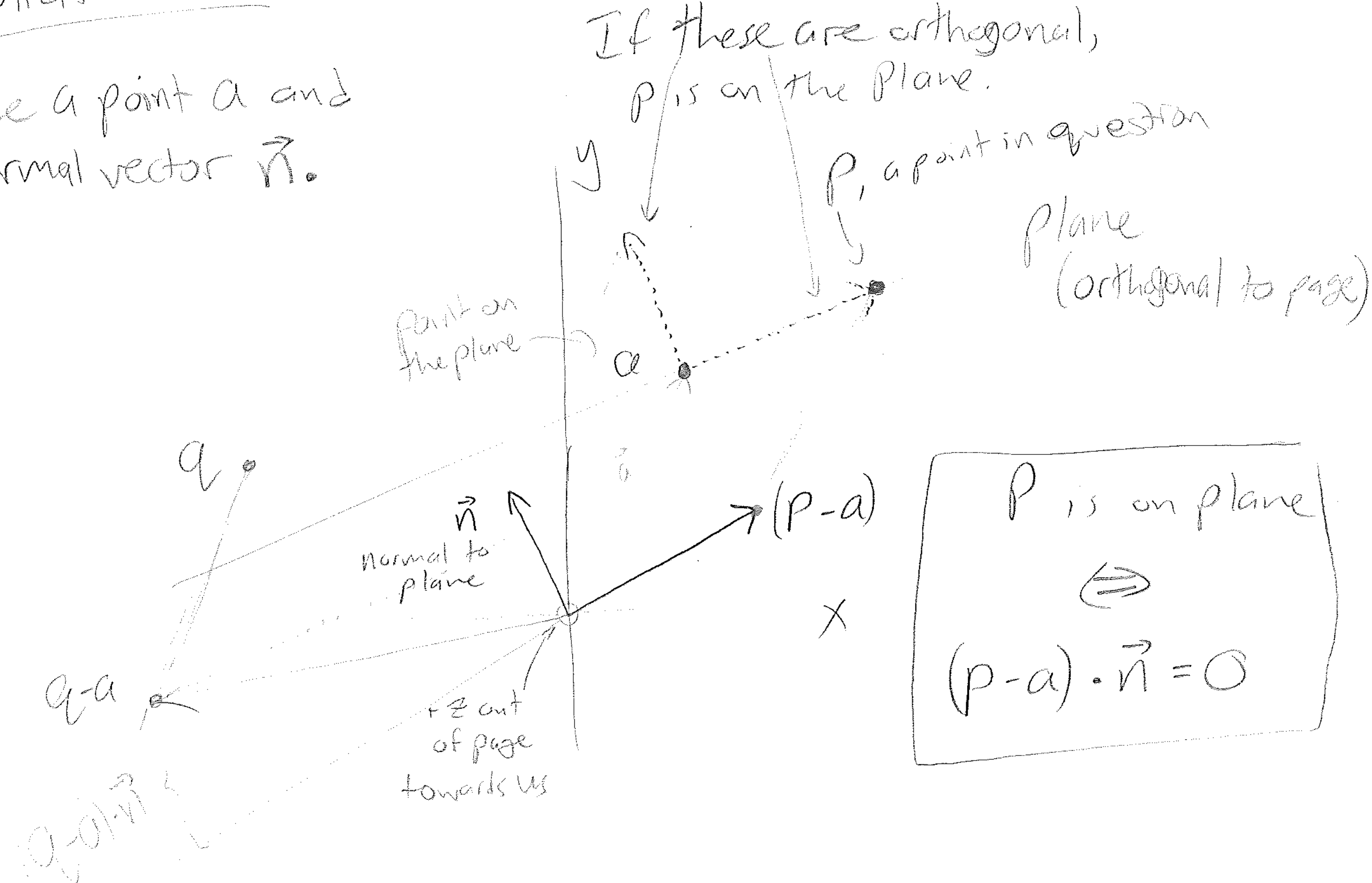
10.1

Implicit Planes

Use a point a and normal vector $\vec{n}$.



p is on plane $\iff$ $(p-a)\cdot\vec{n}=0$

Triangle's plane:

If we have triangle $a\ b\ c$, its normal is $\vec{n}=(b-a)\times(c-a)$, and a is a point on the plane.

Plugging into the above, a point x is on the tri's plane iff $(x-a)\cdot((b-a)\times(c-a))=0$.

To intersect with a ray $p+td$, substitute for x:

$$(p+td-a)\cdot((b-a)\times(c-a))=0.$$

This would give x, y, z coords in the plane. But is it inside the triangle?

Barycentric Coordinates

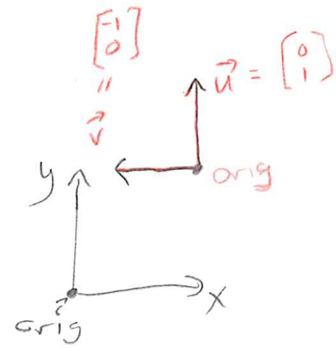
An elegant way to parameterize a plane wrt a triangle.

Coordinate Frames

Let's talk in 2D for now.

A coordinate frame is a coordinate system, made of an origin and a basis

↑ point ↑ axes



Canonical frame: origin $= (0,0,0)$

basis $= \begin{bmatrix}1\\0\end{bmatrix} \begin{bmatrix}0\\1\end{bmatrix}$

Define other frames in terms of this one:

origin in xyz, $\vec{u}$, $\vec{v}$ in xyz coordinates

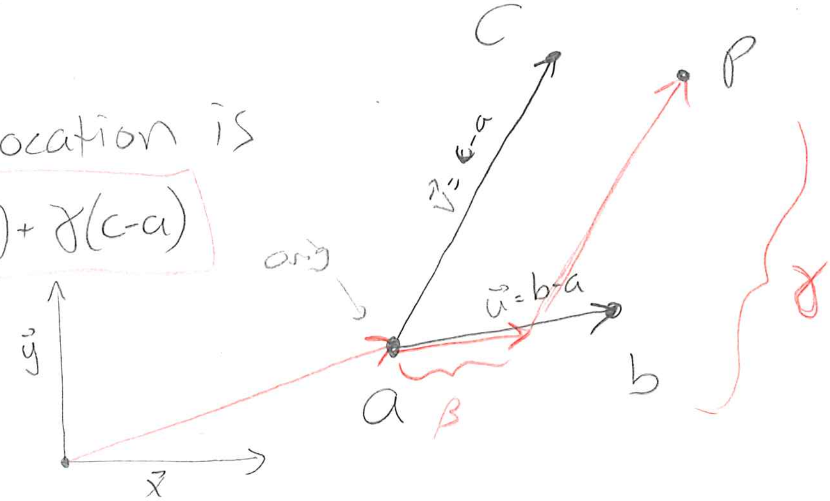
(this is like the camera frames from RT)

Barycentric Frame

Given a triangle, use one corner as origin and two sides as basis!?

A point $\vec{p}$'s location is

$$\boxed{a + \beta(b-a) + \gamma(c-a)}$$



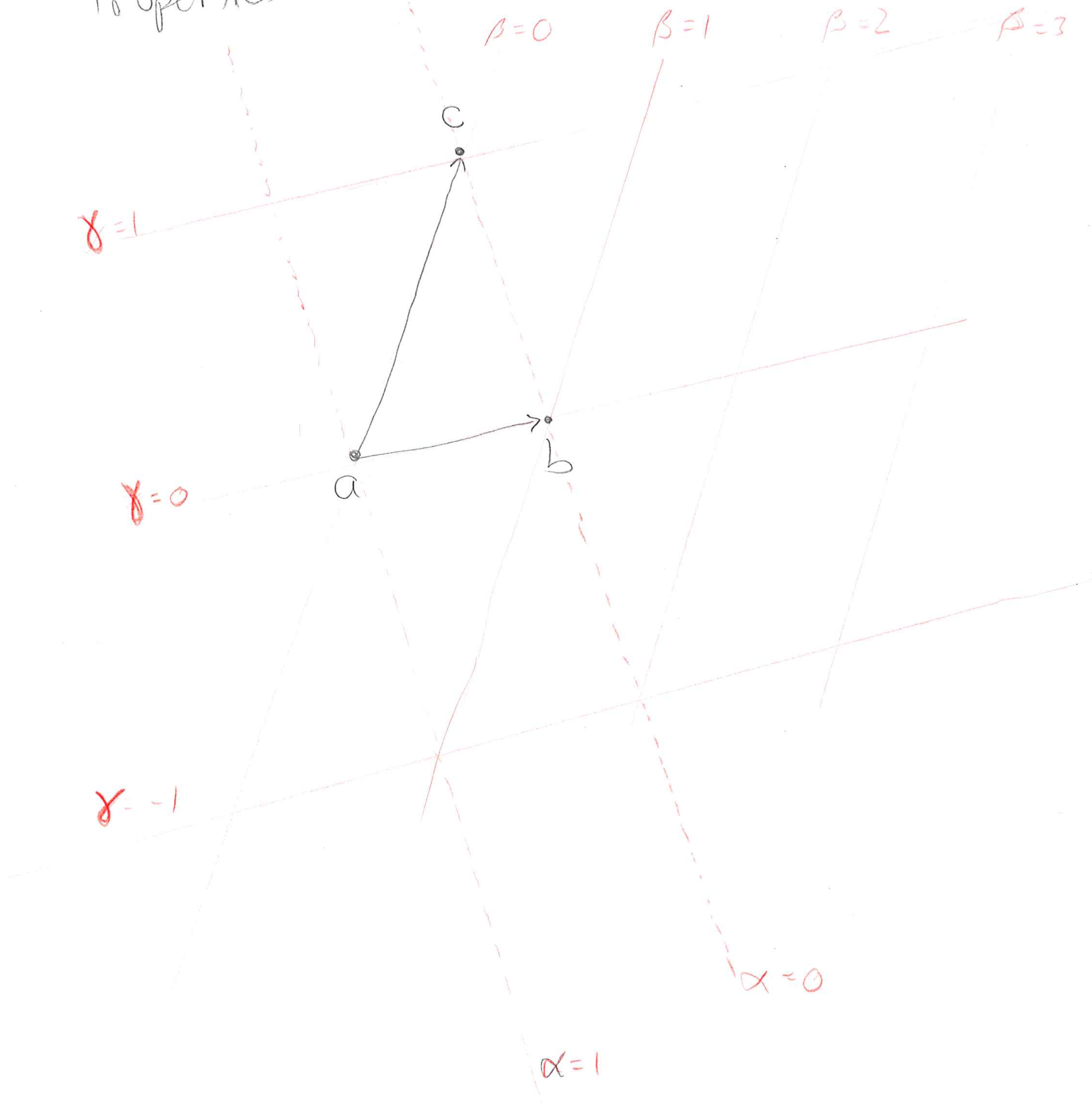
Small rewrite:

$$\begin{aligned} p &= a + \beta(b-a) + \gamma(c-a) \\ &= a - \beta a - \gamma a + \beta b + \gamma c \\ &= (1-\beta-\gamma)a + \beta b + \gamma c \end{aligned}$$

Let $\alpha = 1 - \beta - \gamma$

$$\boxed{p = \alpha a + \beta b + \gamma c}$$

Properties of Barycentric Coordinates



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Properties of Barycentric Coordinates

$p = \alpha a + \beta b + \gamma c$

- $\alpha + \beta + \gamma = 1$, anywhere in the plane.

- α, β, γ give scaled signed distances from edges

  $\alpha = 0$ on edge cd, 1 at a,

  $\beta = 0$ on edge ac, 1 at b

  $\gamma = 0$ on edge ab, 1 at c

- A point is inside the triangle abc iff

  $0 < \alpha < 1$

  $0 < \beta < 1$

  $0 < \gamma < 1$

  If we find α, β, γ, ray-tri intersection is solved!

- Coords are proportional to areas of sub-triangles (slide picture)

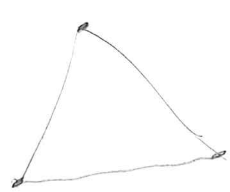
These make interpolation elegant: If I have some vertex data (e.g., color) at each vertex,

RGB_a, RGB_b, RGB_c

Interpolation of vertex data is solved!

The interpolated value is

$\alpha\, RGB_a + \beta\, RGB_b + \gamma\, RGB_c$!



Ray-Triangle Intersection - see slides

Plane: $p(\beta,\gamma) = a + \beta(b-a) + \gamma(c-a)$

Ray: $r(t) = p + td$

Intersect - set equal $p(\beta,\gamma) = r(t)$

$$p + td = a + \beta(b-a) + \gamma(c-a)$$

3D vectors. → 3 eqns and 3 unknowns