CSCI 301 - Lecture 7: Proofs Intro, Direct Proof

Logical Inference

knowing some things what else can you conclude?
(infer)
(deduce)

given this

Duh:

$$\frac{P \wedge Q}{P} \qquad \frac{\begin{matrix}P\\Q\end{matrix}}{P \wedge Q} \qquad \frac{P}{P \vee Q}$$

conclude this

Somewhat less duh:

Modus Ponens:

$$\frac{\begin{matrix}P \Rightarrow Q\\P\end{matrix}}{Q}$$

Modus Tollens:

$$\frac{\begin{matrix}P \Rightarrow Q\\\neg Q\end{matrix}}{\neg P}$$

Elimination:

$$\frac{\begin{matrix}P \vee Q\\\neg P\end{matrix}}{Q}$$

Def An integer n is even if $n = 2a$ for some integer a.

Def An integer n is odd if $n = 2a+1$ for some integer a.

Observation: in a definition, when we say if, we really mean if and only if (iff)

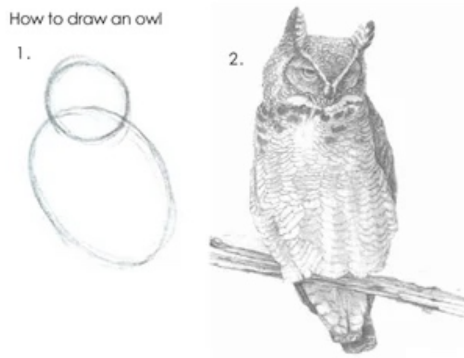
$n \in \mathbb{Z}$ is odd $\Leftrightarrow$ $n = 2a+1$ for some $a \in \mathbb{Z}$

Def Suppose a and b are integers. We say that a divides b, written $a \mid b$, if $b = ac$ for some integer c. In this case, we also say a is a divisor of b, and b is a multiple of a.

Def Two integers have the same parity if they are both even or both odd. Otherwise, they have opposite parity.

Proofs: Given things we know, prove things we didn't.

Very common case: Conditional statements



1. Draw some circles 2. Draw the rest of the [redacted] owl

(ahem) beautiful

Outline for Direct Proof

Proposition If P, then Q.

Proof. Suppose P.

circles

⋮

rest of owl

Therefore Q. ∎

Example: If x is odd, x^2 is odd.

Suppose x is odd

Then $x = 2a+1$ for some $a \in \mathbb{Z}$.

$x^2 = (2a+1)^2 = 4a^2 + 4a + 1$

Let $b =$ $\quad 2(\underbrace{2a^2+2a}_{b})+1$

Then $x^2 = 2b+1$ for $b = 2a^2+2a$.

Therefore x^2 is odd. ∎

Ex #2: Proposition: Suppose $a, b, c \in \mathbb{Z}$. If $a|b$ and $a|c$, then $a|b+c$.

Proof: Because $a|b$ and $a|c$, we can let

$b = ad$ and $c = af$ for $d, f \in \mathbb{Z}$.

Then $b+c = ad + af$

$= a(d+f)$. If we let $p = d+f$,

then we have $b+c = ap$, so $a|b+c$. ∎

Do Ex. A

Cases

If x is an integer, x^2 has the same parity as x.

Case 1: Suppose x is even.
Show: x^2 is even. (proof here)

Case 2: Suppose x is odd.
Show x^2 is odd (proof here)

Do Ex. B

WLOG - Without Loss of Generality

If two integers have opposite parity, then their sum is odd.

a is even b is odd	b is even a is odd
Case 1	Case 2

Without loss of generality, suppose a is even and b is odd.

Case 1: a and b are even. Show: $a+b$ is even.

Case 2: a and b are odd. Show: $a+b$ is odd.