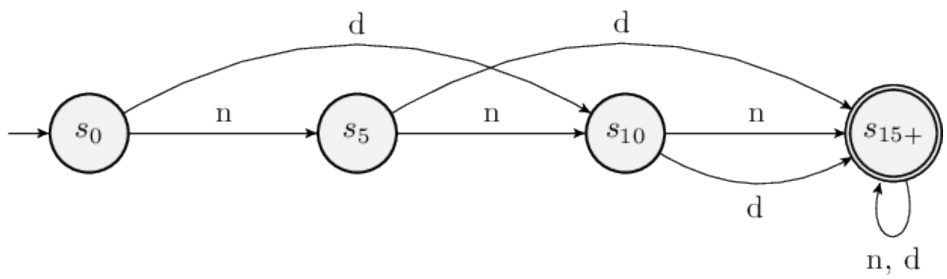
# CSCE 301 - Lecture 22: Languages, Strings, and DFAs

## Computability and complexity

What problems can computers solve, and with how much difficulty?

- "computer" model: automata




- "Problem" model: language acceptance

# Languages and associated definitions

- An alphabet is a finite set whose members are called symbols.

  Examples: $\Sigma = \{0, 1\}$, $\Sigma = \{a, b, c, \ldots, z\}$

- A string $w$ over an alphabet $\Sigma$ is a finite (ordered) sequence of symbols from $\Sigma$.

  $\Sigma = \{0, 1\}$

  strings over $\Sigma$

  | |
  |---|
  | 1 |
  | 100 |
  | 11110 |

- The length of a string $w$, written $|w|$, is the number of symbols in $w$.

  $|1001| = 4$

- The empty string, written $\varepsilon$ or $\epsilon$, is the string with no symbols. Its length is 0.

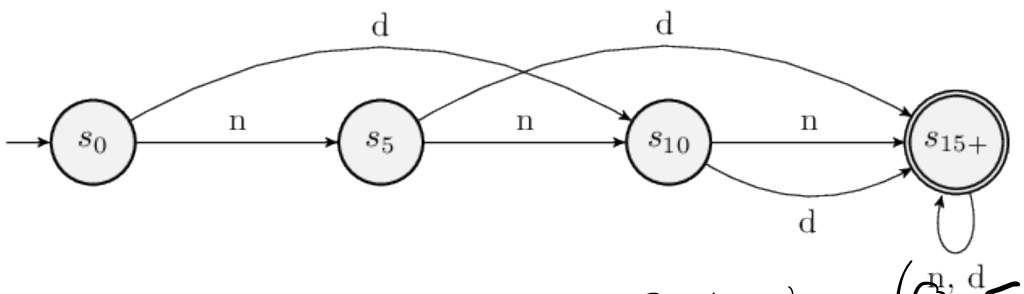
- A language over an alphabet $\Sigma$ is a set of strings over $\Sigma$. $L = \{1, 0, 01, 10\}$ is a language over $\Sigma = \{1, 0\}$.

- The set of all strings over $\Sigma$ is written $\Sigma^*$.

  So $L$ is a language over $\Sigma$ if $L \subseteq \Sigma^*$.

Do Ex. A

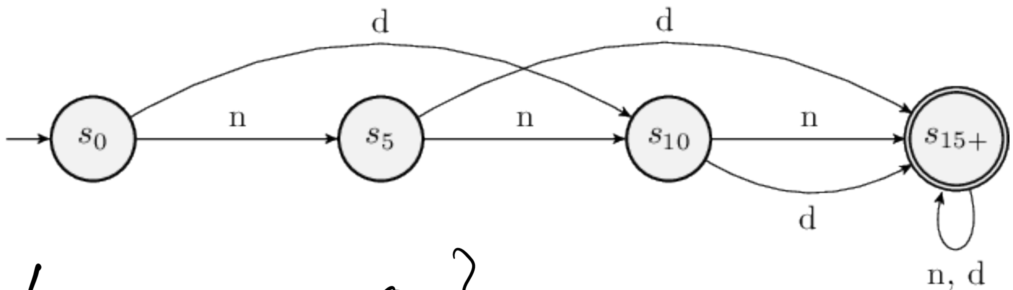
$\Sigma = \{0, 1\}$ $A = \{a, b\}$




A **finite automaton** is a 5-tuple: $(Q, \Sigma, \delta, q, F)$

1. $Q$ is a finite set whose elements are **states**
2. $\Sigma$ is an **alphabet**, whose elements are **symbols**
3. $\delta$ is a function $\delta : Q \times \Sigma \to Q$, called the **transition function**
4. $q$ is the **start state**
5. $F$ is a subset of $Q$ whose members are called **accept states**.

Ex: Write the formal definition of our toll machine.




$Q = \{S_0, S_5, S_{10}, S_{15+}\}$

$\Sigma = \{n, d\}$

$q = S_0$

$F = \{S_{15+}\}$

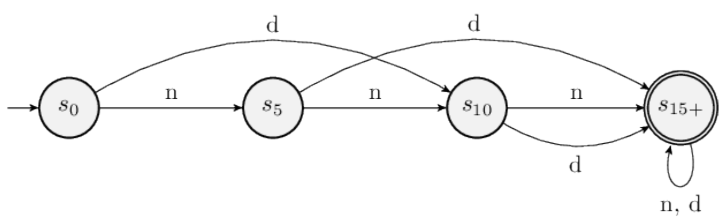
$\delta$:

| | n | d |
|---|---|---|
| $S_0$ | $S_5$ | $S_{10}$ |
| $S_5$ | $S_{10}$ | $S_{15+}$ |
| $S_{10}$ | | |
| $S_{15+}$ | | |

A FA is deterministic if $\delta$ is a function, i.e. $\delta$ always maps $q \in Q$, $s \in \Sigma$ to a single $q'$. Otherwise it's nondeterministic

## The Language Accepted by a DFA

What strings put this machine in an accept state?




nd nnn
dn nnd
dd nnnn

**Definition:** Let $M = (Q, \Sigma, \delta, q, F)$ be a finite automaton and let $w = w_1 w_2 w_3 \dots w_n$ be a string over $\Sigma$.

Define a sequence of states $r_0, r_1, r_2, \dots r_n$ as:

- $r_0 = q$ (start state)
- $r_{i+1} = \delta(r_i, w_{i+1})$ For $i = 1, 2, \dots, n-1$
- if $r_n \in F$, then $M$ accepts $w$.
- if $r_n \notin F$, then $M$ reject (does not accept) $w$

**Definition:** The language accepted by a machine $M$ is the set of all strings accepted by $M$.

$$L(M) = \{ w : w \text{ is a string over } \Sigma \text{ and } M \text{ accepts } w \}$$

Do Ex. B