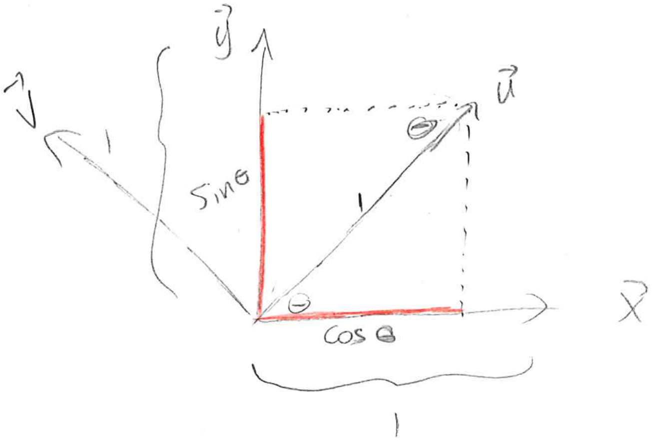
What about Rotation?

Change-of-basis viewpoint:





Need: $\vec{u}$ vector in $\vec{x}, \vec{y}$ basis
$\vec{v}$ vector in $\vec{x}, \vec{y}$ basis.



$$\begin{bmatrix} | & | \\ \vec{u} & \vec{v} \\ | & | \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

from diagram at left

by analogous reasoning for $\vec{v}$ vector