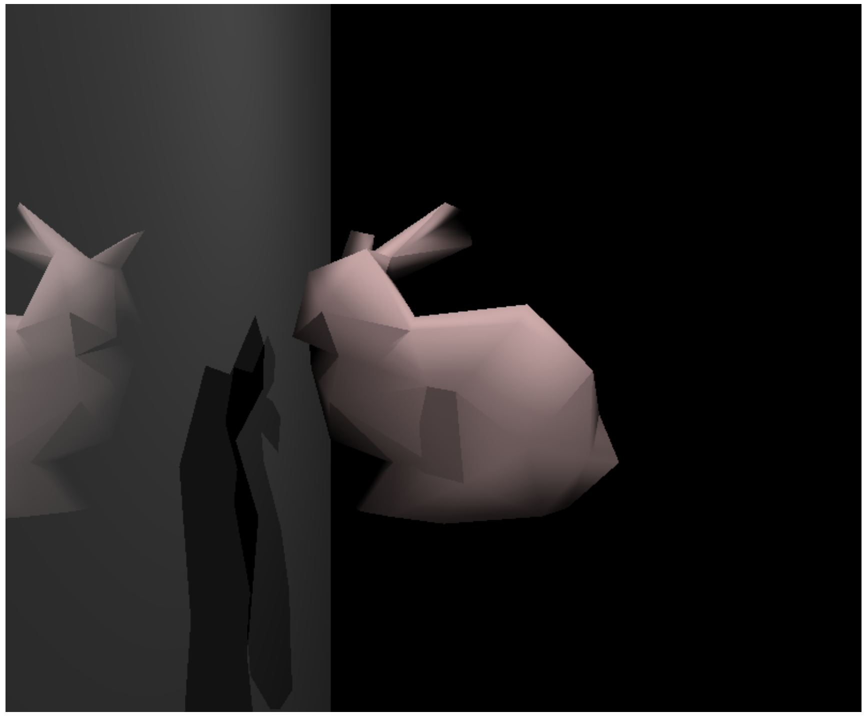
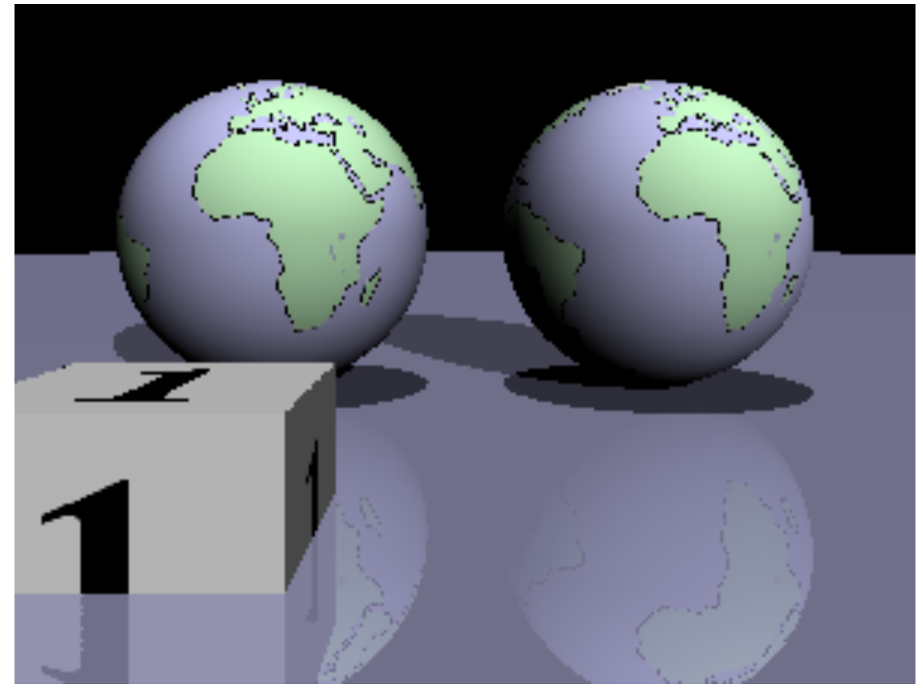
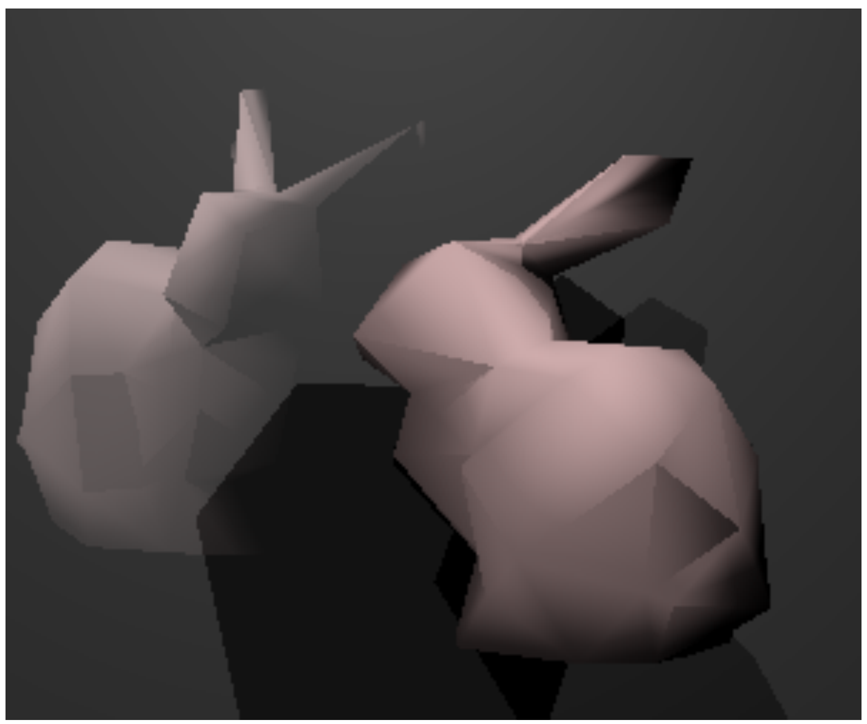
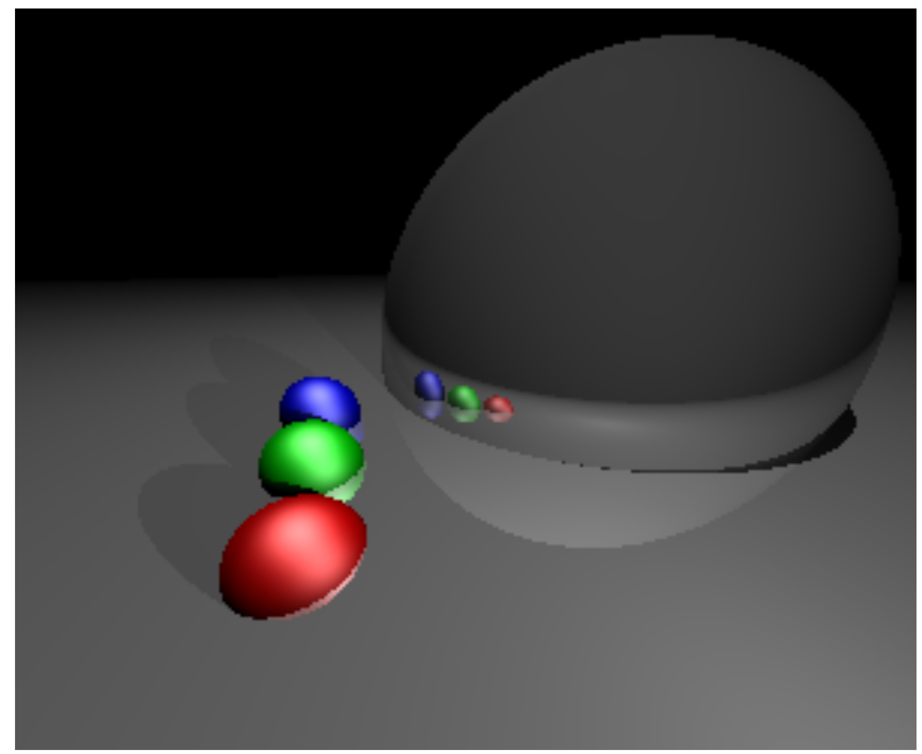





Computer Graphics

Lecture 10

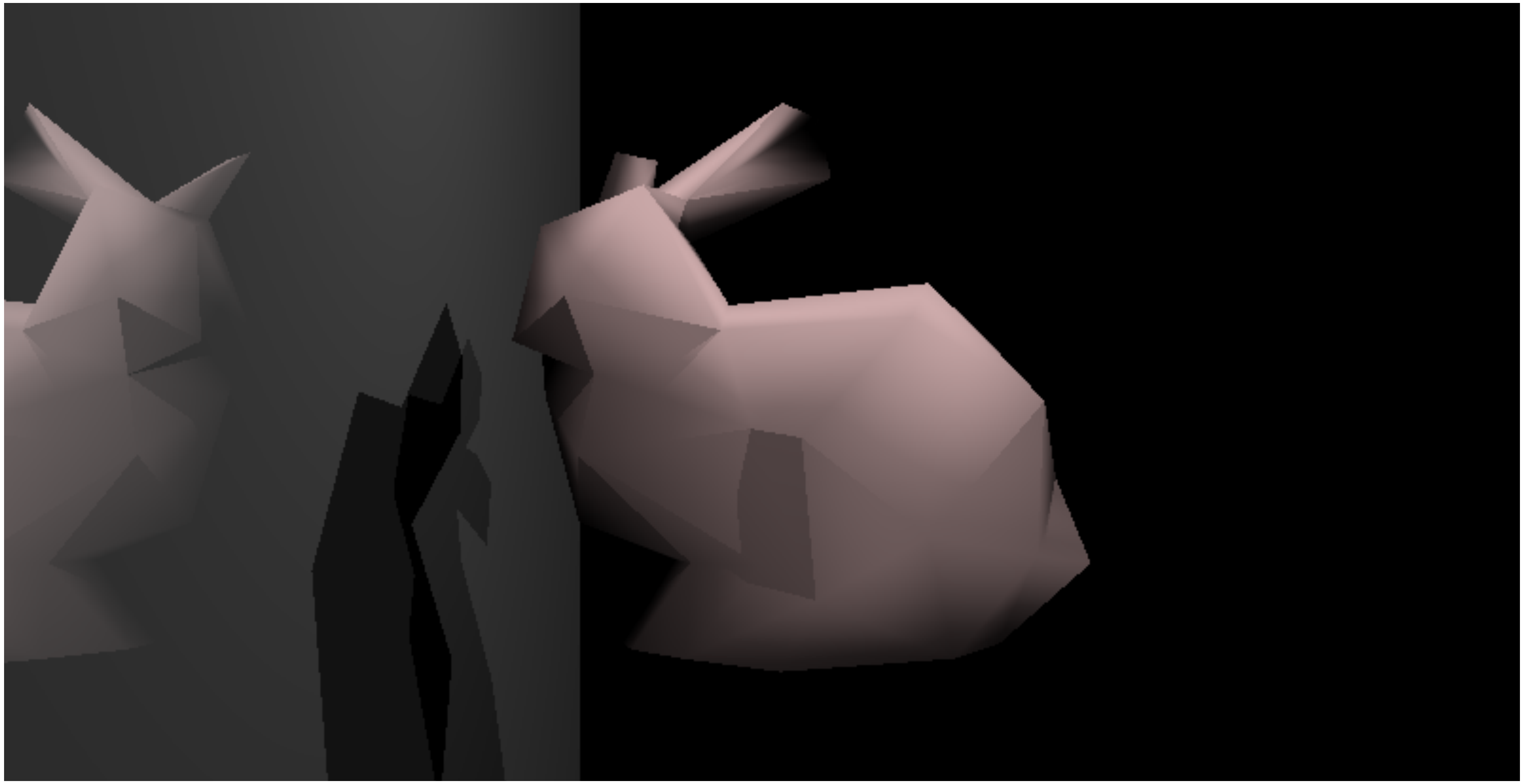
Barycentric Coordinates

Ray-Triangle Intersection

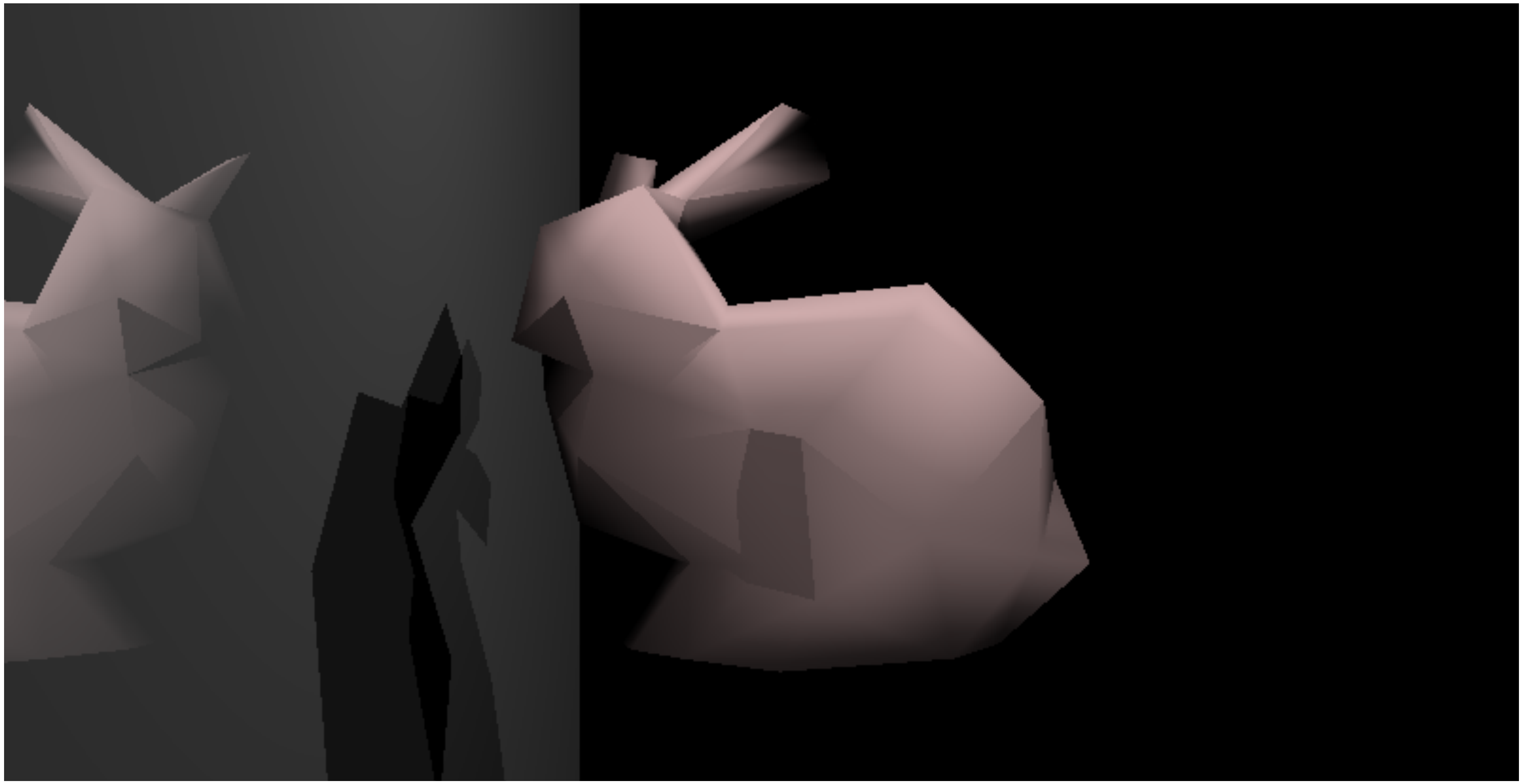
Announcements

Announcements

Let's talk about bunnies.

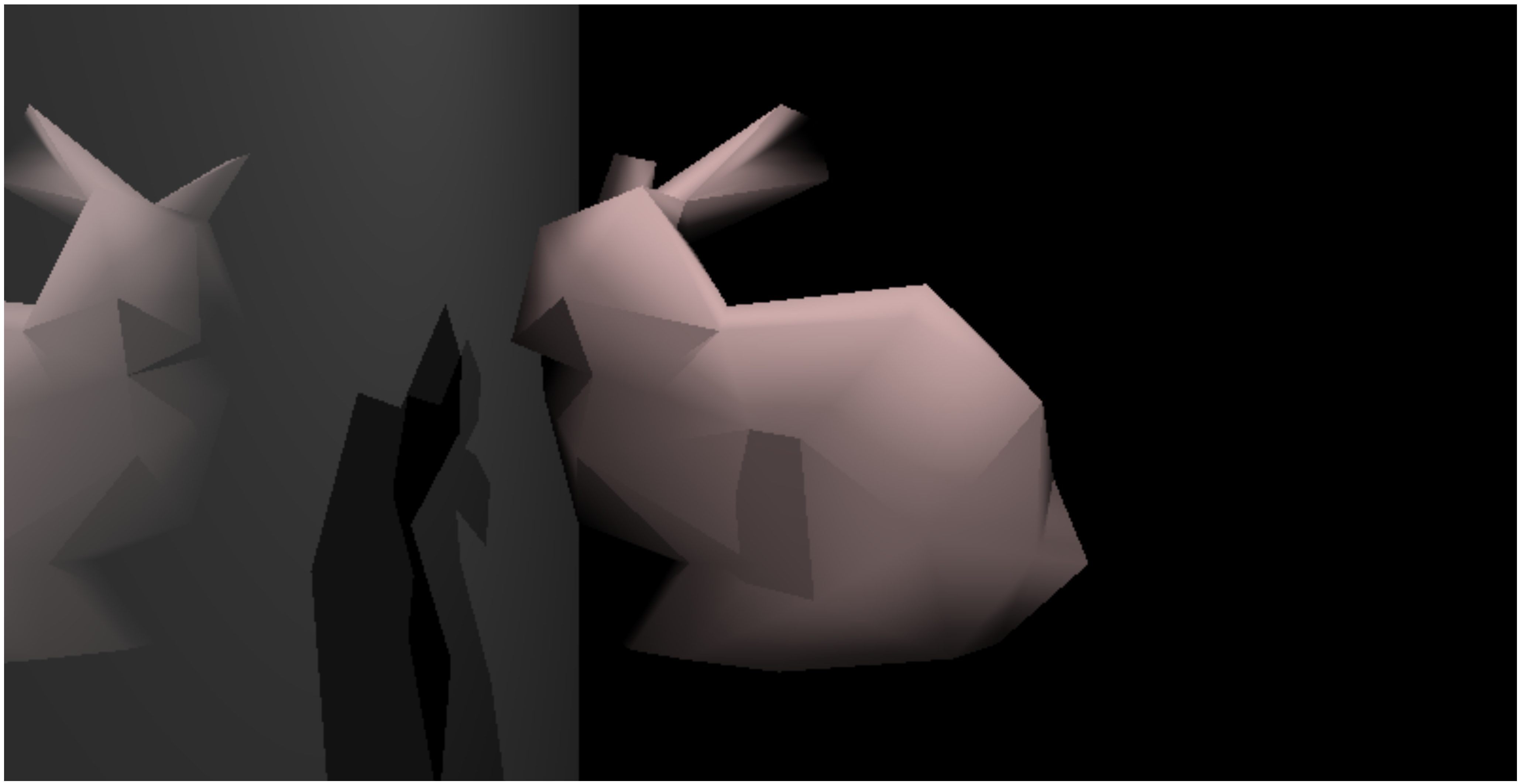


Let's talk about bunnies.



If we want bunnies, we still need to implement

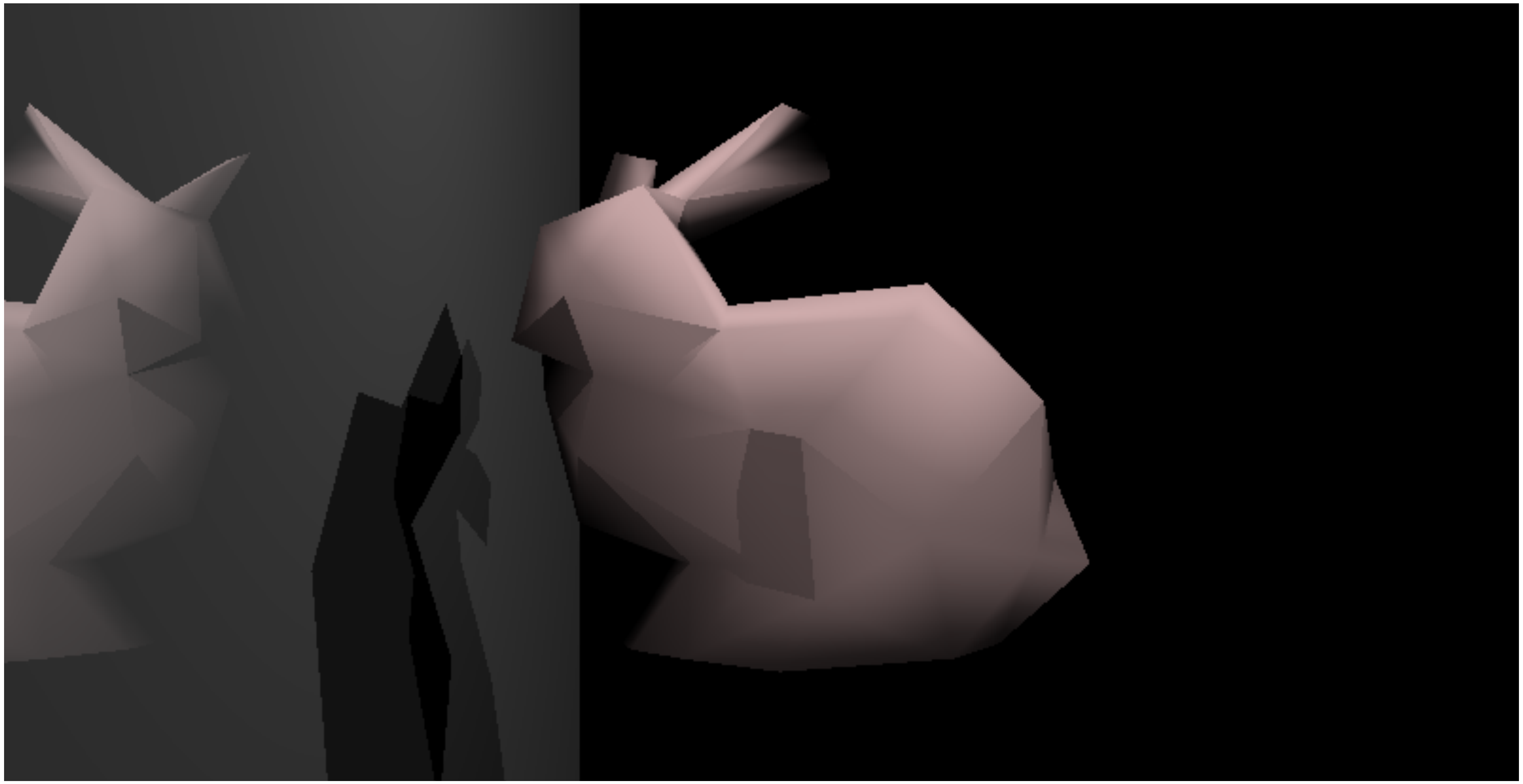
Let's talk about bunnies.



If we want bunnies, we still need to implement

```
function ray_intersect(ray, triangle, tmin, tmax):
```

Let's talk about bunnies.



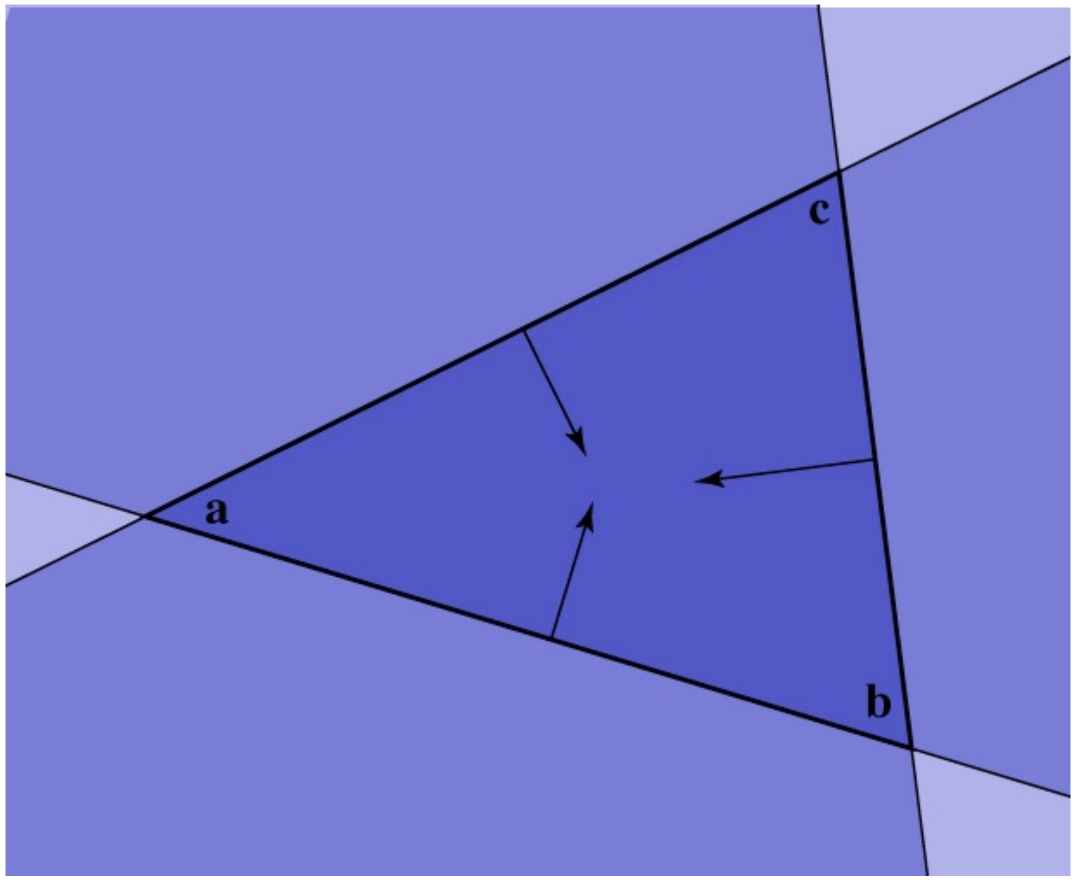
If we want bunnies, we still need to implement

```
function ray_intersect(ray, triangle, tmin, tmax):
```

Then, we can treat a triangle mesh as simply a list of triangles.

Let's talk about triangles.

A triangle is the intersection of three half-planes

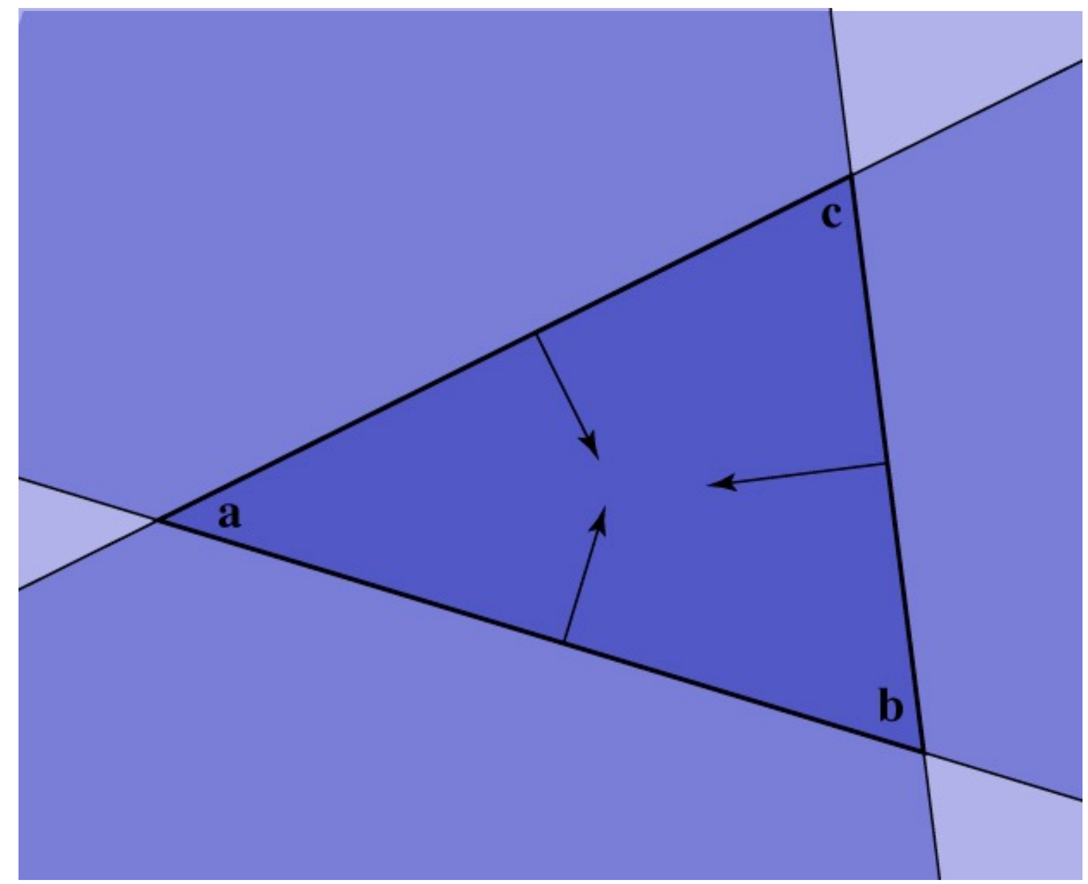


High-level approach:

1. Intersect with the plane
2. Check if intersection is inside the triangle

Let's talk about triangles.

A triangle is the intersection of three half-planes



High-level approach:

1. Intersect with the plane
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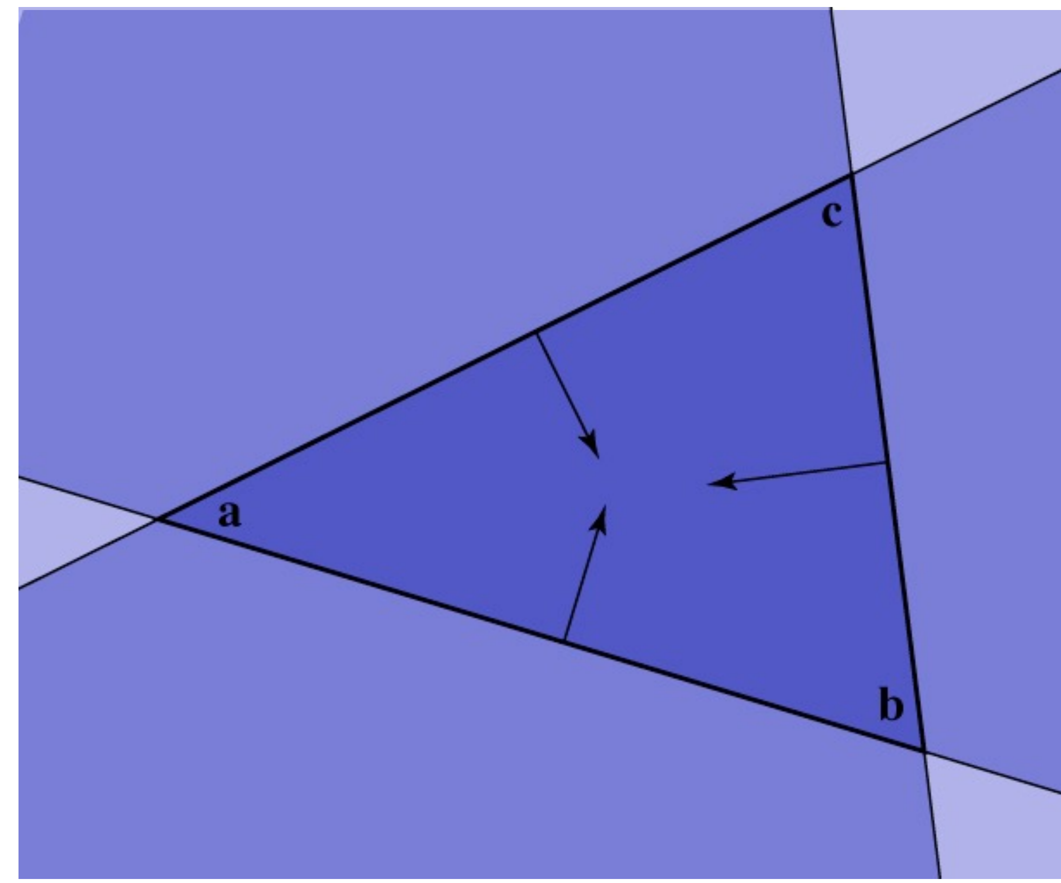


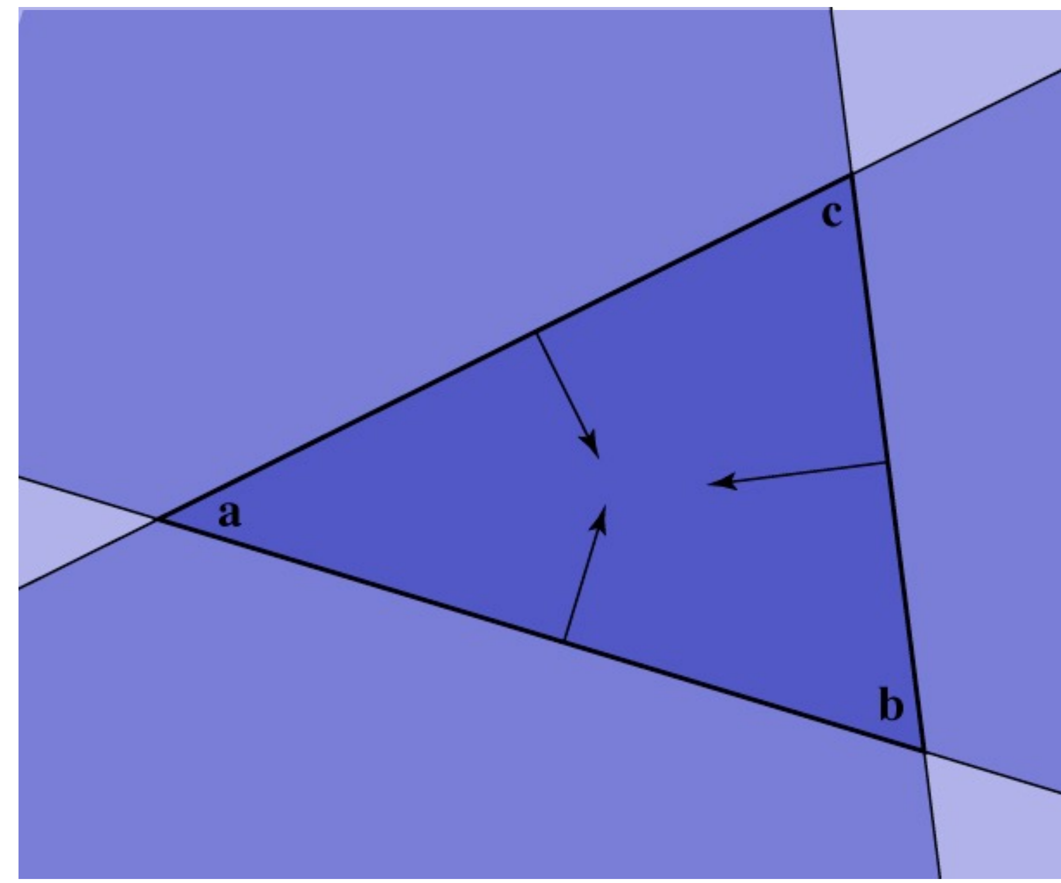
High-level approach:

1. Intersect with the plane
2. **Check if intersection is inside the triangle**

To make this easy, we'll introduce the
weirdest coordinate system you've ever seen.

Let's talk about triangles.

A triangle is the intersection of three half-planes



High-level approach:

1. Intersect with the plane
2. **Check if intersection is inside the triangle**

To make this easy, we'll introduce the
weirdest coordinate system you've ever seen.

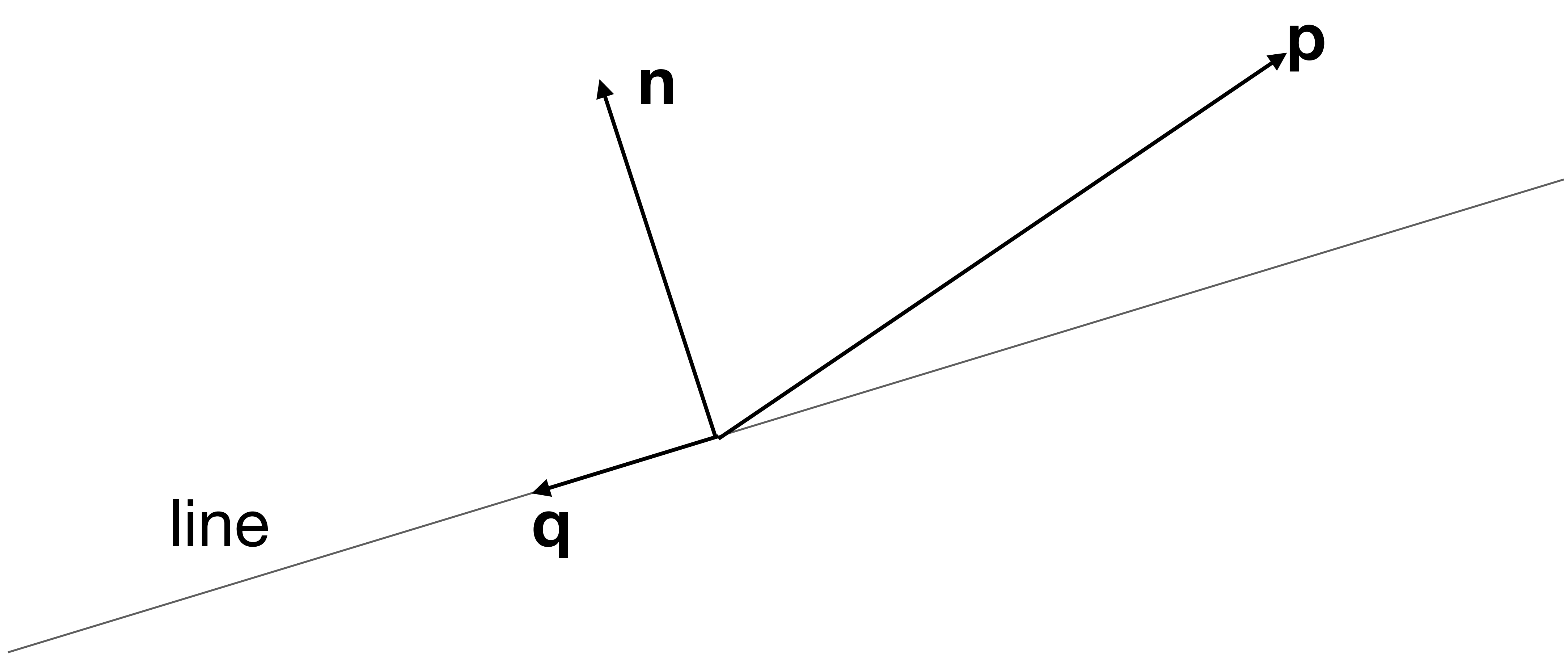
As a bonus, we'll get interpolation of vertex data for free!

Roadmap for today

- Implicit equation for a plane
- Barycentric coordinates
- Finding barycentric coordinates at a ray-plane intersection.

Implicit Planes: Intuition

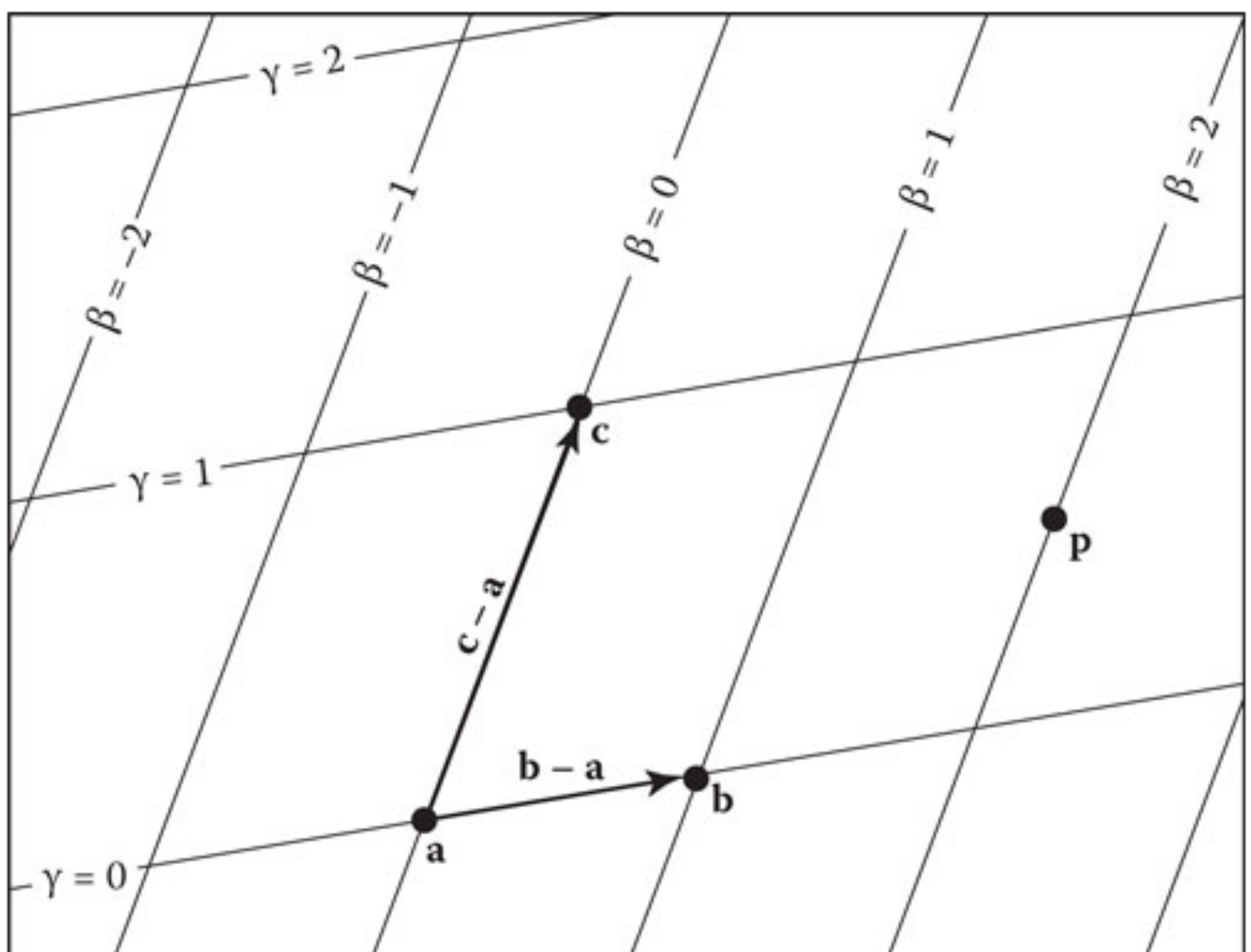
What is true of points on this line,
in terms of its normal vector, **n**?



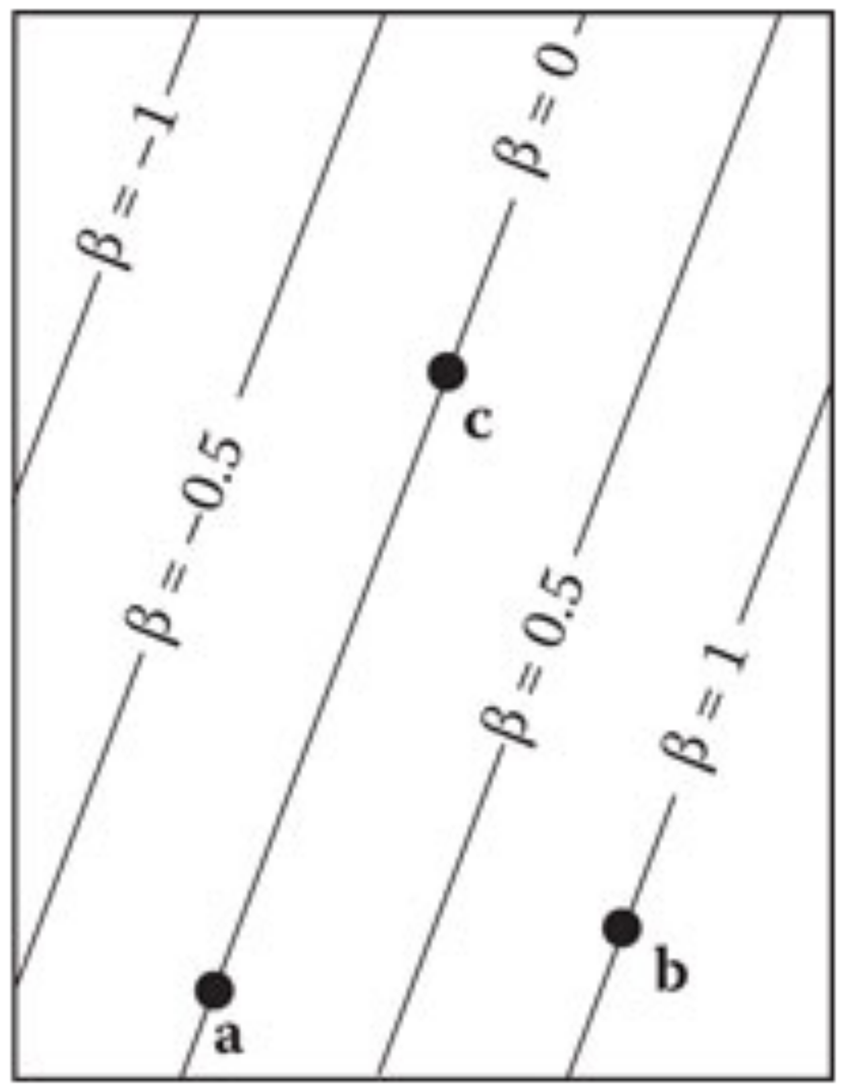
Barycentric Coordinates

A purpose-built coordinate system for talking about points in a specific triangle's plane.

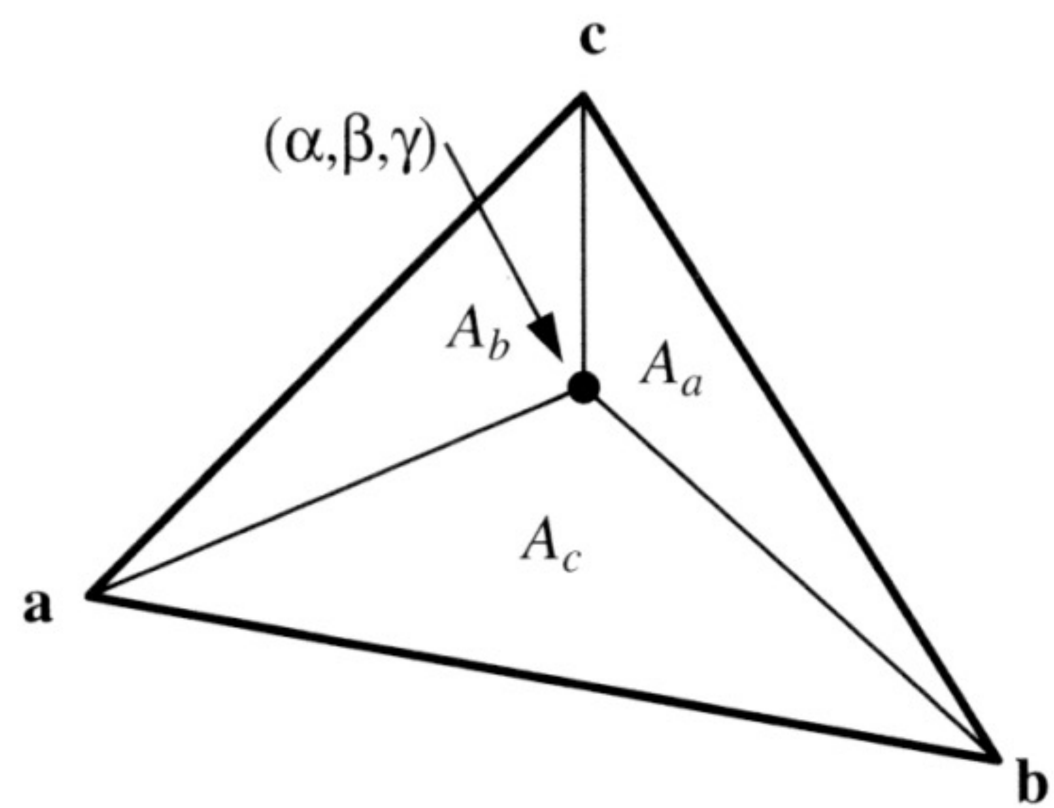
$$\mathbf{p} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$



Properties of Barycentric Coordinates



- Coordinates are proportional to area of subtriangles:



Barycentric ray-triangle intersection

- Every point on the plane can be written in the form:

  $$\mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

  for some numbers β and γ.
- If the point is also on the ray then it is

  $$\mathbf{p} + t\mathbf{d}$$

  for some number t.
- Set them equal: 3 linear equations in 3 variables

  $$\mathbf{p} + t\mathbf{d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

  …solve them to get t, β, and γ all at once!

Barycentric ray-triangle intersection

$$\mathbf{p} + t\mathbf{d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

$$\beta(\mathbf{a} - \mathbf{b}) + \gamma(\mathbf{a} - \mathbf{c}) + t\mathbf{d} = \mathbf{a} - \mathbf{p}$$

$$\begin{bmatrix}\mathbf{a} - \mathbf{b} & \mathbf{a} - \mathbf{c} & \mathbf{d}\end{bmatrix}\begin{bmatrix}\beta \\ \gamma \\ t\end{bmatrix} = \begin{bmatrix}\mathbf{a} - \mathbf{p}\end{bmatrix}$$

$$\begin{bmatrix}x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d\end{bmatrix}\begin{bmatrix}\beta \\ \gamma \\ t\end{bmatrix} = \begin{bmatrix}x_a - x_p \\ y_a - y_p \\ z_a - z_p\end{bmatrix}$$

- This is a linear system: Ax = b
- Various ways to solve, but a fast one uses *Cramer's rule*.
- See 4.4.2 for the TL;DR formula
- See 5.3.2 for an explanation of Cramer's rule