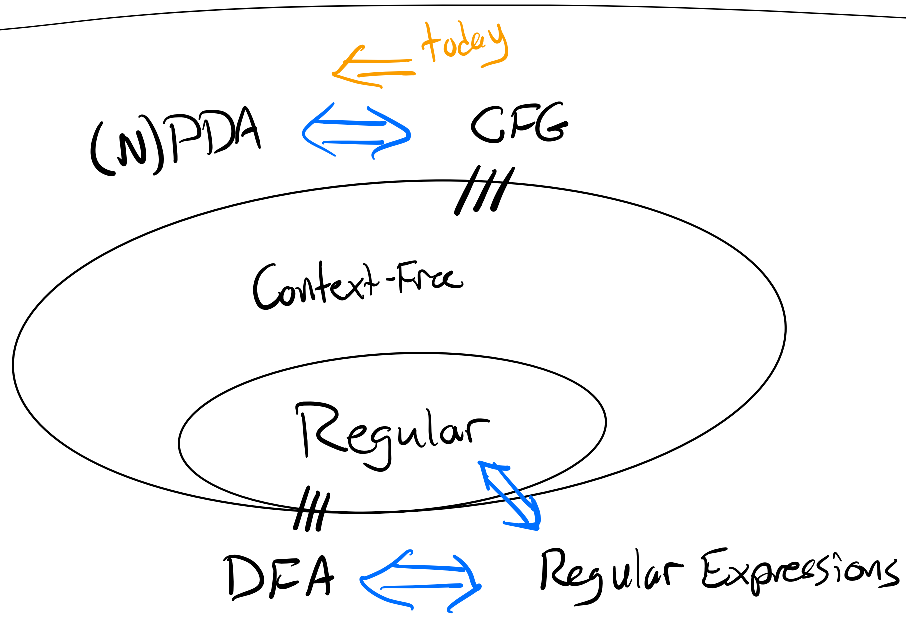
CSCI 301 - Lecture 29: Chomsky Normal Form
Equivalence of PDA and CFG



Theorem: A PDA accepting A exists if and only if a CFG describes A.

PDA $\Leftrightarrow$ CFG

Proof approach:

PDA $\Leftarrow$ CFG 1. Suppose a CFG describes A, and construct a PDA accepting A.

PDA $\Rightarrow$ CFG 2. Suppose a PDA accepts A, and construct a CFG describing A.

Our task: given a CFG, construct a PDA. This sounds... Challenging

A worthwhile investment: Chomsky Normal Form

A CNF grammar is a CFG that only has rules like:

1. $A \to BC$ $\quad B, C \neq S$
2. $A \to a$
3. $S \to \varepsilon$

Theorem: Any Context-Free Grammar can be converted to CNF.

Proof (sketch) by construction:

The conversion can be done in these 5 steps:

1. Eliminate the start symbol from the RHS $\quad S_0 \to S$
2. Eliminate ε-rules $\quad A \to \varepsilon$
3. Eliminate unit rules $\quad A \to B$
4. Eliminate rules with more than 2 RHS symbols $\quad A \to BCD \quad A \to BN,\ N \to CD$
5. Eliminate rules with two symbols that aren't both variables $\quad A \to bC \quad A \to bc$

Example:

$S_0 \to AM \mid SA \mid AS \mid NB \mid a$

$S \to AM \mid SA \mid AS \mid NB \mid a$

$A \to AM \mid SA \mid AS \mid NB \mid a \mid b$

$B \to b$

$M \to SA$

$N \to a$

1. Start Symbol $A \to S$
2. ε-rules $A \to \varepsilon$
3. Unit rules $A \to B$
4. >2 rules $A \to BCD$
5. non-variable pairs $A \to bC$, $A \to bc$

$\boxed{\text{Ex. Part A}}$

S_0

AS

bS

bAS

bbS

bba

S_0

AS

bS

bAS

bbS

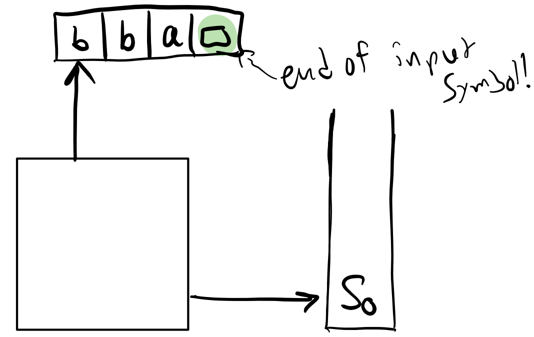
bba

Stack

tape

Given a CFG, construct a PDA.

First, Convert grammar to CNF.



Construct $M = (Q, \Sigma, \Gamma, \delta, q)$ as follows:

- $Q = \{q\}$
- $\Sigma_{PDA} = \Sigma_G$
- $\Gamma = V_G$
- $q = q$
- Construct δ as follows:

  For each rule $A \to BC$,

  $$q * A \to q\, N\, BC$$

  For each rule $A \to b$

  $$q * A \to q\, R\, \varepsilon$$

  If $S_0 \to \varepsilon$, add (Ex. P+B)

  $$q \square S_0 \to q\, N\, \varepsilon$$

Aside: δ rule notation:

state tape stack state move stack string

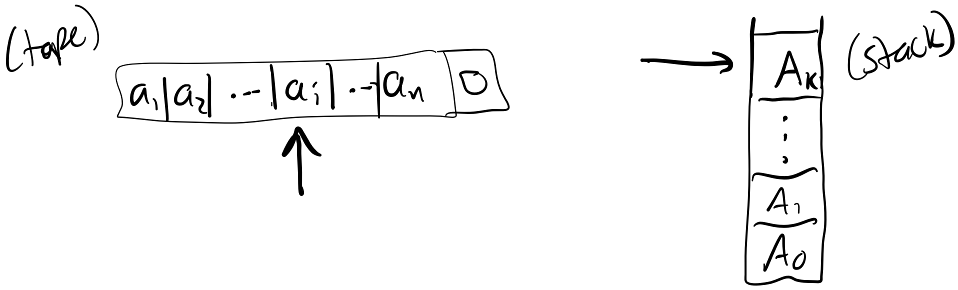
$$q a \$ \to q\, R\, \underline{SS}$$

$$q * \$ \to q\, R\, SS$$

Formally: Prove that M accepts w iff $S_0 \Rightarrow^* w$.

Claim: Let $a_1 a_2 \cdots a_n \in \Sigma^*$ and $A_1 A_2 \cdots A_k \in V^*$,
and let i, k be integers with $1 \le i \le n+1$
and $k \ge 0$.

Then $S_0 \Rightarrow^* a_1 a_2 \cdots a_{i-1} A_k A_{k-1} \cdots A_0$
if and only if M can go from its initial configuration to a state where:



By induction:

Base - Starting configuration

Inductive step: Suppose claim is true for some $k \ge 1$
and show that δ maintains the claim for $A \to BC$ rules and for $A \to b$ rules.