CSCI 301 - Lecture 12: Induction!

Check out this pattern:

S_1 $\quad 1 = 1$

S_2 $\quad 1+3 = 2^2$

S_3 $\quad 1+3+5 = 3^2$

S_4 $\quad 1+3+5+7 = 4^2$

$\vdots$

S_k

$\vdots$

S_k : the first k odd integers sum to k^2.

Proposition: The first n odd numbers sum to n^2.

Proof by induction

"Proof": By induction.

Observe that $1^2 = 1$. (S_1) ← Base case

inductive step (case)

Now suppose that

$$1+3+5+\dots 2k-1 = k^2 \quad (S_k)$$

← inductive hypothesis

Need to show: $S_k \Rightarrow S_{k+1}$, i.e., $1+3+5+\dots+2k-1+2k+1 = (k+1)^2$

$$1+3+5+\dots+2k-1+2k+1 = k^2+2k+1$$

$$= (k+1)^2$$

■

Anatomy of an inductive proof:

Base case, inductive step, inductive hypothesis

- Dominoes graphic (see typed notes)

$\boxed{\text{Try Ex. A \#1}}$

Ex. A #1

$$1+2+3+\dots+n = \frac{n^2+n}{2} \quad \text{for all } n>0.$$

Base case: $1 = \frac{1^2+1}{2}$

Inductive Step:

Show that: $\underline{1+2+\dots+k = \frac{k^2+k}{2}} \Rightarrow 1+2+\dots k+(k+1) = \frac{(k+1)^2+(k+1)}{2}$

$(k^2+2k+1)(k+1)$

$k^3+2k^2+1+k^2+2k+1$

$k^3+3k^2+2k+2+5x+6$

$k^3+5k+6 = 3c$

$k^3+5x+6+k^3+2k+2$

$(k+1)^3+5(k+1)+6$

k^3+

Another Example:

Proposition: $3 \mid (5^{2n} - 1)$ for every integer $n \geq 0$.

Proof: