CSCI 301 - Lecture 6: Logic 3

Logical Equivalence:

Two statements are logically equivalent if their truth tables match exactly. We write this $P \equiv Q$

Example:

$P \Leftrightarrow Q$ is true when P and Q are both true or both false

P	Q	$P \Leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

$$(P \wedge Q) \vee (\neg P \wedge \neg Q)$$

Example: does $\wedge$ distribute over $\vee$?

$$A \wedge (B \vee C) \stackrel{?}{\equiv} (A \wedge B) \vee (A \wedge C)$$

A	B	C	$A \wedge (B \vee C)$	$(A \wedge B) \vee (A \wedge C)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$\boxed{\text{Do Ex. A}}$

P	Q	$P \Rightarrow Q$	$(P \wedge \neg Q)$	$\neg(P \wedge \neg Q)$
T	T	T		T
T	F	F		F
F	T	T		T
F	F	T		T

$\neg(P \wedge \neg Q)$

$P \Rightarrow Q$

Negating Statements

$P \to \neg P$, done!

But there are some tricks.

Ex. #4-5 De Morgan's Laws: Negating $\wedge$ and $\vee$

$$\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$$

$$\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$$

Ex. #6: Negating a conditional. $\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$

$$\neg(P \Rightarrow Q) \not\equiv (Q \Rightarrow P)$$

Quantifiers?

$$\neg(\forall x, P(x)) \equiv \exists x, \neg P(x)$$

$$\neg(\exists x, P(x)) \equiv \forall x, \neg P(x)$$

$\boxed{\text{Do Ex. B}}$

1. x is positive, but y is not positive

2. Every even integer greater than 2 is the sum of two primes.

$\forall x \in \mathbb{Z}, \underbrace{[E(x) \wedge x > 2]}_{P} \Rightarrow \underbrace{[x = y + z \wedge y \text{ is prime} \wedge z \text{ is prime}]}_{Q}$

$\Rightarrow$

$\exists x \in \mathbb{Z}, E(x) \wedge x > 2 \wedge (x \neq y + z \vee y \text{ is not prime} \vee z \text{ is not prime})$

3. At least one of the integers a and b is odd,

4. 2a is even iff a is an integer

5. There exists a real number y for which x < y for any real number x.

Logical Inference

Knowing some things what else can you conclude?
(infer)
(deduce)

Duh:

$$\frac{P \wedge Q}{P} \qquad \frac{\begin{array}{c}P \\ Q\end{array}}{P \wedge Q} \qquad \frac{P}{P \vee Q}$$

Somewhat less duh:

Modus Ponens:

$$\frac{\begin{array}{c}P \Rightarrow Q \\ P\end{array}}{}$$

Modus Tollens:

$$\frac{\begin{array}{c}P \Rightarrow Q \\ \neg Q\end{array}}{}$$

Elimination:

$$\frac{\begin{array}{c}P \vee Q \\ \neg P\end{array}}{}$$

Def An integer n is ______ if $n = 2a$ for some integer a.

Def ______ if $n = 2a + 1$ for some integer a.

Def Suppose a and b are integers. We say that a ______ b, written ______, if $b = ac$ for some integer c. In this case, we also say a is a ______ of b, and b is a ______ of a.

Def Two integers have the ______ if they are both even or both odd. Otherwise, they have ______.