CSCI 301 - Lecture 14: Relations

or: Everything in math is a set, Part K

$(a,b) = \{a, \{a,b\}\}$ $\qquad \emptyset \quad \{\emptyset\} \quad \{\{\emptyset\}\}$

$1 = 1 \qquad 5 \leq 9 \qquad \mathbb{Z} \subseteq \mathbb{R}$

$6 \equiv 2 \pmod 4$

Definition 11.1 A **relation** on a set A is a subset $R \subseteq A \times A$. We often abbreviate the statement $(x,y) \in R$ as xRy. The statement $(x,y) \notin R$ is abbreviated as $x \not R\, y$.

$(x,y) \in \leq \qquad x\,R\,y \qquad x \not R\, y$

$\qquad x \leq y \qquad x \not\leq y$

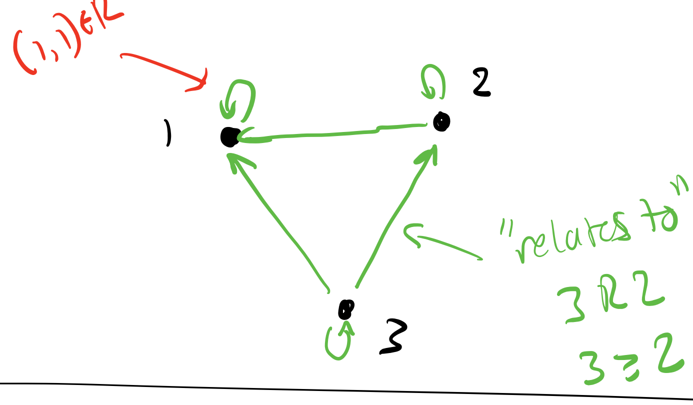
Example

$A = \{1,2,3\}$

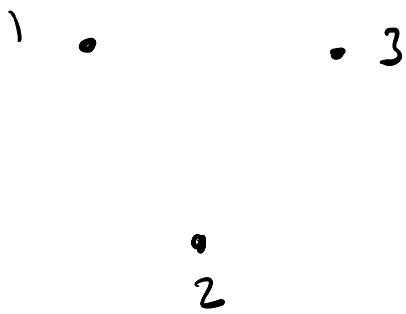
$R = \{(1,1), (2,1), (2,2), (3,3), (3,2), (3,1)\}$

Notice: $R \subseteq A \times A$ What is R? $\boxed{\geq}$

Relations as diagrams.



Do Ex. A



$A = \{1, 2, 3\}$

Same parity $= \{(1,3), (3,1), \ldots$?

$\{(1,1), (1,3), (3,1), (2,2), (3,3)\}$

$(1,2)\ (2,1)\ (3,2), (2,3)$

Properties of Relations

If R is a relation on set A,

R is reflexive if xRx for all $x \in A$

R is symmetric if $xRy \Rightarrow yRx \ \forall x, y \in A$

R is transitive if $xRy \wedge yRz \Rightarrow xRz \ \forall x, y \in A$

Example

$R = \{(1,1), (2,1), (2,2), (3,3), (3,2), (3,1)\}$ ($\geq$, from above)

Reflexive? yes

Symmetric? no

Transitive? yes

Do Ex. B

$A = \{a, b, c\}$

	$<$	$\leq$	$=$	$\mid$	$\nmid$	$\neq$
Reflexive						
Symmetric						
Transitive						

Equivalence Relations

Definition 11.3 A relation R on a set A is an **equivalence relation** if it is reflexive, symmetric and transitive.

Show that $\equiv \pmod 3$ is an equivalence relation.

- Reflexive: $a \equiv a \pmod 3$?

- Symmetric: if $a \equiv b \pmod 3$, then $b \equiv a \pmod 3$

- Transitive: if $a \equiv b \pmod 3$ and $b \equiv c \pmod 3$
  then $a \equiv c \pmod 3$

Definition 11.4 Suppose R is an equivalence relation on a set A. Given any element $a \in A$, the **equivalence class containing** a is the subset $\{x \in A : xRa\}$ of A consisting of all the elements of A that relate to a. This set is denoted as $[a]$. Thus the equivalence class containing a is the set $[a] = \{x \in A : xRa\}$.