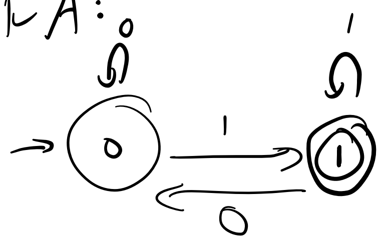
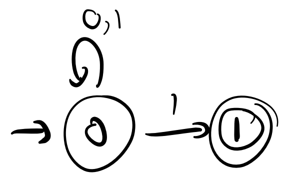
CSCI 301 - Lecture 21: NFA → DFA

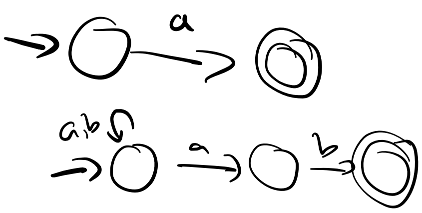
$\Sigma = \{0,1\}$ $\quad L = \{w : w \text{ ends with } 1\}$

DFA:



NFA:





Ex. A: NFA Design warmup

Plot twist / theorem:

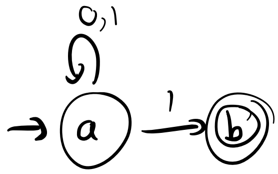
Given an NFA N, there exists a DFA M such that $L(N) = L(M)$

→ NFA's are no more powerful than DFA's!

Proof: By construction.

Suppose $N = (Q, \Sigma, \delta, q, F)$ is an NFA.

Construct a DFA $M = (Q', \Sigma, \delta', q', F')$.

Small Example: 

$\delta: Q \to \mathcal{P}(Q)$

	0	1
a	$\{a\}$	$\{a,b\}$
b	$\emptyset$	$\emptyset$

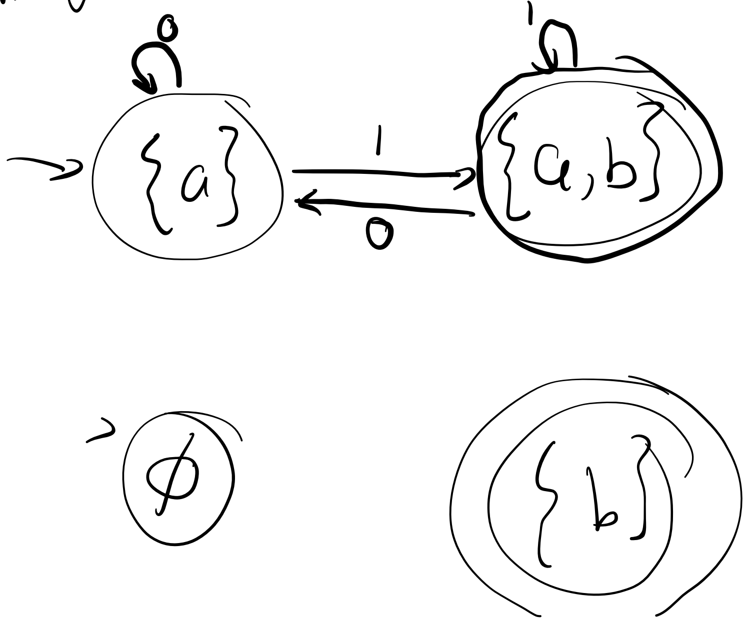
Idea: run all paths through the machine in parallel.

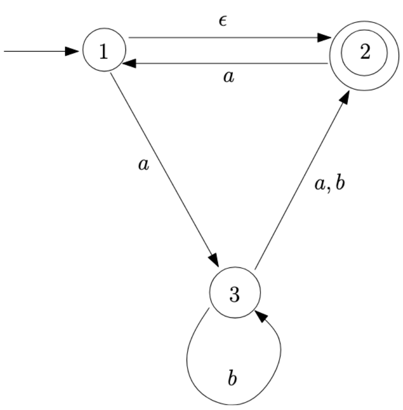
$Q' = \mathcal{P}(Q)$

$\in \Sigma$ $\in \Sigma$

$$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$$

$\in \mathcal{P}(Q) = Q'$ One NFA State





What about ε-transitions?

$\boxed{\text{Ex. 3: NFA Conversion Warmup}}$

Definition: The ε-closure of a state s in Q, written $C_\varepsilon(s)$, is the set of states reachable from s after 0 or more ε-transitions.

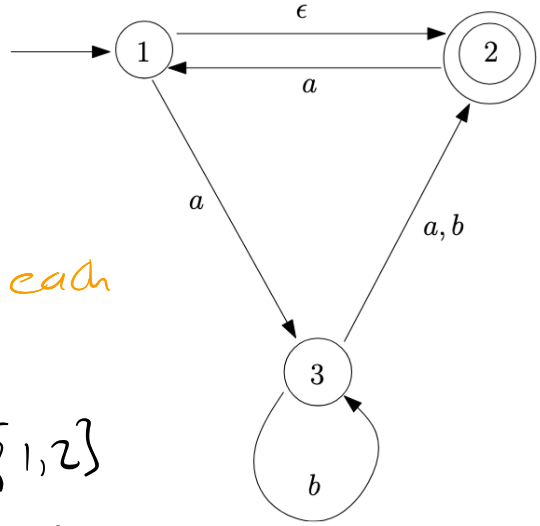
Consider this NFA: $N = (Q, \Sigma, q, \delta, F)$

$Q =$ ______

$\Sigma =$ ______

$q =$ ______

$F =$ ______



Ex.: Find the ε-closure of each state in Q:

$C_\varepsilon(1) = \{1, 2\}$

$C_\varepsilon(2) = \{2\}$

$C_\varepsilon(3) = \{3\}$

$\delta =$

	a	b	ϵ
1	$\{3\}$	$\emptyset$	$\{2\}$
2	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2,3\}$	$\emptyset$

Strategy for running a machine with ε-transitions:

0. Start in state q
1. Make 0 or more ε-transitions

Let $q' = C_\varepsilon(q)$

2. Read a symbol
3. Make 0 or more ε-transitions

one application of δ'

4. Read a symbol

⋮

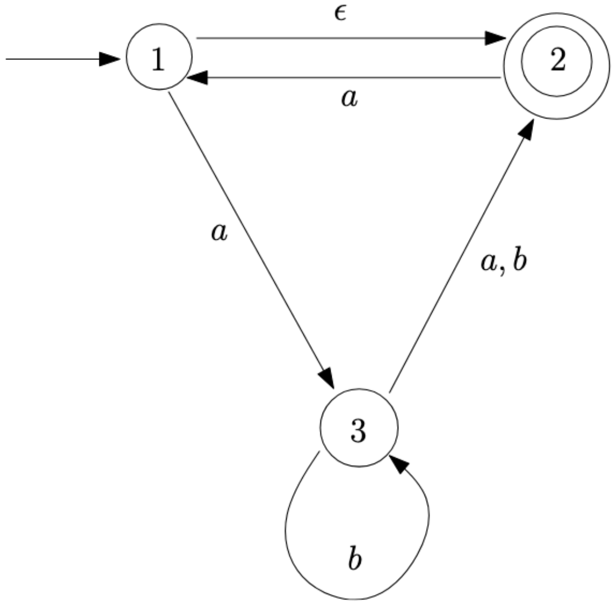
$$\delta'(R, a) = \bigcup_{r \in R} C_\varepsilon(\delta(r, a))$$

$Q' =$ ______

$q' =$ ______

δ':

State	a	b



Ex. C: complete δ' table and determine F'!

$F' =$ ______

Regular Operations

- The **union** of two languages A and B is defined as $A \cup B = \{w : w \in A \text{ or } w \in B\}$.
- The **concatenation** or **product** of two languages A and B is defined as $AB = \{ww' : w \in A \text{ and } w' \in B\}$.
- The **closure** or **star** (or Kleene closure) of a language A is defined as: $A^* = \{u_1 u_2 \dots u_k : k \geq 0 \text{ and } u_i \in A \text{ for all } i = 1, 2, \dots, k\}$

Union: