CSCI 301 - L22:

- NFA → DFA
- Regular Closure
- Regular Expressions

<u>Definition:</u> The ε-closure of a state s in Q, written $C_\varepsilon(s)$, is the set of states reachable from s after 0 or more ε-transitions.

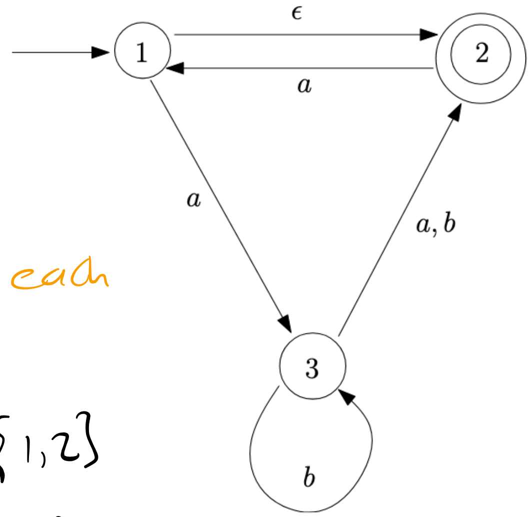
Consider this NFA: $N = (Q, \Sigma, q, \delta, F)$

$Q =$ ______

$\Sigma =$ ______

$q =$ ______

$F =$ ______



$\delta =$

	a	b	ϵ
1	$\{3\}$	$\emptyset$	$\{2\}$
2	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2,3\}$	$\emptyset$

Ex.: Find the ε-closure of each state in Q:

$C_\varepsilon(1) = \{1,2\}$

$C_\varepsilon(2) = \{2\}$

$C_\varepsilon(3) = \{3\}$

Strategy for running a machine with ε-transitions:

0. Start in state q
1. Make 0 or more ε-transitions

Let $q' = C_\varepsilon(q)$

2. Read a symbol
3. Make 0 or more ε-transitions

one application of δ'

4. Read a symbol

⋮

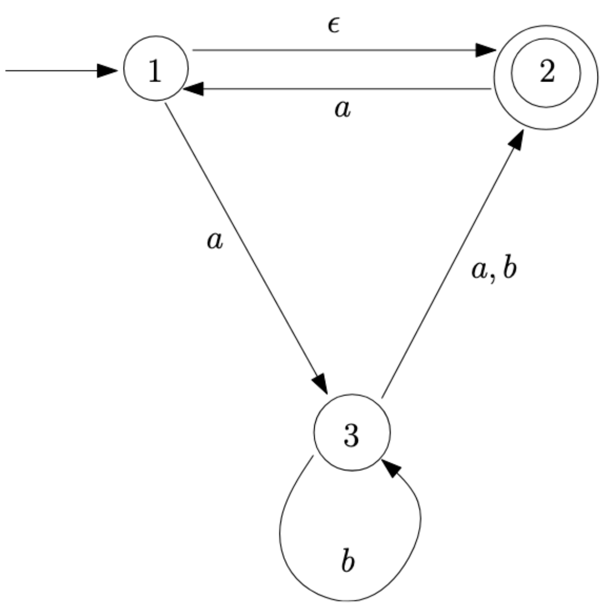
$$\delta'(R, a) = \bigcup_{r \in R} C_\varepsilon(\delta(r, a))$$

$Q' =$ ______

$q' =$ ______

δ':

State	a	b



$F' =$ ______

$C_\varepsilon(1) = \{1, 2\}$

$C_\varepsilon(2) = \{2\}$

$C_\varepsilon(3) = \{3\}$

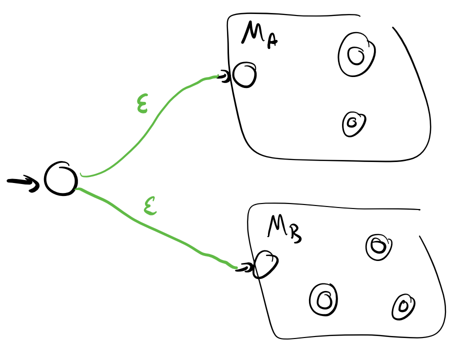
Regular Closure (Finally!)

Prop.: Regular languages are closed under union.

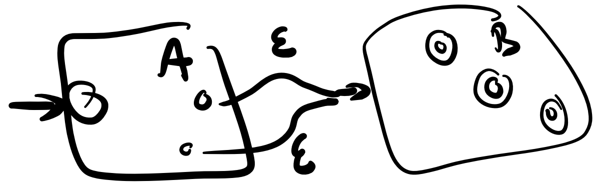
Proof: Let L_A and L_B be regular languages over Σ.
There exist DFA's M_A and M_B with $L(M_A) = L_A$
and $L(M_B) = L_B$.

Construct an NFA N as follows: (sketch)

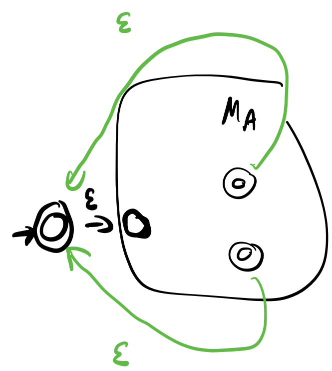
Union: $A \cup B$



Concatenation: AB



Star: A^*



Ex.: Construct an NFA for $L_A L_B$ (concatenation)
and L_A^* (star)

Regular Expressions: A meta-language for regular languages.

Definition: A regular expression over alphabet Σ is defined inductively (recursively) as follows:

Regular Expression	Language described
• ε is a R.E.	$\{\varepsilon\}$
• $\emptyset$ is a R.E.	$\emptyset$
• a is a R.E for all $a \in \Sigma$	$\{a\}$
• If R_1, R_2 are R.E.'s,	
- $R_1 \cup R_2$ is a R.E.	$R_1 \cup R_2$
- $R_1 R_2$ is a R.E.	$R_1 R_2$
• If R is a R.E., R^* is a R.E.	R^*

Examples: Let $\Sigma = \{0, 1\}$

$R = (0 \cup 1)01^*$

$L(R) = \{00, 001, 0011, \ldots, 10, 101, 1011, \ldots\}$

$L(R) = \{w : w \text{ has } 1011 \text{ as a substring}\}$

$R = (0 \cup 1)^* 1011 (0 \cup 1)^*$

Regex Operator Precedence

1. Parens for grouping
2. $*$
3. Concatenation
4. Union

- $01^* = 0(1^*)$
- $01 \cup 0 = (01) \cup 0$
- $1 \cup 0^* = 1 \cup (0^*)$

Ex. Part C

$ab = \{ab\}$

$a \cup b \cup \varepsilon = \{a, b, \varepsilon\}$

$(ab \cup a)b = \{abb, ab\}$

$ab^* = \{a, ab, abb, abbb, \dots\}$