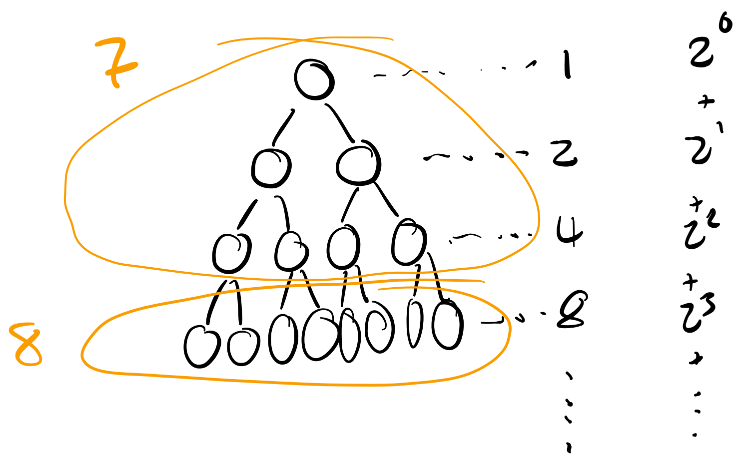
CSCI 301 - Lecture 13: More Induction!

Ex. A #1:

$$2^0 + 2^1 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$$

$2^{n+1} - 1$



Proposition: $2^0 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1 \quad \forall n \in \mathbb{N}$

Proof: $\quad 2^0 = 2^1 - 1$

Base: $2^0 + 2^1 = 1 + 2 = 2^2 - 1$ ✓

Inductive Step:

Suppose $2^0 + 2^1 + 2^2 + \cdots + 2^k = 2^{k+1} - 1$

Show $2^0 + 2^1 + 2^2 + \cdots + 2^k + 2^{k+1} = 2^{k+2} - 1$

$$\underline{2^0 + 2^1 + 2^2 + \cdots + 2^k + 2^{k+1}} = 2^{k+1} - 1 + 2^{k+1}$$

$$= 2^{k+1} + 2^{k+1} - 1$$

$$= 2(2^{k+1}) - 1$$

$$= \underline{2^{k+2} - 1}$$

Fibonacci Sequence

F_0	F_1	F_2	F_3	F_4	F_5	F_6	
1	1	2	3	5	8	13	...

Definition:

$$F_0 = 1$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2}$$

Example: Ex. #2

Should say $F_0 + F_1 + \dots + F_n = F_{n+2} - 1$

⋮

Inductive step:

$$F_0 + F_1 + \dots + F_k + F_{k+1} = \underline{F_{k+2}} - 1 + \underline{F_{k+1}}$$

$$= F_{k+3} - 1$$

Strong Induction

Induction: Prove S_n $\forall n \geq 0$

Base: S_0 (+ maybe others)

Inductive hypothesis:

S_k

Inductive step:

Show $S_k \Rightarrow S_{k+1}$

Strong Induction: Prove S_n $\forall n \geq 0$

Base: S_0 (+ maybe others)

Inductive hypothesis:

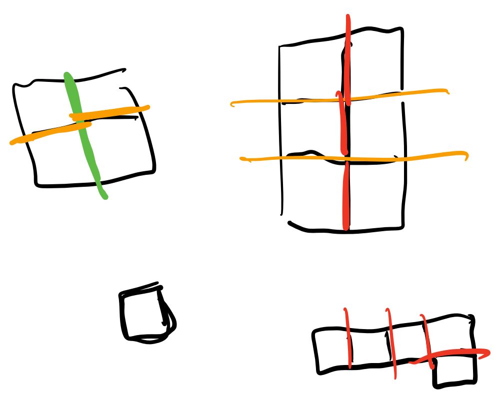
$S_0 \wedge S_1 \wedge \cdots \wedge S_k$

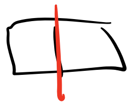
Inductive step:

Show $S_0 \wedge S_1 \wedge \cdots \wedge S_k \Rightarrow S_{k+1}$

Example:

Proposition: it takes $n-1$ breaks to split a chocolate bar with n squares into its individual squares.



Base case: 

Inductive hypothesis:

$p-1$ breaks for p squares if $p \leq k$.

Break a $\underline{k+1}$ bar into 2 pieces.

$a + b = k+1$

$a \leq p$ $\quad a-1$

$b \leq p$ $\quad b-1$

$$(a-1)+(b-1)+1 = \boxed{a+b-1}$$

Proposition: All horses are the same color.