CSCI 301 - Lecture 15: Relations, Functions

	$<$	$\leq$	$=$	$\mid$	$\nmid$	$\neq$
Reflexive						
Symmetric						
Transitive						

Definition: A relation R on a set A is an equivalence relation if it is reflexive, symmetric, and transitive.

Example

Show that $\equiv \pmod 3$ is an equivalence relation.

✓ • Reflexive: $a \equiv a \pmod 3$?

$$3 \mid a - a$$

✓ • Symmetric: if $a \equiv b \pmod 3$, then $b \equiv a \pmod 3$

$$3 \mid a - b \Rightarrow 3 \mid b - a$$

$$a - b = 3c$$

$$b - a = 3(-c)$$

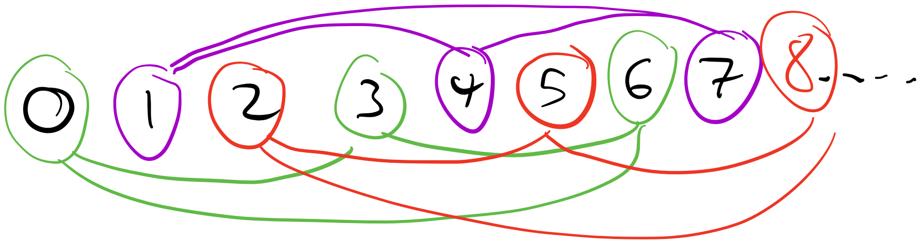
✓ • Transitive: if $a \equiv b \pmod{3}$ and $b \equiv c \pmod{3}$
then $a \equiv c \pmod{3}$

$3 \mid a-b \quad a-b = 3x$ Show: $3 \mid a-c$

$3 \mid b-c \quad +\ b-c = 3y$

$a-b+b-c = 3x+3y$

$a-c = 3(x+y)$

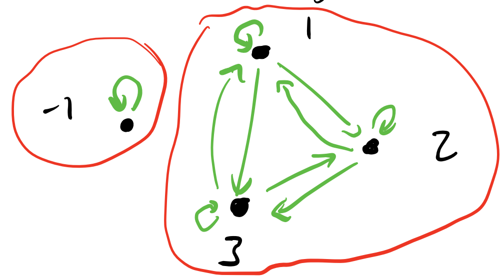


Definition 11.4 Suppose R is an equivalence relation on a set A. Given any element $a \in A$, the **equivalence class containing** a is the subset $\{x \in A : xRa\}$ of A consisting of all the elements of A that relate to a. This set is denoted as $[a]$. Thus the equivalence class containing a is the set $[a] = \{x \in A : xRa\}$.

Def. Suppose R is an equivalence relation on A. Given $a \in A$, the equivalence class containing a, written $[a]$ is the subset: $[a] = \{ x \in A : xRa \}$

Example

$A = \{-1, 1, 2, 3\}$ $R =$ "has the same sign as"



$[-1] = \{-1\}$

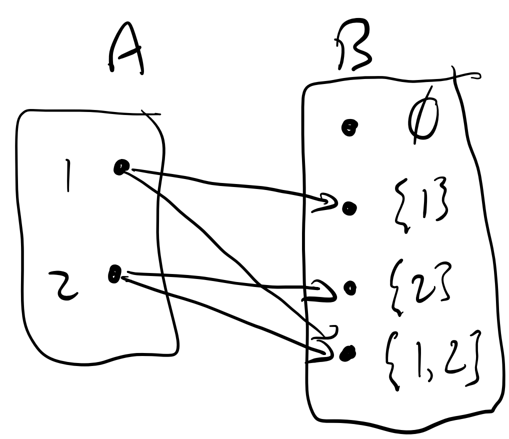
$[1] = \{1, 2, 3\} = [2] = [3]$

$R \subseteq A \times A$

Relations Between 2 Sets: $R \subseteq A \times B$

Example: $A = \{1, 2\}$, $B = \mathcal{P}(A)$

$R = \{(1, \{1\}), (2, \{2\}), (1, \{1,2\}), (2, \{1,2\})\}$



$\in$: Contains!

$A = \{-1, 1, 2, 3, 4\}$

$[-1] = \{-1, 1, 3\}$

$[2] = \{2, 4\}$

Functions (but more formally)

$f(x) = x^2$

Definition: Suppose A, B are sets. A function from A to B, written $f: A \to B$ is a relation $f \subseteq A \times B$ with the property:

f has exactly one element (a, b) for all $a \in A$.

Intuition: all elements of A map to exactly one $b \in B$.

Example: $f: \mathbb{Z} \to \mathbb{N}$ defined as $\{(n, |n|+2) : n \in \mathbb{Z}\}$

or more familiarly, $f(n) = |n| + 2$

Definition(s): If $f: A \to B$, then

- A is the domain of f (the set of possible inputs)
- B is the codomain of f (the set of... things f might map to)
- $\{f(a) : a \in A\}$ is the range of f (the set of... $b \in B$ that f does map to)

$f : A \to B$

Example: $f : \mathbb{Z} \to \mathbb{N} = \{(n, |n|+2) : n \in \mathbb{Z}\}$

$f(-2) = 4$

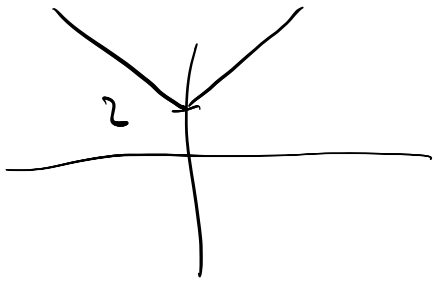
$f(-1) = 3$

$f(0) = 2$

$f(1) = 3$

$f(2) = 4$

- Domain: $\mathbb{Z}$
- Codomain: $\mathbb{N}$
- Range: $\{x \in \mathbb{N} : x > 1\}$



$\{|x|+2 : x \in \mathbb{Z}\}$