CSCI 301 - Lecture 8: Contrapositive Proof

Modular Congruence (equivalence)

Definition: Given integers a, b and $n \in \mathbb{N}$, we say a and b are congruent modulo n if $n \mid (a-b)$.

We write this $a \equiv b \pmod{n}$.

If a and b are not congruent mod n, then we write $a \not\equiv b \pmod{n}$.

Note: this is related to, but is not the mod operator

$\boxed{\text{Do Ex. A}}$

Ex. A

1. $9 \equiv 1 \pmod{4}$

   $\hookrightarrow 4 \overset{?}{\mid} (9-1)$

2.

$$n \overset{?}{\mid} (3n - 0)$$

3.

4. $6 \overset{?}{\mid} (22-8)$ F

Contrapositive Proof

We have seen:

$$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$$

Intuition?

"All humans are mammals."

"If human, then mammal."

$\overset{?}{\equiv}$

"If not mammal, then not human."

Outline for Contrapositive Proof

Proposition If P, then Q.

Proof. Suppose $\sim Q$.

$\vdots$

Therefore $\sim P$. ■

Example

Proposition: Suppose $x, y \in \mathbb{Z}$. If $\boxed{5 \nmid xy}^{P}$, then $\boxed{5 \nmid x \text{ and } 5 \nmid y}^{Q}$

(Proof (Direct): Suppose $5 \nmid xy$)

No! ☹

Proof (contrapositive):

Suppose $\neg(5 \nmid x \wedge 5 \nmid y)$ $\neg Q$

$5 \mid x \vee 5 \mid y$

Show: $5 \mid xy$ $\neg P$

Case 1: Suppose $5 \mid x$.

Then $x = 5c$ for some $c \in \mathbb{Z}$.

$xy = 5cy$.

Let $b = cy$, then $xy = 5b$, thus $5 \mid xy$.

$\boxed{\text{Do Ex. B}}$

1. $x \in \mathbb{R}$. If $x^2 + 5x < 0$, then $x < 0$.

Proof (contrapositive):

Suppose $x \geq 0$.

Then $x + 5 \geq 0$, so

$$x(x+5) = x^2 + 5x \geq 0$$

Direct: Suppose $x^2 + 5x < 0$.

$x^2 \geq 0$, so $5x < 0$

BoP style guide