

CSCI 301 - Lecture 10: Proving Non-Conditional Statements

Disproof

So far, it's (mostly) been

"Proposition: $P \Rightarrow Q$ "

What if it's not?

- Proposition: $P \Leftrightarrow Q$

Outline for If-and-Only-If Proof

Proposition P if and only if Q .

Proof.

[Prove $P \Rightarrow Q$ using direct, contrapositive or contradiction proof.]

[Prove $Q \Rightarrow P$ using direct, contrapositive or contradiction proof.] ■

Do Ex. A

- Proposition: $\exists x, P(x)$ Proof: Let $x = \dots$
 $\therefore$ (show $P(x)$)
 Therefore $\exists x, P(x)$

Suppose $a \in \mathbb{Z}$. $14|a$ iff $7|a \wedge 2|a$

$$14|a \Rightarrow 7|a \wedge 2|a: \text{Suppose } 14|a.$$

$$a = 14n$$

$$= 2(7n) \quad \text{r} \quad 2 \mid a$$

$$= 7(2n) \quad \Leftarrow 7 \mid a$$

$$\underline{7|a \wedge 2|a \Rightarrow 14|a:}$$

$$a = 7b$$

$$2|a, \text{ so } 2|7b \leftarrow \text{let } b = 2c \dots$$

$$\text{so } a = 7b = \underline{2(7c)}$$

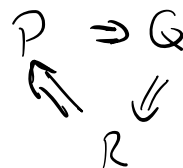
$$a = 14c \quad 14|c$$

Equivalences

Proposition: P, Q, R are equivalent

Proof:

$$P \Leftrightarrow Q \Leftrightarrow R$$



Aside: Constructive vs non-constructive proofs

Claim: $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+, y < x$ Let $y = \frac{x}{2}$

Claim: $\exists x, y \notin \mathbb{Q}, x^y \in \mathbb{Q}$. Let $x = y = \sqrt{2}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational, $\blacksquare$

Else, let $x = \sqrt{2}^{\sqrt{2}}, y = \sqrt{2}$

$$x^y = \sqrt{2}^{\sqrt{2}^{\sqrt{2}}} = \sqrt{2}^2 = 2$$

Disproof - Some things are not true!

How to disprove:

Do Ex. B

- P

How to disprove P : Prove $\sim P$.

- $\forall x \in S, P(x)$

How to disprove $\forall x \in S, P(x)$.

Produce an example of an $x \in S$ that makes $P(x)$ false.

- $P(x) \Rightarrow Q(x)$

How to disprove $P(x) \Rightarrow Q(x)$.

Produce an example of an x that makes $P(x)$ true and $Q(x)$ false.

- P , by contradiction

How to disprove P with contradiction:

Assume P is true, and deduce a contradiction.

Do Ex. C