CSCI 301 - Lecture 10: Proving Non-Conditional Statements

Disproof

So far, it's (mostly) been

"Proposition: $P \Rightarrow Q$"

What if it's not?

- Proposition: $P \Leftrightarrow Q$

Outline for If-and-Only-If Proof

Proposition P if and only if Q.

Proof.
[Prove $P \Rightarrow Q$ using direct, contrapositive or contradiction proof.]
[Prove $Q \Rightarrow P$ using direct, contrapositive or contradiction proof.] ■

Do Ex. A

- Proposition: $\exists x, P(x)$

Proof: Let $x = \ldots$
⋮ (show $P(x)$)
Therefore $\exists x, P(x)$

Suppose $a \in \mathbb{Z}$. $14 | a$ iff $7|a \wedge 2|a$

$14|a \Rightarrow 7|a \wedge 2|a$: Suppose $14|a$.

$a = 14n$
$= 2(7n) \leftarrow 2|a$
$= 7(2n) \leftarrow 7|a$

$7 | a \wedge 2 | a \Rightarrow 14 | a$:

$$a = 7b$$

$2 | a$, so $2 | 7b$ $\leftarrow$ let $b = 2c$ ...

So $a = 7b = 2(7c)$

$a = 14c$ $\quad 14 | c$

Equivalences

Proposition: P, Q, R are equivalent.

Proof:

$$P \Leftrightarrow Q \Leftrightarrow R$$

$$\begin{tikzcd} P \arrow[rr, Rightarrow] & & Q \arrow[dl, Rightarrow] \\ & R \arrow[ul, Rightarrow] & \end{tikzcd}$$

Aside: Constructive vs non-constructive proofs

Claim: $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+, y < x$ $\quad$ Let $y = \frac{x}{2}$

Claim: $\exists x, y \notin \mathbb{Q}, x^y \in \mathbb{Q}$. Let $x = y = \sqrt{2}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational,

Else, let $x = \sqrt{2}^{\sqrt{2}}, y = \sqrt{2}$ $\quad x^y = \sqrt{2}^{\sqrt{2}^{\sqrt{2}}} = \sqrt{2}^2 = 2$

Disproof - Some things are not true!

Do Ex. B

How to disprove:

- P

How to disprove P: Prove $\sim P$.

- $\forall x \in S, P(x)$

How to disprove $\forall x \in S, P(x)$.

Produce an example of an $x \in S$ that makes $P(x)$ false.

- $P(x) \Rightarrow Q(x)$

How to disprove $P(x) \Rightarrow Q(x)$.

Produce an example of an x that makes $P(x)$ true and $Q(x)$ false.

- P, by contradiction

How to disprove P with contradiction:

Assume P is true, and deduce a contradiction.

Do Ex. C